Summary of Lecture 4 – FORCE AND NEWTON'S LAWS

1. Ancient view: objects tend to stop if they are in motion; force is required to keep something moving. This was a natural thing to believe in because we see objects stop moving after some time; frictionless motion is possible to see only in rather special circumstances.

2. Modern view: objects tend to remain in their initial state; force is required to *change* motion. Resistance to changes in motion is called *inertia*. More inertia means it is harder to make a body accelerate or decelerate.

3. Newton's First Law: An object will remain at rest or move with constant velocity unless acted upon by a net external force. (A non-accelerating reference frame is called an inertial frame; Newton's First Law holds only in inertial frames.)

4. More force leads to more acceleration: $\Rightarrow \quad a \propto F$

5. The greater the mass of a body, the harder it is to change its state of motion. More mass means more inertia. In other words, more mass leads to less acceleration:

$$\Rightarrow \quad a \propto \frac{1}{m}$$

Combine both the above observations to conclude that:

$$a \propto \frac{F}{m}$$

6. Newton's Second Law: $a = \dfrac{F}{m}$ (or, if you prefer, write as $F = ma$).

7. $F = ma$ is one relation between three independent quantities (m, a, F). For it to be useful, we must have separate ways of measuring mass, acceleration, and force. Acceleration is measured from observing the rate of change of velocity; mass is a measure of the amount of matter in a body (e.g. two identical cars have twice the mass of a single one). Forces (due to gravity, a stretched spring, repulsion of two like charges, etc) will be discussed later.

8. Force has dimensions of $[\text{mass}] \times [\text{acceleration}] = M\,L\,T^{-2}$. In the MKS system the unit of force is the Newton. It has the symbol N where:
   1 Newton = 1 kilogram.metre/second^2.

9. Forces can be internal or external. For example the mutual attraction of atoms within a block of wood are called internal forces. Something pushing the wood

is an external force. In the application of $F = ma$, remember that F stands for the total external force upon the body.

10. Forces are vectors, and so they must be added vectorially:

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$$

This means that the components in the $\hat{x}$ direction must be added separately, those in the $\hat{y}$ direction separately, etc.

11. Gravity acts directly on the mass of a body - this is a very important experimental observation due to Newton and does not follow from $F = ma$. So a body of mass m_1 experiences a force $F_1 = m_1 g$ while a body of mass m_2 experiences a force $F_2 = m_2 g$, where g is the acceleration with which any body (big or small) falls under the influence of gravity. (Galileo had established this important fact when he dropped different masses from the famous leaning tower of Pisa!)

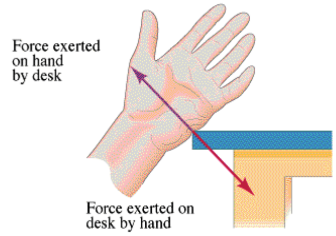
12. The weight of a body W is the force which gravity exerts upon it, $W = mg$. Mass and weight are two completely different quantities. So, for example, if you used a spring balance to weigh a kilo of grapes on earth, the same grapes would weigh only 1/7 kilo on the moon.

13. Newton's Third Law: for every action there is an equal and opposite reaction. More precisely, $F_{AB} = -F_{BA}$, where F_{AB} is the force exerted by body B upon A whereas F_{BA} is the force exerted by body A upon B. Ask yourself what would happen if this was not true. In that case, a system of two bodies, even if it is completely isolated from the surroundings, would have a net force acting upon it because the net force acting upon both bodies would be $F_{AB} + F_{BA} \neq 0$.

14. If action and reaction are always equal, then why does a body accelerate at all? Students are often confused by this. The answer: in considering the acceleration of a body you must consider only the (net) force acting upon that body. So, for example, the earth pulls a stone towards it and causes it to accelerate because there is a net force acting upon the stone. On the other hand, by the Third Law, the stone also pulls the earth towards it and this causes the earth to accelerate towards the stone. However, because the mass of the earth is so large, we are only able to see the acceleration of the stone and not that of the earth.

QUESTIONS AND EXERCISES – 4

Q.1 The picture shown here makes the fairly obvious statement that action and reaction forces in this situation are equal. However, why do you feel pain if there is no net force acting upon your hand?



