Questions for Module#3

Q.1 The position of an object is given by $s(t) = (8-t)(t+6)^{\frac{3}{2}}$. Note that a negative position here simply means that the position is to the left of the "zero position" and is perfectly acceptable. Answer each of the following questions. Solution

(a) Compute (accurate to at least 8 decimal places) the average velocity of the object between $t = 10$ and the following values of t.

(i) 10.5 **(ii)** 10.1 **(iii)** 10.01 **(iv)** 10.001 **(v)** 10.0001

(vi) 9.5 **(vii)** 9.9 **(viii)** 9.99 **(ix)** 9.999 **(x)** 9.9999

(b) Use the information from (a) to estimate the instantaneous velocity of the object at $t = 10$ and determine if the object is moving to the right (*i.e.* the instantaneous velocity is positive), moving to the left (*i.e.* the instantaneous velocity is negative), or not moving (*i.e.* the instantaneous velocity is zero).

Q.2 For the function $f(x) = \dfrac{8-x^3}{x^2-4}$ answer each of the following questions. Solution

(a) Evaluate the function at the following values of x compute (accurate to at least 8 decimal places).

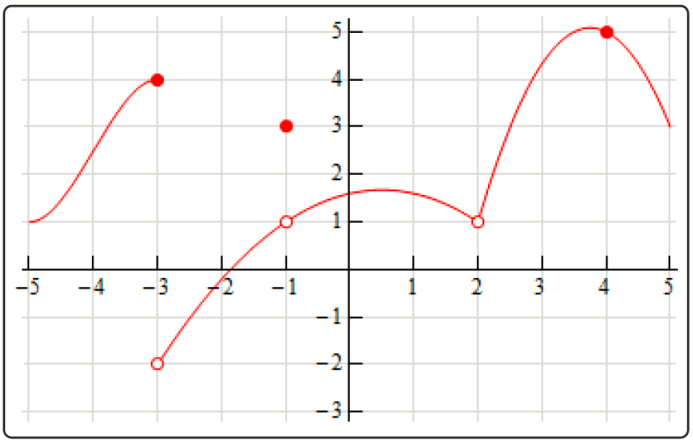
(i) 2.5 **(ii)** 2.1 **(iii)** 2.01 **(iv)** 2.001 **(v)** 2.0001

(vi) 1.5 **(vii)** 1.9 **(viii)** 1.99 **(ix)** 1.999 **(x)** 1.9999

(b) Use the information from **(a)** to estimate the value of $\lim\limits_{x\to 2} \dfrac{8-x^3}{x^2-4}$.

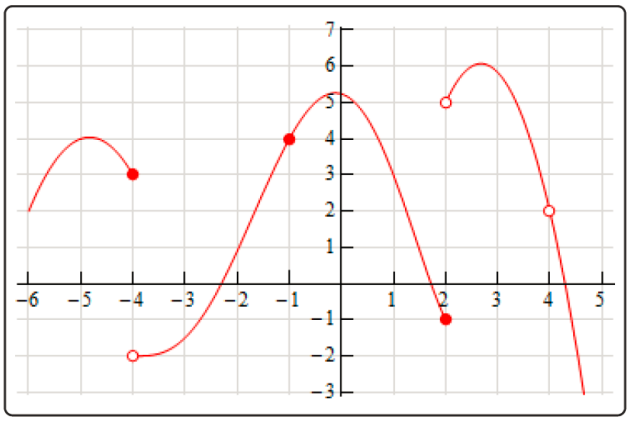
Q.3 Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$ and $\lim\limits_{x\to a} f(x)$. If any of the quantities do not exist clearly explain why. Solution

(a) $a = -3$ **(b)** $a = -1$ **(c)** $a = 2$ **(d)** $a = 4$



Q.4 Below is the graph of $f(x)$. For each of the given points determine the value of $f(a)$, $\lim_{x\to a^-} f(x)$, $\lim_{x\to a^+} f(x)$, and $\lim_{x\to a} f(x)$. If any of the quantities do not exist clearly explain why. Solution

