Summary of Lecture 17 – PHYSICS OF MATERIALS

1. **Elasticity :** the property by virtue of which a body tends to regain its original shape and size when external forces are removed. If a body completely recovers its original shape and size , it is called perfectly elastic. Quartz, steel and glass are very nearly elastic.

2. **Plasticity :** if a body has no tendency to regain its original shape and size , it is called perfectly plastic. Common plastics, kneaded dough, solid honey, etc are plastics.

3. **Stress** characterizes the strength of the forces causing the stretch, squeeze, or twist. It is defined usually as force/unit area but may have different definitions to suit different situations. We distinguish between three types of stresses:
   a) If the deforming force is applied along some linear dimension of a body, the stress is called *longitudinal stress* or *tensile stress* or *compressive stress.*
   b) If the force acts normally and uniformly from all sides of a body, the stress is called *volume stress.*
   c) If the force is applied tangentially to one face of a rectangular body, keeping the other face fixed, the stress is called tangential or shearing stress.

4. Strain: When deforming forces are applied on a body, it undergoes a change in shape or size. The fractional (or relative) change in shape or size is called the strain.

$$\text{Strain} = \frac{\text{change in dimension}}{\text{original dimension}}$$

Strain is a ratio of similar quantities so it has no units. There are 3 different kinds of strain:

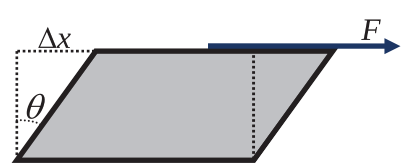
a) *Longitudinal (linear) strain* is the ratio of the change in length (ΔL) to original length (l), i.e., the linear strain $= \dfrac{\Delta l}{l}$.

b) *Volume strain* is the ratio of the change in volume (ΔV) to original volume (V)

Volume strain $= \dfrac{\Delta V}{V}$.

c) *Shearing strain :* The angular deformation (θ) in radians is called shearing stress.

For small θ the shearing strain $\equiv \theta \approx \tan\theta = \dfrac{\Delta x}{l}$.



5. Hooke's Law: for small deformations, stress is proportional to strain.

Stress = E × Strain

The constant E is called the modulus of elasticity. E has the same units as stress because strain is dimensionless. There are three moduli of elasticity.

(a) Young's modulus (Y) for linear strain:

$$\text{Y} \equiv \frac{\text{longitudinal stress}}{\text{longitudinal strain}} = \frac{F/A}{\Delta l/l}$$

(b) Bulk Modulus (B) for volume strain: Let a body of volume V be subjected to a uniform pressure ΔP on its entire surface and let ΔV be the corresponding decrease in its volume. Then,

$$\text{B} \equiv \frac{\text{Volume Stress}}{\text{Volume Strain}} = -\frac{\Delta P}{\Delta V/V}.$$

$1/B$ is called the compressibility. A material having a small value of B can be compressed easily.

(c) Shear Modulus (η) for shearing strain: Let a force F produce a strain θ as in the diagram in point 4 above. Then,

$$\eta \equiv \frac{\text{shearing stress}}{\text{shearing strain}} = \frac{F/A}{\theta} = \frac{F}{A\tan\theta} = \frac{Fl}{A\Delta x}.$$

6. When a wire is stretched, its length increases and radius decreases. The ratio of the lateral strain to the longitudinal strain is called Poisson's ratio, $\sigma = \frac{\Delta r/r}{\Delta l/l}$. Its value lies between 0 and 0.5.