Q.2 Two astronauts are in outer space connected with a rope in a state of complete weightlessness. Each pulls one end of a rope with his hands. Describe what will happen as time goes on.

Q.3 Add together the forces $\vec{F}_1 = 3\hat{x} + 5\hat{y} - \hat{z}$ and $F_2 = -2\hat{x} - 3\hat{y} + 2\hat{z}$ and obtain the magnitude of the resultant force.

Q.4 A cube of constant density ρ is pushed with a pressure P from one side. The cube is place on a smooth level surface. Find the acceleration of the cube, and the distance covered after time t.

Q.5 Describe all the forces acting upon a ladder that is leaning against a wall. Both the floor and the wall are rough.

Q.6 In the above situation, suppose the floor and wall suddenly become perfectly smooth. What will the net force on the ladder become, and what will be the acceleration in the horizontal and vertical directions?

Summary of Lecture 5 – APPLICATIONS OF NEWTON'S LAWS – I

1. An obvious conclusion from $F = ma$ is that if $F = 0$ then $a = 0$! How simple, yet how powerful ! This says that for any body that is not accelerating the *sum of all the forces acting upon it* must vanish.

2. Examples of systems in equilibrium: a stone resting on the ground; a pencil balanced on your finger; a ladder placed against the wall, an aircraft flying at a constant speed and constant height.

3. Examples of systems out of equilibrium: a stone thrown upwards that is at its highest point; a plane diving downwards; a car at rest whose driver has just stepped on the car's accelerator.

4. If you know the acceleration of a body, it is easy to find the force that causes it to accelerate. Example: An aircraft of mass m has position vector,

$$\vec{r} = (at + bt^3)\hat{i} + (ct^2 + dt^4)\hat{j}$$

What force is acting upon it?

SOLUTION:
$$\vec{F} = m\frac{d^2x}{dt^2}\hat{i} + m\frac{d^2y}{dt^2}\hat{j}$$
$$= 6\,bmt\,\hat{i} + m(2c + 12d\,t^2)\,\hat{j}$$

5. The other way around is not so simple: suppose that you know F and you want to find *x*. For this you must solve the equation,

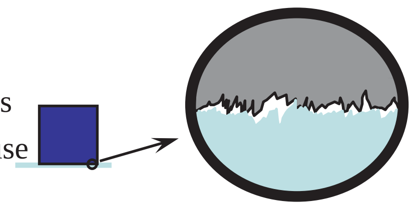
$$\frac{d^2x}{dt^2} = \frac{F}{m}$$

This may or may not be easy, depending upon F (which may depend upon both x as well as t if the force is not constant).

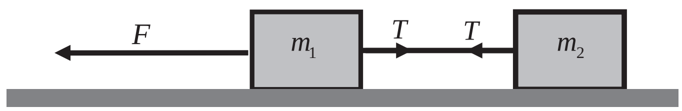
6. Ropes are useful because you can pull from a distance to change the direction of a force. The tension, often denoted by T, is the force you would feel if you cut the rope and grabbed the ends. For a massless rope (which may be a very good approximation in many situations) the tension is the same at every point along the rope. Why? Because if you take any small slice of the rope it weighs nothing (or very little). So if the force on one side of the slice was any different from the force force on the other side, it would be accelerating hugely. All this was for the "ideal rope" which has no mass and never breaks. But this idealization if often good enough.

7. We are all familiar with frictional force. When two bodies rub against each other, the frictional force acts upon each body separately opposite to its direction of motion (i.e it acts to slow down the motion). The harder you press two bodies against each other, the greater the friction. Mathematically, $\vec{F} = \mu\vec{N}$, where $\vec{N}$ is the force with which you press the two bodies against each other (normal force). The quantity μ is called the coefficient of friction (obviously!). It is large for rough surfaces, and small for smooth ones. Remember that $\vec{F} = \mu\vec{N}$ is an empirical relation and holds only approximately. This is obviously true: if you put a large enough mass on a table, the table will start to bend and will eventually break.

