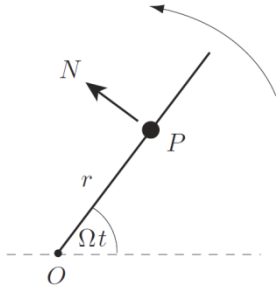
Questions for Module # 15

Q.1 Solution



A particle P of mass m can slide along a *smooth* rigid straight wire. The wire has one of its points fixed at the origin O, and is made to rotate in the (x, y)-plane with angular speed Ω. By using the vector equation of motion of P in polar co-ordinates, show that r, the distance of P from O, satisfies the equation

$$\ddot{r} - \Omega^2 r = 0,$$

and find a second equation involving N, where $N\widehat{\boldsymbol{\theta}}$ is the force the wire exerts on P. [Ignore gravity in this question.]

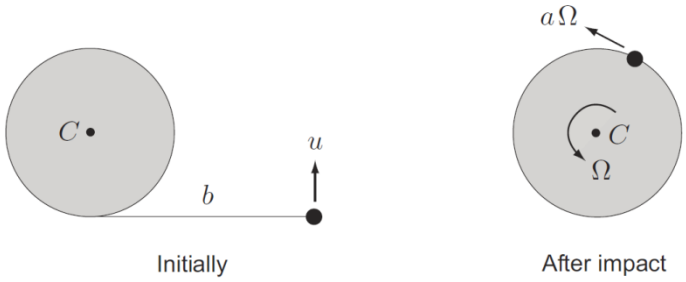
Initially, P is at rest (relative to the wire) at a distance a from O. Find r as a function of t in the subsequent motion, and deduce the corresponding formula for N.

Q.2 A fairground target consists of a uniform circular disk of mass M and radius a that can turn freely about a diameter which is fixed in a vertical position. Initially the target is at rest. A bullet of mass m is moving with speed u along a horizontal straight line at right angles to the target. The bullet embeds itself in the target at a point distance b from the rotation axis. Find the final angular speed of the target. [The moment of inertia of the disk about its rotation axis is $Ma^2/4$.] Solution

Show also that the energy lost in the impact is

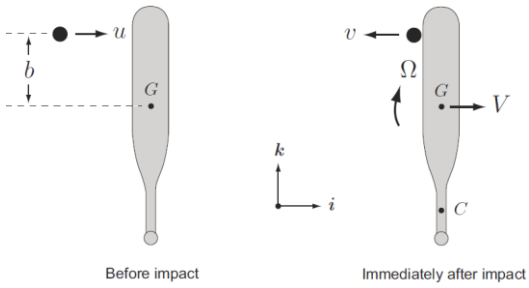
$$\frac{1}{2}mu^2\left(\frac{Ma^2}{Ma^2 + 4mb^2}\right).$$

Q.3 A uniform circular cylinder of mass M and radius a can rotate freely about its axis of symmetry which is fixed in a vertical position. A light string is wound around the cylinder so that it does not slip and a particle of mass m is attached to the free end. Initially the system is at rest with the free string taut, horizontal and of length b. The particle is then projected horizontally with speed u at right angles to the string. The string winds itself around the cylinder and eventually the particle strikes the cylinder and sticks to it. Find the final angular speed of the cylinder. Solution



Q.4 A cloud of interstellar gas of total mass M can move freely in space. Initially the cloud has the form of a uniform sphere of radius a rotating with angular speed Ω about an axis through its centre. Later, the cloud is observed to have changed its form to that of a thin uniform circular disk of radius b which is rotating about an axis through its centre and perpendicular to its plane. Find the angular speed of the disk and the increase in the kinetic energy of the cloud. Solution

Q.5 A baseball bat has mass M and moment of inertia Mk^2 about any axis through its centre of mass G that is perpendicular to the axis of symmetry. The bat is at rest when a ball of mass m, moving with speed u, is normally incident along a straight line through the axis of symmetry at a distance b from G. Show that, whether the impact is elastic or not, there is a point on the axis of symmetry of the bat that is instantaneously at rest after the impact and that the distance c of this point from G is given by $bc = k^2$. In the elastic case, find the speed of the ball after the impact. [Gravity (and the batter!) should be ignored throughout this question.] Solution



Q.6 The bob of a certain pendulum moves on a vertical circle of radius b and, when the string makes an angle θ with the downward vertical, the circumferential velocity v of the bob is given by Solution

$$v^2 = 2gb\cos\theta,$$

where g is a positive constant. Find the acceleration of the bob when the string makes angle θ with the downward vertical.