Questions for Module#4

Q.1 Without using a calculator determine the exact value of $\log_3 81$. Solution

Q.2 Write $\log_4\left(\frac{x-4}{y^2\sqrt[5]{z}}\right)$ in terms of simpler logarithms. Solution

Q.3 $3\ln(t+5) - 4\ln(t) - 2\ln(s-1)$ into a single logarithm with a coefficient of one. Solution

Q.4 Find all the solutions to $4x+1 = (12x+3)\,\mathbf{e}^{x^2-2}$. If there are no solutions clearly explain why. Solution

Q.5 Determine the absolute extrema of $R(t) = 1 + 80t^3 + 5t^4 - 2t^5$ on $[-4.5,\ 4]$. Solution

Q.6 Determine the absolute extrema of $f(y) = \sin\left(\frac{y}{3}\right) + \frac{2y}{9}$ on $[-10, 15]$. Solution

Q.7 For $h(t) = t^2\sqrt[3]{t-7}$ answer each of the following questions. Solution

(a) Identify the critical points of the function.

(b) Determine the intervals on which the function increases and decreases.

(c) Classify the critical points as relative maximums, relative minimums or neither.

Q.8 Given that $f(x)$ is an increasing function and define $h(x) = [f(x)]^2$. Will $h(x)$ be an increasing function? If yes, prove that $h(x)$ is an increasing function. If not, can you determine any other conditions needed on the function $f(x)$ that will guarantee that $h(x)$ will also increase? Solution

Q.9 For $h(t) = t^4 + 12t^3 + 6t^2 - 36t + 2$ answer each of the following questions. Solution

(a) Determine a list of possible inflection points for the function.

(b) Determine the intervals on which the function is concave up and concave down.

(c) Determine the inflection points of the function.

Q.10 Determine all the number(s) c which satisfy the conclusion of Rolle's Theorem for $g(t) = 2t - t^2 - t^3$ on $[-2, 1]$. Solution

Q.11 Determine all the number(s) c which satisfy the conclusion of Mean Value Theorem for $A(t) = 8t + \mathbf{e}^{-3t}$ on $[-2, 3]$. Solution

Q.12 We have 45 m^2 of material to build a box with a square base and no top. Determine the dimensions of the box that will maximize the enclosed volume. Solution

Q.13 Use L'Hospital's Rule to evaluate $\lim\limits_{x\to 2}\dfrac{x^3 - 7x^2 + 10x}{x^2 + x - 6}$. Solution

Q.14 Use L'Hospital's Rule to evaluate $\lim_{t\to\infty}\left[t\ln\left(1+\frac{3}{t}\right)\right]$. Solution

Q.15 Use L'Hospital's Rule to evaluate $\lim_{y\to 0+}[\cos(2y)]^{1/y^2}$. Solution

Q.16 Find the linear approximation to $g(z)=\sqrt[4]{z}$ at $z=2$. Use the linear approximation to approximate the value of $\sqrt[4]{3}$ and $\sqrt[4]{10}$. Compare the approximated values to the exact values. Solution

Q.17 Compute dy and Δy for $y=\mathbf{e}^{x^2}$ as x changes from 3 to 3.01. Solution