Questions for Module # 9

Q.1 Sketch the direction field for the following differential equation. Sketch the set of integral curves for this differential equation. Determine how the solutions behave as $t \to \infty$ and if this behavior depends on the value of $y(0)$ describe this dependency. Solution

$$y' = \left(y^2 - y - 2\right)(1 - y)^2$$

Q.2 Sketch the direction field for the following differential equation. Sketch the set of integral curves for this differential equation. Solution

$$y' = y - x$$

Q.3 Verify that $\cos(x)\,y' + \sin(x)\,y = 2\cos^3(x)\sin(x) - 1 \qquad y\left(\frac{\pi}{4}\right) = 3\sqrt{2}, \qquad 0 \le x < \frac{\pi}{2}$

has the solution, $y(x) = -\frac{1}{2}\cos(x)\cos(2x) - \sin(x) + 7\cos(x)$

Q.4 Find the solution to the following IVP. Solution

$$t\,y' + 2y = t^2 - t + 1 \qquad y(1) = \frac{1}{2}$$

Q.5 Verify that $t\,y' - 2y = t^5\sin(2t) - t^3 + 4t^4 \qquad y(\pi) = \frac{3}{2}\pi^4$

has the solution, $y(t) = -\frac{1}{2}t^4\cos(2t) + \frac{1}{2}t^3\sin(2t) + \frac{1}{4}t^2\cos(2t) - t^3 + 2t^4 + \left(\pi - \frac{1}{4}\right)t^2$

Q.6 Find the solution to the following IVP and determine all possible behaviors of the solution as $t \to \infty$. If this behavior depends on the value of y_0 give this dependence. Solution

$$2y' - y = 4\sin(3t) \qquad y(0) = y_0$$

Q.7 Solve the following IVP and find the interval of validity for the solution. Solution

$$y' = \frac{3x^2 + 4x - 4}{2y - 4} \qquad y(1) = 3$$

Q.8 Solve the following IVP and find the interval of validity of the solution. Solution

$$y' = \mathbf{e}^{-y}(2x - 4) \qquad y(5) = 0$$

Q.9 For the linear homogeneous 2nd order ode $ay'' + by' + cy = 0$ show that if $y_1(t)$ and $y_2(t)$ are two solutions then so is $y(t) = c_1y_1(t) + c_2y_2(t)$. Will this remain true for the equation $p(t)\,y'' + q(t)\,y' + r(t)\,y = 0$?

Q.10 Solve the following IVP: $y'' - 9y = 0 \qquad y(0) = 2 \qquad y'(0) = -1$ Solution

Q.11 Solve the following IVP: $y'' - 8y' + 17y = 0 \quad y(0) = -4 \quad y'(0) = -1$ Solution

Q.12 Solve the following IVP: $y'' - 4y' + 4y = 0 \quad y(0) = 12 \quad y'(0) = -3$ Solution

Q.13 Solve the following IVP: $16y'' - 40y' + 25y = 0 \quad y(0) = 3 \quad y'(0) = -\frac{9}{4}$ Solution

Q.14 Verify that the solution to $2t^2y'' + ty' - 3y = 0, \quad t > 0$ is $y(t) = c_1 t^{-1} + c_2 t^{\frac{3}{2}}$

Q.15 Prove that $y_1(t) = e^{\lambda t}\cos(\mu t)$ and $y_2(t) = e^{\lambda t}\sin(\mu t)$ are a set of fundamental Solution solutions to an equation of the form $ay'' + by' + cy = 0$.