8. Friction is caused by roughness at a microscopic level - if you look at any surface with a powerful microscope you will see unevenness and jaggedness. If these big bumps are levelled somehow, friction will still not disappear because there will still be little bumps due to atoms. More precisely, atoms from the two bodies will interact each other because of the electrostatic interaction between their charges. Even if an atom is neutral, it can still exchange electrons and there will be a force because of surrounding atoms.



9. Consider the two blocks below on a frictionless surface:



We want to find the tension and acceleration: The total force on the first mass is $F - T$ and so $F - T = m_1 a$. The force on the second mass is simply T and so $T = m_2 a$. Solving the above, we get: $T = \frac{m_2 F}{m_1 + m_2}$ and $a = \frac{F}{m_1 + m_2}$.

10. There is a general principle by which you solve equilibrium problems. For equilibrium, the sum of forces in every direction must vanish. So $F_x = F_y = F_z = 0$. You may always choose the x, y, z directions according to your convenience. So, for example, as in the lecture problem dealing with a body sliding down an inclined plane, you can choose the directions to be along and perpendicular to the surface of the plane.

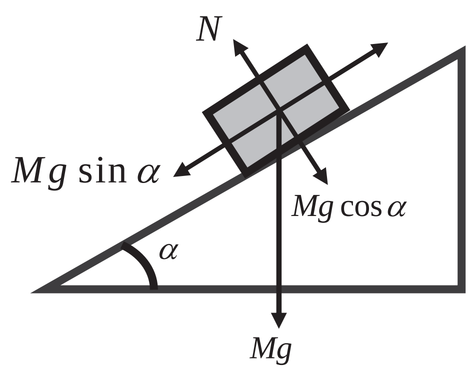
QUESTIONS AND EXERCISES – 5

Q.1 The relation $F = \mu N$ is an approximate one only. Discuss various circumstances in which a)it would be correct to use the relation, b)where it would not be correct.

Q.2 Force is a vector quantity but $F = \mu N$ is written as a relation between the magnitudes of two forces. So what is the direction of the frictional force ?

Q.3 In (9), a frictionless surface was considered. Now assume that the coefficient of friction is μ. What will be the acceleration and the tension now. Assume that the body is pulled with sufficient force so that it moves.

Q.4 For a body resting on an inclined plane as below:

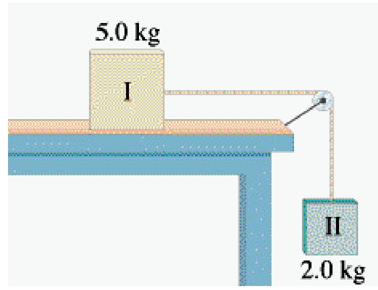


a) Find the minimum angle at which the body starts to slide.
b) If the angle exceeds the above, find the acceleration.
c) Find the time taken to slide a length d down the slope if the body starts at rest.

Q5. Consider the system of two masses shown. Find the acceleration of the two masses (are they the same?) for the following two situations:

a) The table is frictionless.

b) The coefficient of friction $\mu = 2$.



Summary of Lecture 6 – APPLICATIONS OF NEWTON'S LAWS – II

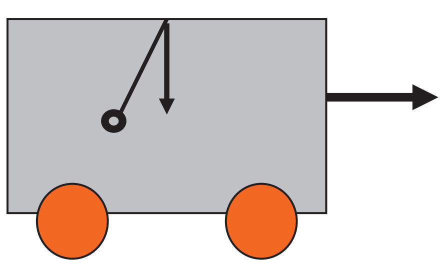
1. As a body moves through a body it displaces the fluid. it has to exert a force on the fluid to push it out of the way. By Newton's third law, the fluid pushes back on the body with an equal and opposite force. The direction of the fluid resistance force on a body is always opposite to the direction of the body's velocity relative to the fluid.

2. The magnitude of the fluid resistance force usually increases with the speed of the body through the fluid. Typically, $f = k\mathrm{v}$ (an empirical law!). Imagine that you drop a ball bearing into a deep container filled with oil. After a while the ball bearing will approach its maximum (terminal) speed when the forces of gravity and friction balance each other: $mg = k\mathrm{v}$ from which $\mathrm{v}_{\mathrm{final}} = mg/k$.

