Name: **Entrance Test** NIC number:

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The Black Hole, Islamabad

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1 Physics

1. Aircraft A and aircraft B are flying parallel to each other at the same altitude. A is flying twice as fast as B. Both release bombs at exactly the same time. Ignore air resistance. Which of the following is false?

   (a) Both bombs will reach the ground simultaneously.

   (b) B's bomb will travel a smaller total distance than A's bomb.

   (c) B's bomb will hit the ground before A's bomb.

   (d) The vertical distances traveled by the two bombs are equal.

   (e) The horizontal distance traveled by A's bomb will be twice that of B's bomb.

   Solution
   Let's analyze each statement to determine which one is false:

   (a) Both bombs will reach the ground simultaneously. - This statement is true. Both aircraft release their bombs at the same time, and we're ignoring air resistance. Since they are at the same altitude and there is no air resistance, both bombs will fall under the influence of gravity and will reach the ground simultaneously.

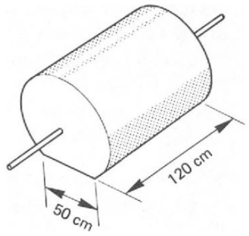
   (b) B's bomb will travel a smaller total distance than A's bomb. - This statement is true. Aircraft A is flying twice as fast as B, so when they both release their bombs at the same time, A's bomb will have a higher horizontal velocity. Since the time in the air is the same, A's bomb will travel a greater horizontal distance than B's bomb.

   (c) B's bomb will hit the ground before A's bomb. - This statement is false. Since both bombs are released at the same time and are subject to the same gravitational acceleration, they will hit the ground simultaneously, assuming there is no air resistance.

   (d) The vertical distances traveled by the two bombs are equal. - This statement is true. Both aircraft release their bombs at the same altitude, and the time of flight is the same. In the absence of air resistance, both bombs will fall vertically downward, covering the same vertical distance.

   (e) The horizontal distance traveled by A's bomb will be twice that of B's bomb. - This statement is true. Aircraft A is flying twice as fast as B, and when both bombs are released at the same time, A's bomb will have a higher horizontal velocity. Since time is the same, the horizontal distance traveled by A's bomb will be twice that of B's bomb.

   So, the false statement is **(c)** B's bomb will hit the ground before A's bomb.



2. A vehicle designed for carrying heavy loads across mud has four wide low-pressure tyres, each of which is 120 cm wide. When the vehicle and its load have a combined mass of 12000 kg each tyre flattens so that 50 cm of tyre is in contact with the mud as shown.The pressure exerted on the mud is,

   (a) 7N/cm^2
   (b) 5kN/cm^2
   (c) 9N/m^2
   (d) 5N/cm^2
   (e) 13N/m^2

   Solution

   To calculate the pressure exerted on the mud, you can use the formula for pressure, which is defined as force divided by area:

$$\text{Pressure (P)} = \frac{\text{Force (F)}}{\text{Area (A)}}$$

   Given:

   - The vehicle and its load have a combined mass of 12,000 kg.
   - Each tire has a width of 120 cm.
   - When the vehicle and load are on the mud, each tire flattens so that 50 cm of the tire is in contact with the mud.

   First, calculate the area of contact of one tire with the mud. Since the tire has a width of 120 cm, and 50 cm of the tire is in contact, the area is:

$$A = 120\,\text{cm} \times 50\,\text{cm} = 6000\,\text{cm}^2$$

   So the total area of contact for all 4 tires combined is:

$$A_{\text{total}} = 6000 \times 4\,\text{cm}^2 = 24000\,\text{cm}^2$$

   Now, you need to calculate the force exerted on the tires due to the weight of the vehicle and its load. This force is equal to the weight (mass times gravitational acceleration) of the vehicle and load:

$$\text{Weight (F)} = m \times g$$

Where:

$$m = 12,000\,\text{kg}$$
$$g = 10\,\text{m/s}^2$$

Thus,

$$F = 12,000\,\text{kg} \times 10\,\text{m/s}^2 = 120,000\,\text{N}$$

Now, we have both the force (F) and the area (A). We can calculate the pressure (P):

$$P = \frac{F}{A}$$

$$P = \frac{120,000\,\text{N}}{24,000\,\text{cm}^2} = 5\,\text{N/cm}^2$$

So the correct option is **(d)**.

