Questions for Module # 11

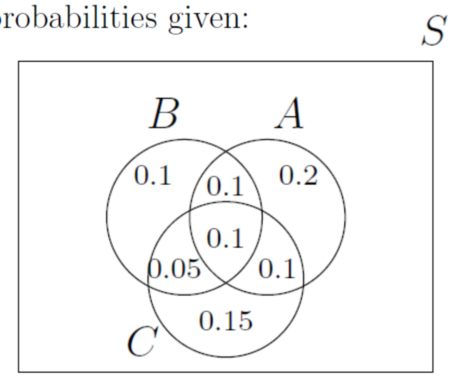
Q.1 Suppose that the universal set S is defined as $S = \{1, 2, \cdots, 10\}$ and $A = \{1, 2, 3\}$, $B = \{x \in S : 2 \leq x \leq 7\}$, and $C = \{7, 8, 9, 10\}$. Solution

(a) Find $A \cup B$
(b) Find $(A \cup C) - B$
(c) Find $\bar{A} \cup (B - C)$
(d) Do A, B, and C form a partition of S?

Q.2 I roll a fair die twice and obtain two numbers. X_1 = result of the first roll, X_2 = result of the second roll. Solution
(a) Find the probability that $X_2 = 4$.
(b) Find the probability that $X_1 + X_2 = 7$.
(c) Find the probability that $X_1 \neq 2$ and $X_2 \geq 4$.

Q.3 Let A, B, and C be three events with probabilities given: Solution

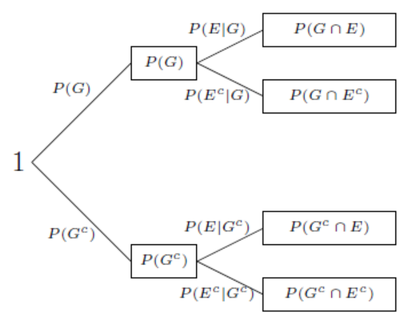
(a) Find $P(A|B)$
(b) Find $P(C|B)$
(c) Find $P(B|A \cup C)$
(d) Find $P(B|A, C) = P(B|A \cap C)$



Q.4 Consider a communication system. At any given time, the communication channel is in good condition with probability 0.8 and is in bad condition with probability 0.2. An error occurs in a transmission with probability 0.1 if the channel is in good condition and with probability 0.3 if the channel is in bad condition. Let G be the event that the channel is in good condition and E be the event that there is an error in transmission. Solution

(a) Complete the following tree diagram:

(b) Using the tree find $P(E)$.
(c) Using the tree find $P(G|E^c)$.



Q.5 A coffee shop has 4 different types of coffee. You can order your coffee in a small, medium, or large cup. You can also choose whether you want to add cream, sugar, or milk (any combination is possible. For example, you can choose to add all three). In how many ways can you order your coffee? Solution

Q.6 There are 20 black cell phones and 30 white cell phones in a store. An employee takes 10 phones at random. Find the probability that Solution

(a) there will be exactly 4 black cell phones among the chosen phones.

(b) there will be less than 3 black cell phones among the chosen phones.

Q.7 You have a biased coin for which $P(H) = p$. You toss the coin 20 times. What is the probability that: Solution

(a) You observe 8 heads and 12 tails?

(b) You observe more than 8 heads and more than 8 tails?

Q.8 There are two coins in a bag. For coin 1, $P(H) = \frac{1}{2}$ and for coin 2, $P(H) = \frac{1}{3}$. Your friend chooses one of the coins at random and tosses it 5 times. Solution

(a) What is the probability of observing at least 3 heads?

(b) You ask your friend, "did you observe at least three heads?" Your friend replies, "yes." What is the probability that coin 2 was chosen?

Q.9 Let X be a discrete random variable with the following PMF Solution

$$P_X(x) = \begin{cases} \frac{1}{2} & \text{for } x = 0 \\ \frac{1}{3} & \text{for } x = 1 \\ \frac{1}{6} & \text{for } x = 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find R_X, the range of the random variable X.

(b) Find $P(X \geq 1.5)$.

(c) Find $P(0 < X < 2)$.

(d) Find $P(X = 0|X < 2)$

Q.10 For each of the following random variables, find $P(X > 5)$, $P(2 < X \leq 6)$ and $P(X > 5|X < 8)$. You do not need to provide the numerical values for your answers. In other words, you can leave your answers in the form of sums. Solution

(a) $X \sim Geometric(\frac{1}{5})$

(b) $X \sim Binomial(10, \frac{1}{3})$

(c) $X \sim Poisson(5)$

Q.11 The number of emails that I get in a weekday (Monday through Friday) can be modeled by a Poisson distribution with an average of $\frac{1}{6}$ emails per minute. The number of emails that I receive on weekends (Saturday and Sunday) can be modeled by a Poisson distribution with an average of $\frac{1}{30}$ emails per minute. Solution

1. What is the probability that I get no emails in an interval of length 4 hours on a Sunday?
2. A random day is chosen (all days of the week are equally likely to be selected), and a random interval of length one hour is selected in the chosen day. It is observed that I did not receive any emails in that interval. What is the probability that the chosen day is a weekday?

Q.12 The number of customers arriving at a grocery store is a Poisson random variable. On average 10 customers arrive per hour. Let X be the number of customers arriving from $10am$ to $11:30am$. What is $P(10 < X \leq 15)$? Solution

Q.13 Determine whether each of the following can serve as a probability mass function of a discrete random variable: Solution

(a) $f(x) = \frac{1}{2}(x-2)$, $x = 1, 2, 3, 4$. (b) $g(x) = \frac{1}{10}(x+1)$, $x = 0, 1, 2, 3$.

(c) $h(x) = \frac{1}{20}x^2$, $x = 0, 1, 2, 3, 4$.

Q.14 Let X be a discrete random variable with the following PMF Solution

$$P_X(k) = \begin{cases} \frac{1}{4} & \text{for } k = -2 \\ \frac{1}{8} & \text{for } k = -1 \\ \frac{1}{8} & \text{for } k = 0 \\ \frac{1}{4} & \text{for } k = 1 \\ \frac{1}{4} & \text{for } k = 2 \\ 0 & \text{otherwise} \end{cases}$$

I define a new random variable Y as $Y = (X+1)^2$.

a. Find the range of Y.
b. Find the PMF of Y.

Q.15 The cumulative distribution function (CDF) of random variable X is defined as Solution

$$F_X(x) = P(X \leq x), \text{ for all } x \in \mathbb{R}.$$

I toss a coin twice. Let X be the number of observed heads. Find the CDF of X.