3. The above was a simple example of equilibrium under two forces. In general, while solving problems you should a)draw a diagram, b)define an origin for a system of coordinates, c)identify all forces (tension, normal, friction, weight, etc) and their x and y components, d)Apply Newton's law separately along the x and y axes. e) find the accelerations, then velocities, then displacements. This sounds very cook-book, and in fact it will occur to you naturally how to do this while solving actual problems.

4. Your weight in a lift: suppose you are in a lift that is at rest or moving at constant velocity. In either case $a=0$ and the normal force N and the force due to gravity are exactly equal, $N - Mg = 0 \Rightarrow N = Mg$. But if the lift is accelerating downwards then $Mg - N = Ma$ or $N = M(g-a)$. So now the normal force (i.e. the force with which the floor of the lift is pushing on you) is decreased. Note that if the lift is accelerating downwards with acceleration a (which it will if the cable breaks!) then $N=0$ and you will experience weightlessness just like astronauts in space do. Finally, if the lift is accelerating upwards then a is negative and you will feel heavier.

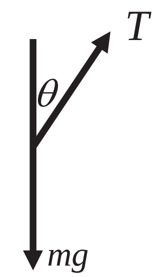
5. Imagine that you are in a railway wagon and want to know how much you are accelerating. You are not able to look out of the windows. A mass is hung from the roof. Find the acceleration of the car from the angle made by the mass.



We first balance the forces vertically: $T\cos\theta = mg$
And then horizontally: $T\sin\theta = ma$

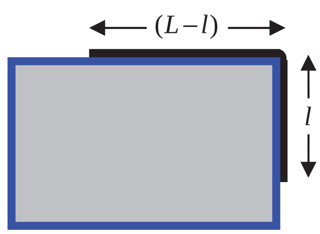
From these two equations we find that: $\tan\theta = \dfrac{a}{g}$

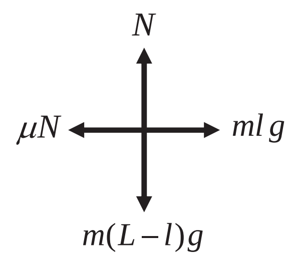
Note that the mass m doesn't matter - it cancels out!



6. Friction is a funny kind of force. It doe not make up its mind which way to act until some other force compels it to decide. Imagine a block lying on the floor. If you push it forward, friction will act backward. And if you push it to the left, friction will act to the right. In other words, the direction of the frictional force is always in the opposite direction to the applied force.

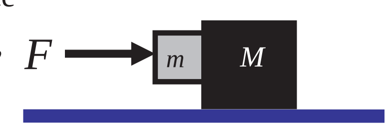
7. Let us solve the following problem: a rope of total length L and mass per unit length m is put on a table with a length l hanging from one edge. What should be l such that the rope just begins to slip?





To solve this, look at the balance of forces in the diagram below: in the vertical direction, the normal force balances the weight of that part of the rope that lies on the table: $N = m(L-l)g$. In the horizontal direction, the rope exerts a force mlg to the right, which is counteracted by the friction that acts to the left. Therefore $\mu N = mlg$. Substituting N from the first equation we find that $l = \dfrac{\mu L}{\mu + 1}$. Note that if μ isvery small then even a small piece of string that hangs over the edge will cause the entire string to slip down.

9. In this problem, we would like to calculate the minimum force F such that the small block does not slip downwards. Clearly, since the 2 bodies move together, $F = (m+M)a$. This gives $a = \dfrac{F}{(m+M)}$. We want the friction μN to be at least as large as the downwards force, mg.



So, we put $N = ma = m\left(\dfrac{F}{(m+M)}\right)$ from which the minimum horizontal force needed to prevent slippage is $F = \dfrac{(m+M)g}{\mu}$.

QUESTIONS AND EXERCISES – 6

Q.1 a) Why is it necessary to choose a reference frame?

b) If you used spherical coordinates to solve a problem instead of rectangular cartesian coordinates, would it make a difference?

c) Why do we pretend that the earth is an inertial frame when we know it is not?

d) Without making measurements by looking outside, can you know whether or not your frame is inertial?

e) Look up the meaning of "inertial guidance".

f) Newton thought that one frame that was fixed to the distant stars was the best (or most preferred frame). Was this a sensible thought at that time? Discuss.