7. We can calculate the work done in stretching a wire. Obviously, we must do work against a force. If x is the extension produced by the force F in a wire of length l, then $F = \frac{YA}{l}x$. The work done in extending the wire through Δl is given by,

$$W = \int_0^{\Delta l} F dx = \frac{YA}{l}\int_0^{\Delta l} x dx = \frac{YA}{l}\frac{(\Delta l)^2}{2}$$

$$= \frac{YA}{l}\frac{(\Delta l)^2}{2} = \frac{1}{2}(Al)\left(\frac{Y\Delta l}{l}\right)\left(\frac{\Delta l}{l}\right) = \frac{1}{2} \times \text{volume} \times \text{stress} \times \text{strain}$$

Hence, Work / unit volume $= \frac{1}{2} \times \text{stress} \times \text{strain}$. We can also write this as,

$$W = \frac{1}{2}\left(\frac{YA\Delta l}{l}\right)\Delta l = \frac{1}{2} \times \text{load} \times \text{extension}.$$

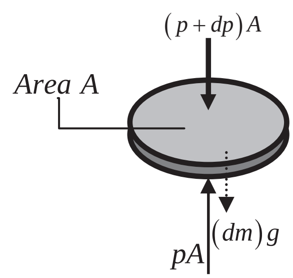
8. A fluid is a substance that can flow and does not have a shape of its own. Thus all liquids and gases are fluids. Solids possess all the three moduli of elasticity whereas a fluid possess only the bulk modulus (B). A fluid at rest cannot sustain a tangential force. If such force is applied to a fluid, the different layers simply slide over one another. Therefore the forces acting on a fluid at rest have to be normal to the surface. This implies that the free surface of a liquid at rest, under gravity, in a container, is horizontal.

9. The normal force per unit area is called pressure, $P = \frac{\Delta F}{\Delta A}$. Pressure is a scalar quantity.

Its unit is Newtons/metre^2, or Pascal (Pa). Another scalar is density, $\rho = \frac{\Delta m}{\Delta V}$, where Δm is the mass of a small piece of the material and ΔV is the volume it occupies.

10. Let us calculate how the pressure in a fluid changes with depth. So take a small element of fluid volume submerged within the body of the fluid: $dm = \rho dV = \rho A dy \quad \therefore \; (dm)g = \rho g A dy$

Now let us require that the sum of the forces on the fluid element is zero: $pA - (p+dp)A - \rho g A dy = 0 \quad \Rightarrow \frac{dp}{dy} = -\rho g.$



Note that we are taking the origin ($y = 0$) at the bottom of the liquid. Therefore as the elevation increases (dy positive), the pressure decreases (dp negative). The quantity ρg is the weight per unit volume of the fluid. For liquids, which are nearly incompressible, ρ is practically constant.

$$\therefore \rho g = \text{constant} \Rightarrow \frac{dp}{dy} = \frac{\Delta p}{\Delta y} = \frac{p_2 - p_1}{y_2 - y_1} = -\rho g \Rightarrow \; p_2 - p_1 = -\rho g\left(y_2 - y_1\right).$$

11. **Pascal's Principle**: Pressure applied to an enclosed fluid is transmitted to every portion of the fluid and to the walls of the containing vessel. This follows from the above: if h is the height below the liquid's surface, then $p = p_{ext} + \rho g h$. Here p_{ext} is the pressure at the surface of the liquid, and so the difference in pressure is $\Delta p = \Delta p_{ext} + \Delta(\rho g h)$. Therefore, $\Delta(\rho g h) = 0 \; \Rightarrow \Delta p = \Delta p_{ext}$. (I have used here the fact that liquids are incompressible). So the pressure is everywhere the same.

QUESTIONS AND EXERCISES – 17

Q.1 Give 3 examples (other than those given in the lecture or the summary above) of

a) Nearly elastic materials.

b) Nearly plastic materials.

Would you consider glass to be elastic or plastic? Why?

Q.2 In the summary above, I wrote that "the forces acting on a fluid at rest have to be normal to the surface. This implies that the free surface of a liquid at rest, under gravity, in a container, is horizontal.". Be sure that you understand this, and then explain what happens as:

a) You slowly tilt a glass. Why does the surface stay horizontal?

b) You stir a cup of tea with a spoon. Why does the surface of the liquid rise up as you move away from the centre?