3. A pendulum of mass m reaches a maximum height h relative to the lowest height while it swings. We can ignore friction and air resistance. What is the speed of the pendulum when it is at height $h/4$ above the lowest point?

   (a) $\sqrt{mgh^2/4}$
   (b) $gh/2$
   (c) $\sqrt{3gh/2}$
   (d) $\frac{1}{2}\sqrt{gh}$
   (e) $\sqrt{mgh/2}$

Solution

To find the speed of the pendulum when it is at a height of $\frac{h}{4}$ above the lowest point, you can use the principle of conservation of mechanical energy. At the highest point (h), the pendulum has only gravitational potential energy, and at the given height ($\frac{h}{4}$), it will have a combination of gravitational potential energy and kinetic energy.

The total mechanical energy (E) of the pendulum is conserved and remains constant:

$$E = \text{Potential Energy} + \text{Kinetic Energy}$$

At the highest point (h), the potential energy (PE) is at its maximum, and the kinetic energy (KE) is zero:

$$E_{\text{max}} = PE_{\text{max}} + KE_{\text{max}}$$
$$E_{\text{max}} = mgh + 0$$

At the point where the pendulum is at a height of $\frac{h}{4}$, the potential energy (PE) is:

$$PE_{\frac{h}{4}} = mg\left(\frac{h}{4}\right)$$

The kinetic energy (KE) at this point can be calculated as the difference between the total energy and the potential energy:

$$KE_{\frac{h}{4}} = E_{\text{max}} - PE_{\frac{h}{4}}$$

$$KE_{\frac{h}{4}} = mgh - mg\left(\frac{h}{4}\right)$$
$$KE_{\frac{h}{4}} = mg\left(h - \frac{h}{4}\right)$$
$$KE_{\frac{h}{4}} = mg\left(\frac{3h}{4}\right)$$

To find the speed (v) at this point, you can use the formula for kinetic energy:

$$KE = \frac{1}{2}mv^2$$

Setting the kinetic energy we found equal to this formula:

$$\frac{1}{2}mv^2 = mg\left(\frac{3h}{4}\right)$$

Now, solve for v:

$$v^2 = 2g\left(\frac{3h}{4}\right)$$

$$v = \sqrt{\frac{3gh}{2}}$$

So the correct option is **(c)**.

4. The fabric of a balloon designed for high altitude research has a mass of 50g. When filled with hydrogen of density 0.09 kg/m^3 it has a volume of 1 m^3. If the density of air is 1.29 kg/m^3 what is the maximum weight of instruments it can carry?

   (a) 3N
   (b) 4N
   (c) 9.5N
   (d) 11.5N
   (e) 12.5N

Solution

To find the maximum weight of instruments the balloon can carry, we can use the concept of buoyant force. The buoyant force on the balloon when filled with hydrogen is given by the formula:

$$\text{Buoyant Force} = \text{Density of Air} \times \text{Volume Displaced} \times \text{Acceleration Due to Gravity}$$

Given information:

- Mass of the fabric of the balloon $m_{\text{fabric}} = 50\,\text{g} = 0.05\,\text{kg}$
- Density of hydrogen, $\rho_{\text{hydrogen}} = 0.09\,\text{kg/m}^3$
- Volume of the balloon when filled with hydrogen, $V_{\text{balloon}} = 1\,\text{m}^3$
- Density of air, $\rho_{\text{air}} = 1.29\,\text{kg/m}^3$
- Acceleration due to gravity, $g = 10\,\text{m/s}^2$

First, calculate the buoyant force on the balloon when filled with hydrogen:

$$\text{Buoyant Force} = \rho_{\text{air}} \cdot V_{\text{balloon}} \cdot g = 1.29\,\text{kg/m}^3 \cdot 1\,\text{m}^3 \cdot 10\,\text{m/s}^2 = 12.9\,\text{N}$$