Q.2 For a particular liquid and a certain shape of body the relation between the frictional resistance and velocity is of the form $F = a\text{v} + b\text{v}^2$.

a) What are the dimensions of a and b?

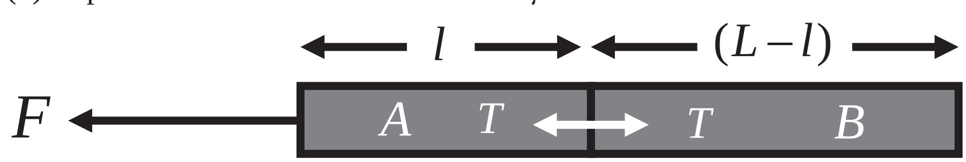
b) If a body of mass M is dropped into a deep vessel containing the liquid, what is the maximum speed that it will attain?

c) What will be the initial acceleration just as it enters the liquid?

Q.3 A rope is pulled with force F on the floor.

(a) Find its acceleration and the tension T at distance l.

(b) Repeat if the coefficient of friction is μ.



Solution to part (a):

Take a small piece of the rope and look at the forces acting to the left and the right.

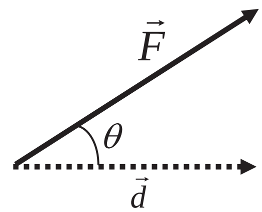
The net force on part A of the rope gives its acceleration: $F - T = m\,l\,a$

Similarly, the net force on part B of the rope gives its acceleration: $T = m(L-l)a$

From these two equations it follows that T depends on l : $T = F\left(1 - \frac{l}{L}\right)$

Summary of Lecture 7 – WORK AND ENERGY

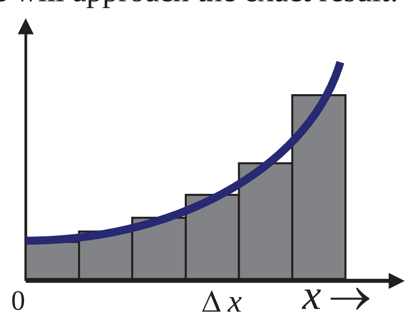
1. Definition of work: force applied in direction of displacement × displacement. This means that if the force F acts at an angle θ with respect to the direction of motion, then $W = \vec{F} \cdot \vec{d} = Fd\cos\theta$



2. a) Work is a scalar - it has magnitude but no direction.
   b) Work has dimensions: $M \times (L\ T^{-2}) \times L\ =\ M\ L^2 T^{-2}$
   c) Work has units: 1 Newton × 1 Metre ≡ 1 Joule (J)

3. Suppose you lift a mass of 20 kg through a distance of 2 metres. Then the work you do is 20 kg × 9.8 Newtons × 2 metres = 39.2 Joules. On the other hand, the force of gravity is directed opposite to the force you exert and the work done by gravity is -39.2 Joules.

4. What if the force varies with distance (say, a spring pulls harder as it becomes longer). In that case, we should break up the distance over which the force acts into small pieces so that the force is approximately constant over each bit. As we make the pieces smaller and smaller, we will approach the exact result:



Now add up all the little pieces of work:

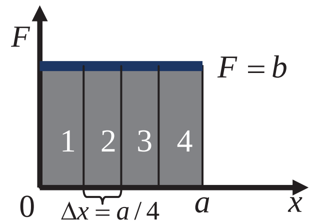
$$W = \Delta W_1 + \Delta W_2 + \cdots + \Delta W_N = F_1\Delta x + F_2\Delta x + \cdots + F_N\Delta x \equiv \sum_{n=1}^{N} F_n \Delta x$$

To get the exact result let $\Delta x \rightarrow 0$ and the number of intervals N $\rightarrow \infty$: $W = \lim_{\Delta x \rightarrow 0} \sum_{n=1}^{\infty} F_n \Delta x$

Definition: $W = \lim_{\Delta x \rightarrow 0} \sum_{n=1}^{\infty} F_n\ \Delta x \equiv \int_{x_i}^{x_f} F(x)dx$ is called the integral of F with respect to x from

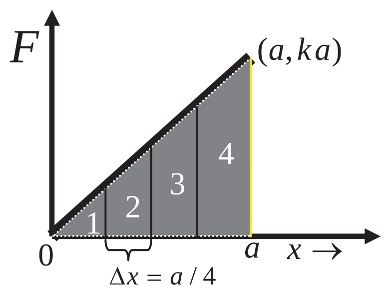
from x_i to x_f. This quantity is the work done by a force, constant or non-constant. So if the force is known as a function of position, we can always find the work done by calculating the definite integral.