Q.3 To what height should a cylindrical vessel be filled with a homogeneous liquid to make the force with which the liquid presses on the sides of the wall equal to the force exerted by the liquid on the bottom of the vessel? [Hint: the force exerted by the liquid on the bottom $= (hg\rho)\pi r^2$; mean pressure on the wall $= \frac{1}{2}(hg\rho)$. Answer: the liquid should be filled up to the height equal to the radius of the cylinder. Work out the remaining details!]

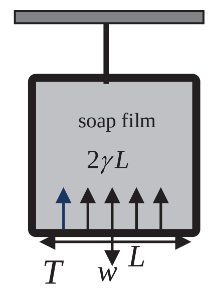
Q.4 As discussed in the lecture, Archimedes Principle states that a body wholly or partially immersed in a fluid is buoyed up by a force (or upthrust) equal in magnitude to the weight of the fluid displaced by the body. Show how this follows for a cube immersed in water using the equation derived earlier, namely, $p_2 - p_1 = -\rho g\left(y_2 - y_1\right)$.

Summary of Lecture 18 – PHYSICS OF FLUIDS

1. A fluid is matter that has no definite shape and adjusts to the container that it is placed in. Gases and liquids are both fluids. All fluids are made of molecules. Every molecules attracts other molecules around it.

2. Liquids exhibit surface tension. A liquid has the property that its free surface tends to contract to minimum possible area and is therefore in a state of tension. The molecules of the liquid exerts attractive forces on each other, which is called cohesive forces. Deep inside a liquid, a molecule is surrounded by other molecules in all directions. Therefore there is no net force on it. At the surface, a molecule is surrounded by only half as many molecules of the liquid, because there are no molecules above the surface.

3. The force of contraction is at right angles to an imaginary line of unit length, tangential to the surface of a liquid, is called its surface tension: $\gamma = \frac{F}{L}$. Here F is the force exerted by the "skin" of the Liquid. The SI unit of the surface tension is N/m.



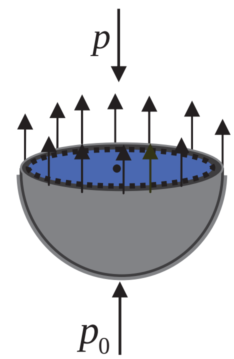
4. Quantitative measurement of surface tension: let w be the weight of the sliding wire, $T =$ force with which you pull the wire downward. Obviously, $T + w = F =$ net downward force. Since film has both front and back surfaces, the force F acts along a total length of $2L$. The surface tension in the film is defined as, $\gamma = \frac{F}{2L} \quad \Rightarrow F = 2\gamma L.$ Hence, the surface tension is $\gamma = \frac{w+T}{2L}$.

5. Let's ask how much work is done when we stretch the skin of a liquid. If we move the sliding wire through a displacement $\Delta x,$ the work done is $F\Delta x$. Now F is a conservative force, so there is potential energy $\Delta U = F\Delta x = \gamma L\Delta x$ where L is the length of the surface layer $L\Delta x = \Delta A =$ change in area of the surface. Thus $\gamma = \frac{\Delta U}{\Delta A}$. So we see that surface tension is the surface potential energy per unit area !

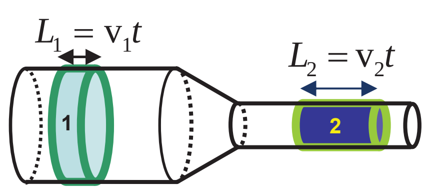
6. When liquids come into contact with a solid surface, the liquid's molecules are attracted by the solid's molecules (called "adhesive" forces). If these adhesive forces are stronger than the cohesive forces, the liquid's molecules are pulled towards the solid surface and

liquid surface becomes curved inward (e.g. water in a narrow tube). On the other hand, if cohesive forces are stronger the surface becomes curved outwards (e.g. with mercury instead). This also explains why certain liquids spread when placed on the solid surface and wet it (e.g., water on glass) while others do not spread but form globules (e.g., mercury on glass).