(a) $a=-4$ **(b)** $a=-1$ **(c)** $a=2$ **(d)** $a=4$



Q.5 Given $\lim_{x\to -4} f(x)=1$, $\lim_{x\to -4} g(x)=10$ and $\lim_{x\to -4} h(x)=-7$ use the limit properties given in this section to compute each of the following limits. If it is not possible to compute any of the limits clearly explain why not. Solution

(a) $\lim_{x\to -4}\left[\dfrac{f(x)}{g(x)}-\dfrac{h(x)}{f(x)}\right]$

(b) $\lim_{x\to -4}\left[f(x)\,g(x)\,h(x)\right]$

(c) $\lim_{x\to -4}\left[\dfrac{1}{h(x)}+\dfrac{3-f(x)}{g(x)+h(x)}\right]$

(d) $\lim_{x\to -4}\left[2h(x)-\dfrac{1}{h(x)+7f(x)}\right]$

Q.6 Evaluate $\lim_{x\to -5}\dfrac{x^2-25}{x^2+2x-15}$, if it exists. Solution

Q.7 Evaluate $\lim_{h\to 0}\dfrac{(6+h)^2-36}{h}$, if it exists. Solution

Q.8 Evaluate $\lim_{x\to -3}\dfrac{\sqrt{2x+22}-4}{x+3}$, if it exists. Solution

Q.9 Given Solution

$$h(z)=\begin{cases} 6z & z\le -4 \\ 1-9z & z>-4 \end{cases}$$

Evaluate the following limits, if they exist.

(a) $\lim_{z\to 7} h(z)$ **(b)** $\lim_{z\to -4} h(z)$

Q.10 For $h(t) = \sqrt[3]{t} + 12t - 2t^2$ evaluate each of the following limits. Solution

(a) $\lim_{t \to -\infty} h(t)$ **(b)** $\lim_{t \to \infty} h(t)$

Q.11 Determine where the following function is discontinuous. Solution

$$f(x) = \frac{x^2 - 9}{3x^2 + 2x - 8}$$

Q.12 Use the definition of the derivative to find the derivative of, Solution

$$g(x) = x^3 - 2x^2 + x - 1$$

Q.13 Sketch the graph of a function that satisfies $f(1) = 3$, $f'(1) = 1$, $f(4) = 5$, $f'(4) = -2$. Solution

Q.14 Answer the following questions about the function $W(z) = 4z^2 - 9z$. Solution

(a) Is the function increasing or decreasing at $z = -1$?

(b) Is the function increasing or decreasing at $z = 2$?

(c) Does the function ever stop changing? If yes, at what value(s) of z does the function stop changing?

Q.15 Suppose that the volume of water in a tank for $0 \le t \le 6$ is given by $Q(t) = 10 + 5t - t^2$. Solution

(a) Is the volume of water increasing or decreasing at $t = 0$?

(b) Is the volume of water increasing or decreasing at $t = 6$?

(c) Does the volume of water ever stop changing? If yes, at what times(s) does the volume stop changing?

Q.16 Find the derivative of $f(x) = 10\sqrt[5]{x^3} - \sqrt{x^7} + 6\sqrt[3]{x^8} - 3$. Solution

Q.17 Use the Product Rule to find the derivative of $y = \left(1 + \sqrt{x^3}\right)\left(x^{-3} - 2\sqrt[3]{x}\right)$. Solution

Q.18 Use the Quotient Rule to find the derivative of $f(x) = \dfrac{\sqrt{x} + 2x}{7x - 4x^2}$. Solution

Q.19 Determine where $f(x) = \dfrac{x - x^2}{1 + 8x^2}$ is increasing and decreasing. Solution

Q.20 Where in the range $[-2, 7]$ is the function $f(x) = 4\cos(x) - x$ increasing and decreasing. Solution