5. Just to check what our result looks like for a constant force, let us calculate W if $F = b$,



$$\frac{1}{4}a(b)+\frac{1}{4}a(b)+\frac{1}{4}a(b)+\frac{1}{4}a(b)=ab \qquad \therefore \int_0^a F dx = ab$$

6. Now for a less trivial case: suppose that *F=kx*, i.e. the force increases linearly with *x*.



$$\text{Area of shaded region} = \frac{1}{2}(a)(ka) = k\frac{a^2}{2} \qquad \therefore \int_0^a F dx = k\frac{a^2}{2}$$

7. Energy is the capacity of a physical system to do work:
   - it comes in many forms – mechanical, electrical, chemical, nuclear, etc
   - it can be stored
   - it can be converted into different forms
   - it can never be *created* or *destroyed*

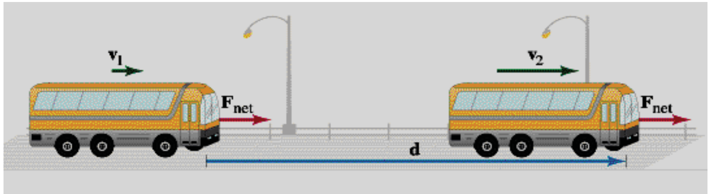
8. Accepting the fact that energy is conserved, let us derive an expression for the kinetic energy of a body. Suppose a *constant force* accelerates a mass *m* from speed *0* to speed v over a *distance d*. What is the work done by the force? Obviously the answer is: $W = Fd$. But $F = ma$ and $v^2 = 2ad$. This gives $W = (ma)d = \frac{mv^2}{2d}d = \frac{1}{2}mv^2$. So, we conclude that the work done by the force has gone into creating kinetic energy. and that the amount of kinetic energy possessed by a body moving with speed v is $\frac{1}{2}mv^2$.

9. The work done by a force is just the force multiplied by the distance – it does not depend upon time. But suppose that the same amount of work is done in half the time. We then say that the *power* is twice as much. We define:

$$\text{Power} = \frac{\text{Work done}}{\text{Time taken}}$$

If the force does not depend on time: $\frac{\text{Work}}{\text{Time}} = \frac{F\,\Delta x}{\Delta t} = F\text{ v}$. Therefore, Power = F v.

10.Let's work out an example. A constant force accelerates a bus (mass m) from speed v_1 to speed v_2 over a distance d. What work is done by the engine?



Recall that for constant acceleration, $v_2^2 - v_1^2 = 2a\,(x_2 - x_1)$ where: v_2 = final velocity, x_2 = final position, v_1 = initial velocity, x_1 = initial position. Hence, $a = \frac{v_2^2 - v_1^2}{2d}$. Now calculate the work done: $W = Fd = mad = m\frac{v_2^2 - v_1^2}{2d}\,d = \frac{1}{2}mv_2{}^2 - \frac{1}{2}mv_1{}^2$. So the work done has resulted in an increase in the quantity $\frac{1}{2}mv^2$, which is kinetic energy.

QUESTIONS AND EXERCISES – 7

Q.1 A stone tied to a string is whirled around. Suppose the string has tension T. How much work will be done when the stone goes around one complete revolution?

Q.2 In (6) above, calculate the areas 1,2,3,4 separately and then add them up. Is your answer equal to $k\frac{a^2}{2}$?

Q.3 A *1000 kg* trolley is pulled up a *45°* inclined plane at 1.5 m/sec. How much power is is needed?

Q. 4 In the previous question, if the coefficient of friction is $\mu = 0.5$ what will be the power needed now?

Q.5 Suppose the air friction acting on a car increases as kv^2. What is the engine power needed to keep the car moving at speed v?

Q.6 In the example solved in (10) above, calculate the power of the bus engine.