Questions for Module # 6

Solutions to Q.1-Q.5 are in the notes. Attempt Q.1-Q.20 before seeking the solutions.

Q.1 Find the values of the real numbers x and y in each of the following:

(a) $\dfrac{x}{1+i}+\dfrac{y}{1-2i}=1$ (b) $\dfrac{x}{2-i}+\dfrac{yi}{i+3}=\dfrac{2}{1+i}$

Q.2 Write the following numbers in $[r,\theta]$ form: (i) $7+2i$ (ii) $3-i$ (iii) $-4+6i$

Q.3 Prove that $\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$.

Q.4 If $z=\cos\theta+i\sin\theta$ then use de Moivre's theorem to show that:

(a) $z+\dfrac{1}{z}=2\cos\theta$ (b) $z^2+\dfrac{1}{z^2}=2\cos 2\theta$ (c) $z^n+\dfrac{1}{z^n}=2\cos n\theta$

Q.5 The point P represents the complex number z on an Argand diagram. Describe the locus geometrically and obtain a cartesian equation for the locus in the cases

(a) $|z|=|z-4|$

(b) $|z|+|z-4|=6$

(c) $|z|=2|z-4|$

Q.6 Determine if the following sequence is increasing, decreasing, not monotonic, bounded below, bounded above and/or bounded. Solution

$$\left\{\frac{4-n}{2n+3}\right\}_{n=1}^{\infty}$$

Q.7 Given that $\displaystyle\sum_{n=0}^{\infty}\frac{1}{n^3+1}=1.6865$ determine the value of $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n^3+1}$. Solution

Q.8 Show that the following series is divergent. $\displaystyle\sum_{n=0}^{\infty}\frac{3n\,\mathbf{e}^n}{n^2+1}$ Solution

Q.9 Determine if the series converges or diverges. If the series converges give its value. Solution

$$\sum_{n=0}^{\infty}3^{2+n}\,2^{1-3n}$$

Q.10 Determine if the series converges or diverges. If the series converges give its value. Solution

$$\sum_{n=1}^{\infty}\frac{(-6)^{3-n}}{8^{2-n}}$$

Q.11 Determine if the series converges or diverges. If the series converges give its value. Solution

$$\sum_{n=1}^{\infty} \frac{3}{n^2+7n+12}$$

Q.12 Determine if the following series converges or diverges. $\sum_{n=0}^{\infty} \frac{2}{3+5n}$ Solution

Q.13 Determine if the following series converges or diverges. $\sum_{n=2}^{\infty} \frac{n-1}{\sqrt{n^6+1}}$ Solution

Q.14 For the following power series determine the interval and radius of convergence. Solution

$$\sum_{n=0}^{\infty} \frac{4^{1+2n}}{5^{n+1}}(x+3)^n$$

Q.15 Write the following function as a power series and give the interval of convergence. Solution

$$f(x) = \frac{x^3}{3-x^2}$$

Q.16 Give a power series representation for the integral of the following function. Solution

$$h(x) = \frac{x^4}{9+x^2}$$

Q.17 Find the Taylor Series for $f(x) = \ln(3+4x)$ about $x = 0$. Solution

Q.18 Use the Binomial Theorem to expand $(9-x)^4$. Solution

Q.19 Write down the first four terms in the binomial series for $\sqrt[3]{8-2x}$. Solution

Q.20 Determine a Taylor Series about $x = 0$ for the following integral. Solution

$$\int \frac{\mathbf{e}^x - 1}{x}\, dx$$