Now, let's calculate the downward force on the balloon due to the weight of the fabric and the weight of hydrogen it is carrying inside:

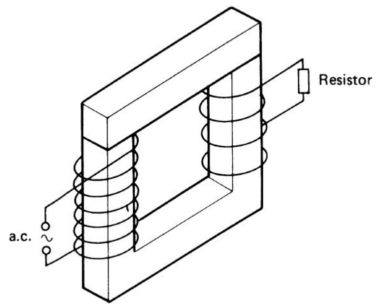
$$\begin{aligned}
\text{Downward Force} &= \text{Weight of Fabric} + \text{Weight of Hydrogen in Balloon} \\
&= m_{\text{fabric}} \cdot g + \rho_{\text{hydrogen}} \cdot V_{\text{balloon}} \cdot g \\
&= (0.05 \times 10) + (0.09 \times 1 \times 10) \\
&= 1.4\,\text{N}
\end{aligned}$$

The maximum weight (W_{max}) of instruments the balloon can carry is the buoyant force minus the downward force, which gives:

$$W_{\text{max}} = \text{Buoyant Force} - \text{Downward Force} = 12.9 - 1.4 = 11.5\,\text{N}$$

So the correct option is **(d)**.

5. The ideal lossless transformer shown below has 600 turns in the primary coil and



200 turns in the secondary coil. The amount of power consumed in the 100 ohm resistor is 36 watts. What is the primary voltage?

(a) 14V
(b) 76V
(c) 150V
(d) 180V
(e) 240V

Solution

To find the primary voltage in the ideal lossless transformer, we first use the transformer voltage and current relationship to relate the primary and secondary voltages and the number of turns in the coils:

$$\frac{V_{\text{primary}}}{V_{\text{secondary}}} = \frac{N_{\text{primary}}}{N_{\text{secondary}}}$$

Where:

- V_{primary} is the primary voltage
- $V_{\text{secondary}}$ is the secondary voltage
- N_{primary} is the number of turns in the primary coil (600 turns)
- $N_{\text{secondary}}$ is the number of turns in the secondary coil (200 turns).

Rearranging the formula to solve for V_{primary}:

$$V_{\text{primary}} = \frac{N_{\text{primary}}}{N_{\text{secondary}} \cdot V_{\text{secondary}}}$$

Substituting the given values:

$$V_{\text{primary}} = 3V_{\text{secondary}}$$

Now, we need to find $V_{\text{secondary}}$. We can use the formula for power ($P = V^2/R$). The resistor has a resistance of 100 ohms, and the power is 36 watts:

$$P_{\text{secondary}} = V_{\text{secondary}}^2/\text{R} \implies V_{\text{secondary}} = \sqrt{PR}$$

Putting in the values:

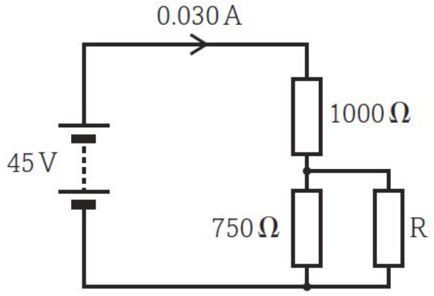
$$V_{\text{secondary}} = \sqrt{36 \times 100} = \sqrt{3600} = 60\,\text{V}$$

Now, plug this value into the equation for V_{primary}:

$$V_{\text{primary}} = 3V_{\text{secondary}} = 3 \times 60 = 180\,\text{V}$$

So the correct option is **(d)**.

