Questions for Module # 10

Q.1 Use Gauss' method to solve each system or conclude 'many solutions' or 'no solutions'.

(a) $\begin{aligned} 2x + 2y &= 5 \\ x - 4y &= 0 \end{aligned}$ **(b)** $\begin{aligned} -x + y &= 1 \\ x + y &= 2 \end{aligned}$ **(c)** $\begin{aligned} x - 3y + z &= 1 \\ x + y + 2z &= 14 \end{aligned}$

(d) $\begin{aligned} -x - y &= 1 \\ -3x - 3y &= 2 \end{aligned}$ **(e)** $\begin{aligned} 4y + z &= 20 \\ 2x - 2y + z &= 0 \\ x + z &= 5 \\ x + y - z &= 10 \end{aligned}$ **(f)** $\begin{aligned} 2x + z + w &= 5 \\ y - w &= -1 \\ 3x - z - w &= 0 \\ 4x + y + 2z + w &= 9 \end{aligned}$

Q.2 **(a)** Can the equation $3x - 2y = 5$ be derived, by a sequence of Gaussian reduction steps, from the equations in this system? Solution

$$\begin{aligned} x + y &= 1 \\ 4x - y &= 6 \end{aligned}$$

(b) Can the equation $5x - 3y = 2$ be derived, by a sequence of Gaussian reduction steps, from the equations in this system?

$$\begin{aligned} 2x + 2y &= 5 \\ 3x + y &= 4 \end{aligned}$$

(c) Can the equation $6x - 9y + 5z = -2$ be derived, by a sequence of Gaussian reduction steps, from the equations in the system?

$$\begin{aligned} 2x + y - z &= 4 \\ 6x - 3y + z &= 5 \end{aligned}$$

Q.3 Show that the simultaneous equations given here,

$$\begin{aligned} x + z &= 1 \\ 2x + y + z &= 0 \\ x + y + 2z &= 1 \end{aligned}$$

can be reduced to: $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 1 \\ -2 \\ 2 \end{Bmatrix}$

Q.4 Make up a four equations/four unknowns system having Solution

(a) a one-parameter solution set;

(b) a two-parameter solution set;

(c) a three-parameter solution set.

Q.5 Solve the equations below and give the solution in vector notation. Identify the particular solution and the solution of the homogeneous equation. Solution

(a) $\begin{aligned} 2x + y - z &= 1 \\ 4x - y &= 3 \end{aligned}$ **(b)** $\begin{aligned} x - z &= 1 \\ y + 2z - w &= 3 \\ x + 2y + 3z - w &= 7 \end{aligned}$ **(c)** $\begin{aligned} x - y + z &= 0 \\ y + w &= 0 \\ 3x - 2y + 3z + w &= 0 \\ -y - w &= 0 \end{aligned}$

(d) $\begin{aligned} a + 2b + 3c + d - e &= 1 \\ 3a - b + c + d + e &= 3 \end{aligned}$

Q.6 Is the given vector in the set generated by the given set?

(a) $\begin{pmatrix}2\\3\end{pmatrix}, \{\begin{pmatrix}1\\4\end{pmatrix}, \begin{pmatrix}1\\5\end{pmatrix}\}$

(b) $\begin{pmatrix}-1\\0\\1\end{pmatrix}, \{\begin{pmatrix}2\\1\\0\end{pmatrix}, \begin{pmatrix}1\\0\\1\end{pmatrix}\}$

(c) $\begin{pmatrix}1\\3\\0\end{pmatrix}, \{\begin{pmatrix}1\\0\\4\end{pmatrix}, \begin{pmatrix}2\\1\\5\end{pmatrix}, \begin{pmatrix}3\\3\\0\end{pmatrix}, \begin{pmatrix}4\\2\\1\end{pmatrix}\}$

(d) $\begin{pmatrix}1\\0\\1\\1\end{pmatrix}, \{\begin{pmatrix}2\\1\\0\\1\end{pmatrix}, \begin{pmatrix}3\\0\\0\\2\end{pmatrix}\}$

Q.7 Evaluate the determinant of each. Solution

(a) $\begin{pmatrix}2 & 0\\-1 & 3\end{pmatrix}$ **(b)** $\begin{pmatrix}2 & 1 & 1\\0 & 5 & -2\\1 & -3 & 4\end{pmatrix}$ **(c)** $\begin{pmatrix}2 & 3 & 4\\5 & 6 & 7\\8 & 9 & 1\end{pmatrix}$

Q.8 Singular or nonsingular? Use the determinant to decide. Solution

(a) $\begin{pmatrix}2 & 1 & 1\\3 & 2 & 2\\0 & 1 & 4\end{pmatrix}$ **(b)** $\begin{pmatrix}1 & 0 & 1\\2 & 1 & 1\\4 & 1 & 3\end{pmatrix}$ **(c)** $\begin{pmatrix}2 & 1 & 0\\3 & -2 & 0\\1 & 0 & 0\end{pmatrix}$

Q.9 The *cross product* of the vectors

$$\vec{x} = \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} \qquad \vec{y} = \begin{pmatrix}y_1\\y_2\\y_3\end{pmatrix}$$

is the vector computed as this determinant.

$$\vec{x} \times \vec{y} = \det(\begin{pmatrix}\vec{e}_1 & \vec{e}_2 & \vec{e}_3\\x_1 & x_2 & x_3\\y_1 & y_2 & y_3\end{pmatrix})$$

Note that the first row is composed of vectors, the vectors from the standard basis for $\mathbb{R}^3$. Show that the cross product of two vectors is perpendicular to each vector.

Q.11 Show that each of these is a vector space.

(a) The set of linear polynomials $\mathcal{P}_1 = \{a_0 + a_1 x \mid a_0, a_1 \in \mathbb{R}\}$ under the usual polynomial addition and scalar multiplication operations

(b) The set of 2×2 matrices with real entries under the usual matrix operations

(c) The set of three-component row vectors with their usual operations

(d) The set

$$L = \{\begin{pmatrix}x\\y\\z\\w\end{pmatrix} \in \mathbb{R}^4 \mid x + y - z + w = 0\}$$

under the operations inherited from $\mathbb{R}^4$