Questions for Module # 7

Q.1 Determine the average value of each of the following functions on the given interval. Solution

(a) $f(t) = t^2 - 5t + 6\cos(\pi t)$ on $\left[-1, \frac{5}{2}\right]$

(b) $R(z) = \sin(2z)\, e^{1-\cos(2z)}$ on $[-\pi, \pi]$

Q.2 Use the method of disks/rings to determine the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$, $y = 3$ and the y-axis about the y-axis. Solution

Q.3 Use the method of disks/rings to determine the volume of the solid obtained by rotating the region bounded by $x = y^2 - 6y + 10$ and $x = 5$ about the y-axis. Solution

Q.4 Find the volume of a pyramid of height h whose base is an equilateral triangle of length L. Solution

Q.5 A cable with mass $\frac{1}{2}$ kg/meter is lifting a load of 150 kg that is initially at the bottom of a 50 meter shaft. How much work is required to lift the load ¼ of the way up the shaft? Solution

Q.6 Determine the length of $x = \frac{2}{3}(y-1)^{\frac{3}{2}}$ between $1 \le y \le 4$. Solution

Q.7 Determine the length of $y = \ln\left(\sec(x)\right)$ between $0 \le x \le \frac{\pi}{4}$. Solution

Q.8 Determine the surface area of the solid obtained by rotating $y = \sqrt{9 - x^2}$, $-2 \le x \le 2$ about the x-axis. Solution

Q.9 Eliminate the parameter for the following set of parametric equations, sketch the graph of the parametric curve and give any limits that might exist on x and y. Solution

$$x = 4 - 2t \qquad y = 3 + 6t - 4t^2$$

Q.10 Eliminate the parameter for the following set of parametric equations, sketch the graph of the parametric curve and give any limits that might exist on x and y. Solution

$$x = 3\sin(t) \qquad y = -4\cos(t) \qquad 0 \le t \le 2\pi$$

Q.11 Compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the following set of parametric equations. Solution

$$x = 4t^3 - t^2 + 7t \qquad y = t^4 - 6$$

Q.12 Determine the area of the region below the parametric curve given by the following set of parametric equations. You may assume that the curve traces out exactly once from right to left for the given range of t. You should only use the given parametric equations to determine the answer. Solution

$$x = 4t^3 - t^2 \qquad y = t^4 + 2t^2 \qquad 1 \le t \le 3$$

Q.13 Set up, but do not evaluate, an integral that gives the length of the parametric curve given by the following set of parametric equations. You may assume that the curve traces out exactly once for the given range of t's. Solution

$$x = 2 + t^2 \qquad y = e^t \sin(2t) \qquad 0 \le t \le 3$$

Q.14 Set up, but do not evaluate, an integral that gives the surface area of the object obtained by rotating the parametric curve about the given axis. You may assume that the curve traces out exactly once for the given range of t's. Solution

$$\text{Rotate } x = 1 + \ln\left(5 + t^2\right) \qquad y = 2t - 2t^2 \qquad 0 \le t \le 2 \text{ about the } x\text{-axis}$$

Q.15 For the point with polar coordinates $\left(2, \frac{\pi}{7}\right)$ determine three different sets of coordinates for the same point all of which have angles different from $\frac{\pi}{7}$ and are in the range $-2\pi \le \theta \le 2\pi$. Solution

Q.16 Convert the following equation into an equation in terms of Cartesian coordinates. Solution

$$\frac{2}{r} = \sin(\theta) - \sec(\theta)$$

Q.17 Find the area inside the inner loop of $r = 3 - 8\cos(\theta)$. Solution

Q.18 Find the area that is inside $r = 3 + 3\sin(\theta)$ and outside $r = 2$. Solution

Q.19 Using $n = 6$ approximate the value of $\int_{-1}^{2} \sqrt{e^{-x^2} + 1}\, dx$ using Solution

(a) the Midpoint Rule,

(b) the Trapezoid Rule, and

(c) Simpson's Rule.

Use at least 6 decimal places of accuracy for your work.

Q.20 Using $n = 6$ approximate the value of $\int_{1}^{7} \frac{1}{x^3 + 1}\, dx$ using Solution

(a) the Midpoint Rule,

(b) the Trapezoid Rule, and

(c) Simpson's Rule.

Use at least 6 decimal places of accuracy for your work.