6. Find R in the circuit below.



(a) R=100 Ω
(b) R=500 Ω
(c) R=750 Ω
(d) R=1000 Ω
(e) R=1500 Ω

Solution
To find the value of resistor R in the circuit, we can analyze the circuit's current and voltage relationships. Since the battery provides a total voltage of 45V and the total current is 0.030 A, we can use Ohm's law to find the total resistance in the circuit. The total resistance, or the equivalent resistance, in a series circuit is the sum of the individual resistances. In this case, the series section of the circuit consists of a 1000-ohm resistor and the parallel combination of R (the unknown resistor) and a 750-ohm resistor. Therefore, the total resistance is:

$$R_{\text{eq}} = 1000\,\Omega + \left(\frac{1}{R} + \frac{1}{750\,\Omega}\right)^{-1}$$

Now, we can use Ohm's law to find the total resistance of the circuit:

$$V = I \cdot R_{\text{eq}}$$

Where:

- V is the total voltage (45V)
- I is the total current (0.030 A)
- R_{eq} is the total or equivalent resistance

Solve for R_{eq}:

$$R_{\text{eq}} = \frac{V}{I} = \frac{45\,\text{V}}{0.030\,\text{A}} = 1500\,\Omega$$

Now putting this in the equation for the equivalent resistance,

$$R_{\text{eq}} = 1500\,\Omega = 1000\,\Omega + \left(\frac{1}{R} + \frac{1}{750\,\Omega}\right)^{-1}$$

$$\left(\frac{1}{R} + \frac{1}{750\,\Omega}\right)^{-1} = 1500\,\Omega - 1000\,\Omega = 500\,\Omega$$

Now, take the reciprocal of both sides:

$$\frac{1}{R} + \frac{1}{750\,\Omega} = \frac{1}{500\,\Omega}$$

$$\frac{1}{R} = \frac{1}{500\,\Omega} - \frac{1}{750\,\Omega}$$

$$\frac{1}{R} = \frac{3}{1500\,\Omega} - \frac{2}{1500\,\Omega}$$

$$\frac{1}{R} = \frac{1}{1500\,\Omega}$$

Thus,

$$R = 1500\,\Omega$$

So the correct option is **(e)**.

2 Mathematics

1. Two dice are rolled simultaneously. What is the probability that the sum of the numbers obtained equals 9?

   (a) $\frac{1}{16}$
   (b) $\frac{1}{12}$
   (c) $\frac{1}{9}$
   (d) $\frac{1}{6}$
   (e) $\frac{2}{9}$

   Solution

   To find the probability that the sum of the numbers obtained when two dice are rolled simultaneously equals 9, we can calculate the ratio of the number of favorable outcomes to the total number of possible outcomes.

   Each die has 6 sides with numbers from 1 to 6. When two dice are rolled, there are $6 \times 6 = 36$ equally likely outcomes.

   The favorable outcomes where the sum of the numbers on the two dice equals 9 are:

   - (3, 6)
   - (4, 5)
   - (5, 4)
   - (6, 3)

   There are 4 favorable outcomes.

   The probability of getting a sum of 9 is the ratio of favorable outcomes to the total number of possible outcomes:

   $$\text{Probability} = \frac{\text{Number of Favorable Outcomes}}{\text{Total Number of Possible Outcomes}} = \frac{4}{36} = \frac{1}{9}$$

   So the correct option is **(c)**

2. The numerator of a fraction is five less than the denominator. If the numerator and denominator are each increased by nine, the value of the new fraction formed is $\frac{3}{4}$. Find the original fraction.

   (a) $\frac{1}{5}$
   (b) $\frac{2}{7}$
   (c) $\frac{1}{9}$
   (d) $\frac{6}{13}$
   (e) $\frac{6}{11}$

   Solution

   Let the original numerator of the fraction be N and the original denominator be D.

   From the given information, we have the following equations:

   1. The numerator of the fraction is 5 less than the denominator:

   $$N = D - 5$$

   2. If both the numerator and denominator are increased by 9, the new fraction is $\frac{3}{4}$:

   $$\frac{N+9}{D+9} = \frac{3}{4}$$

   Substituting the value of N from the first equation into the second equation:

   $$\frac{(D-5)+9}{D+9} = \frac{3}{4}$$

   Simplifying the equation:

   $$\frac{D+4}{D+9} = \frac{3}{4}$$

   Cross-multiplying:

   $$4(D+4) = 3(D+9)$$

   Expanding and solving for D:

   $$4D + 16 = 3D + 27$$

   Subtracting $3D$ from both sides:

   $$D + 16 = 27$$

   Subtracting 16 from both sides:

   $$D = 11$$

   Now, we can find the value of N (the numerator) using the first equation:

$$N = D - 5 = 11 - 5 = 6$$

So, the original fraction is $\frac{6}{11}$.
The correct option is **(e)**.