7. The surface tension causes a pressure difference between the inside and outside of a soap bubble or a liquid drop. A soap bubble consists of two spherical surface films with a thin layer of liquid between them. Let p = pressure exerted by the upper half, and p_0 = external pressure. $\therefore$ force exerted due to surface tension is $2(2\pi r\gamma)$ (the "2" is for two surfaces). In equilibrium the net forces must be equal: $\pi r^2 p = 2(2\pi r\gamma) + \pi r^2 p_0$. So the excess pressure is $p - p_0 = \frac{4\gamma}{r}$. For a liquid drop, the difference is that there is only surface and so, excess pressure = $p - p_0 = \frac{2\gamma}{r}$.



8. From the fact that liquids are incompressible, equal volumes of liquid must flow in both sections in time t, i.e. $V_1/t = V_2/t \Rightarrow V_1 = V_2$. But you can see that $V_1 = A_1L_1 = A_1 v_1 t$ and similarly that $V_2 = A_2L_2 = A_2 v_2 t$. Hence $A_1 v_1 = A_2 v_2$. This means that liquid will flow faster when the tube is narrower, and slower where it is wider.



9. **Bernoulli's Equation :** When a fluid flows steadily, it obeys the equation:

$$p + \frac{1}{2}\rho v^2 + \rho g y = \text{ constant}$$

This famous equation, due to Daniel Bernoulli about 300 years ago, tells you how fast a fluid will flow when there is also a gravitational field acting upon. For a derivation, see any of the suggested references. In the following, I will only explain the meaning of the various terms in the formula and apply it to a couple of situations.

10. Let us apply this to water flowing in a pipe whose crossection decreases along its length. (A_1 is area of the wide part, etc).There is no change in the height so $y_1 = y_2$ and

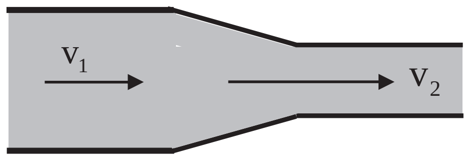
the gravitational potential cancels,

$$p_1 + \frac{1}{2}\rho v_1^2 + \cancel{\rho g y_1} = p_2 + \frac{1}{2}\rho v_2^2 + \cancel{\rho g y_2} \Rightarrow p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2.$$

Now, since the liquid is incompressible, it flows faster in the narrower part:

$$A_1 v_1 = A_2 v_2 \Rightarrow v_2 = \left(\frac{A_1}{A_2}\right) v_1 \Rightarrow p_2 = p_1 - \frac{1}{2}\rho\left(v_2^2 - v_1^2\right).$$

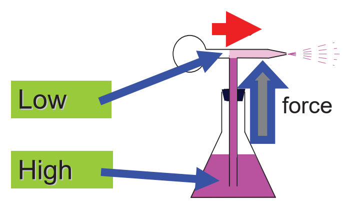
This means that the pressure is smaller where the fluid is flowing faster!



This is exactly why an aircraft flies: the wing shape is curved so that when the aircraft moves through the air, the air moves faster on the top part of the wing than on the lower part. Thus, the air pressure is lower on the top compared to the bottom and there is a net pressure upwards. This is called lift.

QUESTIONS AND EXERCISES – 18

Q.1 This perfume dispenser works on Bernoulli's principle. When you press the rubber bulb, the gas in it rushes out a high speed. Explain how this creates a force and causes the perfume to rise.



Q.2 When water is poured on to a concrete floor from a height of 10 cm, it breaks up into many drops. About how many more drops will be made if it is poured from 30 cms instead? Assume that the size of the drops is the same in both cases.

Q.3 Does it take more energy to create two bubbles of radius of radius 0.5cm or one bubble of radius 1.0cm? Find the ratio of the two energies.

Q.4 Apply Bernoulli's equation to find the difference in pressures between the lower and upper parts of the tube below. An incompressible fluid flows in it. [Hint: $v_1 = v_2$. Why?]