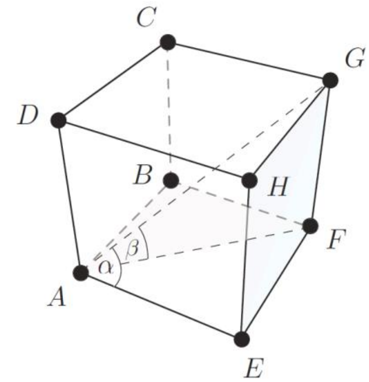
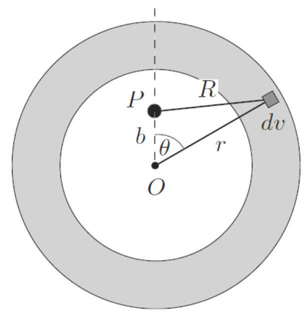
Questions for Module # 20



Q.1 Eight particles, each of mass m, are situated at the corners of a cube of side a. Find the gravitational force exerted on any one of the particles by the other seven. Deduce the total gravitational force exerted on the four particles lying on one face of the cube by the four particles lying on the opposite face.

Solution



Q.2 Show that the gravitational force exerted on a particle *inside* a hollow symmetric sphere is zero. [*Hint.* The proof is the same as for a particle *outside* a symmetric sphere, except in one detail.]

Q.3 A narrow hole is drilled through the centre of a *uniform* sphere of mass M and radius a. Find the gravitational force exerted on a particle of mass m which is inside the hole at a distance r from the centre.

Answer: $\left(\frac{mMG}{a^3}\right)r$.

Q.4 A particle P of mass m moves under the gravitational attraction of a mass M fixed at the origin O. Initially P is at a distance a from O when it is projected with the *critical* escape speed $(2MG/a)^{1/2}$ directly away from O. Find the distance of P from O at time t, and confirm that P escapes to infinity.

Solution

Q.5 If the Earth were suddenly stopped in its orbit, how long would it take for it to collide with the Sun? [Regard the Sun as a *fixed* point mass. You may make use of the formula for the period of the Earth's orbit.]

Solution

Q.6 Show that Newton's laws of motion in a central force field $F = mf(r)\widehat{\boldsymbol{r}}$ reduce to,

$$\ddot{r} - r\dot{\theta}^2 = f(r), \quad r^2\dot{\theta} = L$$

Also that the total energy is,

$$\frac{1}{2}\left(\dot{r}^2 + (r\dot{\theta})^2\right) + V(r) = E$$

Q.7 Instead of a gravitational potential suppose we were to take a hydrogen atom with an attractive Coulomb potential between the proton and electron. In what way would the effective potential,

$$V^*(r) = V(r) + \frac{L^2}{2r^2}$$

change? Discuss bound and unbound electron states in the context of this potential.

Q.8 Derive the path equation $\frac{d^2u}{d\theta^2} + u = -\frac{f(1/u)}{L^2u^2}$ for motion under the central force field, $\boldsymbol{F} = mf(r)\widehat{\boldsymbol{r}}$

Q.9 Write down the equation in Q.8 for a proton that is shot at a nucleus that is so heavy that it may be considered fixed. Consider first the case of a head-on collision and then of a peripheral collision. Though not necessary here, please try solving your equation.