3. Find the values of a, p and q when $y = 2x^2 + 4x - 1$ is written in the form $y = a[(x+p)^2 + q]$.
   (a) $a = -2, p = 1, q = -\frac{3}{2}$
   (b) $a = 3, p = 2, q = \frac{3}{2}$
   (c) $a = 2, p = 1, q = -\frac{3}{2}$
   (d) $a = 1, p = 1, q = \frac{2}{3}$
   (e) $a = 2, p = 3, q = \frac{3}{2}$

Solution
To express the quadratic equation $y = 2x^2 + 4x - 1$ in the form $y = a[(x+p)^2 + q]$, we first expand out the latter:

$$y = a[(x+p)^2 + q] = a(x^2 + p^2 + 2xp + q) = ax^2 + ap^2 + 2axp + aq$$

Now we group terms:

$$y = (a)x^2 + (2ap)x + a(p^2 + q)$$

Comparing it with the original equation $y = 2x^2 + 4x - 1$, we get:

$$a = 2,\ 2ap = 4,\ a(p^2 + q) = -1$$

Solving for p:

$$2ap = 4 \implies p = \frac{4}{2a} = \frac{4}{2 \cdot 2} \implies p = 1$$

Solving for q:

$$a(p^2 + q) = -1 \implies q = -\frac{1}{a} - p^2 = -\frac{1}{2} - 1^2 \implies q = -\frac{3}{2}$$

So, we have

$$a = 2,\ p = 1,\ q = -\frac{3}{2}$$

The correct option is **(c)**.

4. Given that $f(x) = 2x, g(x) = x^2, h(x) = \frac{1}{x}$ find $h[f[g[x]]]$

   (a) $\frac{3}{2x^2}$

   (b) $2x$

   (c) $2x^2$

   (d) $\frac{1}{2x^2}$

   (e) $\frac{1}{\sqrt{2x}}$

   Solution

   To find $h(f(g(x)))$, we start by evaluating $g(x)$:

   $$g(x) = x^2$$

   Next, we compute $f(g(x))$ by substituting $g(x)$ into $f(x) = 2x$:

   $$f(g(x)) = 2(x^2) = 2x^2$$

   Finally, we calculate $h(f(g(x)))$ by substituting $f(g(x))$ into $h(x) = \frac{1}{x}$:

   $$h(f(g(x))) = \frac{1}{2x^2}$$

   So the correct option is **(d)**.

5. By eliminating θ from $x = 4\cos\theta$, $y = 3\sin\theta$ one gets,

(a) $x^3 + 2xy + y^3 = 1$

(b) $x^2 + 2xy + y^2 = 1$

(c) $\frac{x^2}{4} + 2xy + \frac{y^2}{9} = 1$

(d) $\frac{x^2}{16} + \frac{y^2}{9} = 1$

(e) $\frac{x^2}{9} + \frac{y^2}{25} = 1$

Solution

To eliminate θ from the parametric equations $x = 4\cos\theta$ and $y = 3\sin\theta$, we start by squaring both equations:

$$x^2 = (4\cos\theta)^2 = 16\cos^2\theta \quad \text{and} \quad y^2 = (3\sin\theta)^2 = 9\sin^2\theta$$

Rearranging, we get:

$$\frac{x^2}{16} = \cos^2\theta \quad \text{and} \quad \frac{y^2}{9} = \sin^2\theta$$

Now add the two equations and use the identity $\cos^2\theta + \sin^2\theta = 1$:

$$\frac{x^2}{16} + \frac{y^2}{9} = \cos^2\theta + \sin^2\theta = 1$$

Thus we have,

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

So the correct option is **(d)**.

6. The points A, B, C have co-ordinates $(2,3),(0,4),(-2,1)$. What is the vector $\overrightarrow{CB}-\overrightarrow{BA}$?

