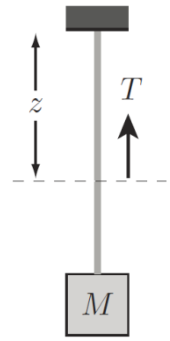
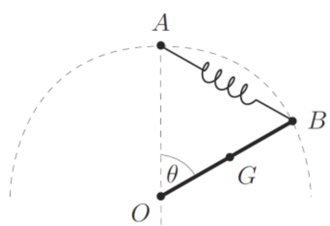
Questions for Module # 14

Q.1 A body of mass M is suspended from a fixed point O by an inextensible uniform rope of mass m and length b. Find the tension in the rope at a distance z below O. The point of support now begins to rise with acceleration $2g$. What now is the tension in the rope? Solution



Q.2 Two uniform lead spheres each have mass 5000 kg and radius 47 cm. They are released from rest with their centres 1 m apart and move under their mutual gravitation. Show that they will collide in *less* than 425 s. [$G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$.] Solution

Q.3 A uniform rod of length $2a$ has one end smoothly pivoted at a fixed point O. The other end is connected to a fixed point A, which is a distance $2a$ vertically above O, by a light elastic spring of natural length a and modulus $\frac{1}{2}mg$. The rod moves in a vertical plane through O. Show that there are two equilibrium positions for the rod, and determine their stability. [The vertically upwards position for the rod would compress the spring to zero length and is excluded.] Solution



Q.4 The internal gravitational potential energy of a system of masses is sometimes called the **self energy** of the system. (The reference configuration is taken to be one in which the particles are all a great distance from each other.) Show that the self energy of a uniform sphere of mass M and radius R is $-3M^2G/5R$. [Imagine that the sphere is built up by the addition of successive thin layers of matter brought in from infinity.] Solution

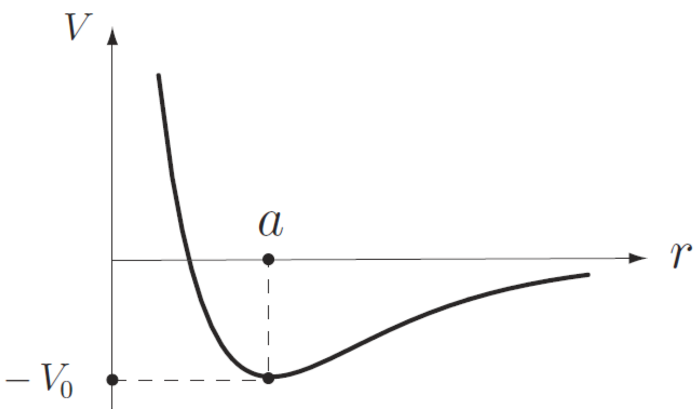
Q.5 Two blocks of mass m_1 and m_2, with initial velocity v_1 and v_2, respectively, meet in a perfectly inelastic collision. Then, a third block of mass m_3 and velocity v_3 hits the two blocks in a perfectly inelastic collision. Using conservation of momentum, find the velocity of the three blocks, assuming no external forces (one dimensional case). Solution

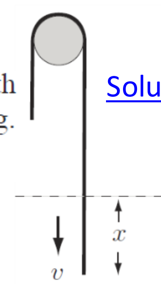
Q.6 The internal potential energy function for a diatomic molecule is approximated by the **Morse potential** Solution

$$V(r) = V_0\left(1 - e^{-(r-a)/b}\right)^2 - V_0,$$

where r is the distance of separation of the two atoms, and V_0, a, b are positive constants. Make a sketch of the Morse potential.

Suppose the molecule is restricted to *vibrational* motion in which the centre of mass G of the molecule is fixed, and the atoms move on a fixed straight line through G. Show that there is a single equilibrium configuration for the molecule and that it is stable. If the atoms each have mass m, find the angular frequency of small vibrational oscillations of the molecule.

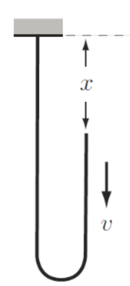




Q.7 A heavy uniform rope of length $2a$ is draped symmetrically over a *thin* smooth horizontal peg. The rope is then disturbed slightly and begins to slide off the peg. Find the speed of the rope when it finally leaves the peg. Solution

Q.8 A uniform heavy rope of length a is held at rest with its two ends close together and the rope hanging symmetrically below. (In this position, the rope has two long vertical segments connected by a small curved segment at the bottom.) One of the ends is then released. Find the velocity of the free end when it has descended by a distance x. Solution

Deduce a similar formula for the acceleration of the free end and show that it always *exceeds* g. Find how far the free end has fallen when its acceleration has risen to $5g$.



Q.9 Show that, if a system moves periodically, then the average of the total external force over a period of the motion must be zero. Solution

A juggler juggles four balls of masses M, $2M$, $3M$ and $4M$ in a periodic manner. Find the time average (over a period) of the total force he applies to the balls. The juggler wishes to cross a shaky bridge that cannot support the combined weight of the juggler and his balls. Would it help if he juggles his balls while he crosses?

Q.10 Find the center of mass of [Solution]

a. A cone of radius R and height h, with its base on the xy plane.
b. A hemisphere of radius R, with its base on the xy plane.