Questions for Module # 16

Solutions can be found in Classical Mechanics by R. Douglas Gregory

Q.1 When a body is suspended from a fixed point by a certain linear spring, the angular frequency of its vertical oscillations is found to be Ω_1. When a different linear spring is used, the oscillations have angular frequency Ω_2. Find the angular frequency of vertical oscillations when the two springs are used together (i) in parallel, and (ii) in series. Show that the first of these frequencies is at least twice the second.

Q.2 An overdamped harmonic oscillator satisfies the equation

$$\ddot{x} + 10\dot{x} + 16x = 0.$$

At time $t = 0$ the particle is projected from the point $x = 1$ towards the origin with speed u. Find x in the subsequent motion.

Show that the particle will reach the origin at some later time t if

$$\frac{u-2}{u-8} = e^{6t}.$$

How large must u be so that the particle will pass through the origin?

Q.3 A block of mass M is connected to a second block of mass m by a linear spring of natural length $8a$. When the system is in equilibrium with the first block on the floor, and with the spring and second block vertically above it, the length of the spring is $7a$. The upper block is then pressed down until the spring has half its natural length and is then resleased from rest. Show that the lower block will leave the floor if $M < 2m$. For the case in which $M = 3m/2$, find when the lower block leaves the floor.

Q.4 A particle P of mass $3m$ is suspended fron a fixed point O by a light linear spring with strength α. A second particle Q of mass $2m$ is in turn suspended from P by a second spring of the same strength. The system moves in the vertical straight line through O. Find the normal frequencies and the form of the normal modes for this system. Write down the form of the general motion.

Q.5 Two particles P and Q, each of mass m, are secured at the points of trisection of a light string that is stretched to tension T_0 between two fixed supports a distance $3a$ apart. The particles undergo small *transverse* oscillations perpendicular to the equlilibrium line of the string. Find the normal frequencies, the forms of the normal modes, and the general motion of this system. [Note that the forms of the modes could have been deduced from the symmetry of the system.] Is the general motion periodic?