   (a) $(0,0)$
   (b) $(4,-4)$
   (c) $(4,2)$
   (d) $(2,-3)$
   (e) $(3,3)$

Solution
Given the coordinates of points A, B, and C:
$A:(2,3)$ $B:(0,4)$ $C:(2,1)$
To find $\vec{CB}$, subtract the coordinates of C from the coordinates of B:

$$\vec{CB} = \begin{bmatrix} 0-2 \\ 4-1 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

Next, find $\vec{BA}$ by subtracting the coordinates of B from the coordinates of A:

$$\vec{BA} = \begin{bmatrix} 2-0 \\ 3-4 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

Now, subtract $\vec{BA}$ from $\vec{CB}$ to find $\vec{CB}-\vec{BA}$:

$$\vec{CB}-\vec{BA} = \begin{bmatrix} -2 \\ 3 \end{bmatrix} - \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -4 \\ 4 \end{bmatrix}$$

So, the vector $\vec{CB}-\vec{BA}$ is $\begin{bmatrix} -4 \\ 4 \end{bmatrix}$.
None of the options had the correct answer due to a printing mistake. We have given marks to all those who chose $(4,-4)$ as the correct option since that is the closest to the real answer.

3 Analytical Reasoning

1. Alif: Physics research provides us with new technologies that contribute to human betterment. However, still more worthwhile is its role in expanding knowledge and providing new, unexplored ideas.

   Bay: Your priorities are mistaken. Technology is what counts most. Without physics research, technology would not be as advanced as it is.

   Alif and Bay disagree on whether physics research:

   (a) is important for providing new technologies.
   (b) expands the boundaries of our knowledge of the world.
   (c) should have technology as the most important goal.
   (d) leads to technological applications.
   (e) has no value apart from generating new technologies.

   Solution

   Alif and Bay disagree on the priority of physics research:

   (c) should have technology as the most important goal.

   Alif emphasizes the value of expanding knowledge and generating new, unexplored ideas through physics research, indicating that technology is important but not the most important goal. Bay, on the other hand, argues that technology is what counts most, suggesting that technology should be the primary focus of physics research.

   But their disagreement can be construed as being about whether physics research has *any* value apart from generating new technologies.

   So option **(c)** and **(e)** both are correct.

2. If there are no dancers that are not slim and no singers that are not dancers, then which statements are always true?

   (a) There is not one slim person that isn't a dancer

   (b) All singers are slim

   (c) Anybody slim is also a singer

   (d) To be slim you have to be a dancer

   (e) None of the above

   Solution

   Given,

   - Statement 1: "There are no dancers that are not slim"
   - Statement 2: "There are no singers that are not dancers"

   Statement 1 implies that all dancers are slim. Statement 2 implies that all singers are dancers. If all singers are dancers and all dancers are slim, then all singers *must* be slim.

   So the correct option is **(b)**.

3. Apples are harvested and the record shows that 80% of apples collected were heavy (over 250g), 10% of apples were green, 60% were red and 50% were soft. Which of the following statements must be false?
   (a) All red apples were not big
   (b) 30% of red apples were big
   (c) There were no apples that were both green and big
   (d) Half of the apples were small
   (e) All statements above are false

Solution
Given information:

- 80% of apples are heavy (big)
- 10% of apples are green
- 60% of apples are red
- 50% of apples are soft

Let's look at each option one by one.
Option (a) says that "all red apples are not big", which means there are some red apples that are small. Since 20% of the apples are small and there is no restriction that they be of a specific colour, it can be the case that some of them are red. So it is not the case that option (a) must be false.
Option (b) says that "30% of red apples were big". Since 80% of the apples are big and 60% are red, it can be the case that the overlap between the two categories is exactly 30%. Thus it is not the case that option (b) must be false.
Option (c) says that "there were no apples that were both green and big". Must green apples always be small and all big apples non-green? There is no indication in the given information to this effect. It is very much possible that we may find apples in our sample that are both green and big. So it is not the case that option (c) must be false.
Option (d) says that "half of the apples were small". According to the given information, 80% of the apples collected are big, which means only 20% of them are small. It cannot be the case that half, or 50%, of the apples are small. Thus, option (d) *must* be false.
So the correct option is **(d)**.