Summary of Lecture 19 – PHYSICS OF SOUND

1. Sound waves correspond to longitudinal oscillations of density. So if sound waves are moving from left to right, as you look along this direction you will find the density of air greater in some places and less in others. Sound waves carry energy. The minimum energy that humans can hear is about 10^{-12} watts per cm^3 (This is called I_0, the threshold of hearing.)

2. To measure the intensity of sound, we use *decibels* as the unit. Decibels (db) are a relative measure to compare the intensity of different sounds with one another,

$$R \equiv \text{relative intensity of sound } I = \log_{10}\frac{I}{I_0} \text{ (decibels)}$$

Typically, on a street without traffic the sound level is about 30db, a pressure horn creates about 90db, and serious ear damage happens around 120db.

3. A sound wave moving in the x direction with speed v is described by

$$\rho(x,t) = \rho_m \sin\frac{2\pi}{\lambda}(x - \text{v}t)$$

where $\rho(x,t)$ is the density of air at a point x at time t. Let us understand various aspects of this formula.

a) Suppose that as time t increases, we move in such a way as to keep $x - \text{v}t$ constant. So if at $t = 0$ the value of x is 0.23 (say), then at $t = 1$ the value of x would be v+0.23, etc. In other words, to keep the density $\rho(x,t)$ constant, we would have to move with the speed of sound , i.e. v.

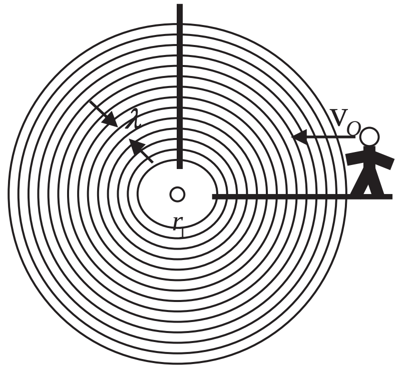
b) In the expression for $\rho(x,t)$, replace x by $x + \lambda$. What happens? Answer: nothing, because $\sin\frac{2\pi}{\lambda}(x + \lambda - \text{v}t) = \sin\frac{2\pi}{\lambda}(x - \text{v}t)$. This is why we call λ the "wavelength", meaning that length after which a wave repeats itself.

c) In the expression for $\rho(x,t)$, replace t by $t + T$ where $T = \lambda / \text{v}$. What happens? Again the answer: nothing. T is called the time period of the sound wave, meaning that time after which it repeats itself. The frequency is the number of cycles per second and is obviously related to T through $\nu = 1/T$.

d) It is also common to introduce the *wavenumber* k the and *angular frequency* ω:

$$k = \frac{2\pi}{\lambda} \quad \text{and} \quad \omega = \frac{2\pi}{T} = 2\pi\nu$$

4. **Doppler Effect**: The relative motion between source and observer causes the observer to receive a frequency that is different from that emitted by the source. One must distinguish between two cases:



Moving observer, source at rest. If the observer was at rest, the number of waves she would receive in time t would be t/T (or $\mathrm{v}t/\lambda$). But if she is moving towards the source with speed v_0 (as in the above figure), the additional number of waves received is obviously $\mathrm{v}_0 t/\lambda$. By definition,

$$\nu' = \text{ frequency actually heard} = \frac{\text{number of waves received}}{\text{unit time}}$$

$$\therefore \nu' = \frac{\frac{\mathrm{v}t}{\lambda} + \frac{\mathrm{v}_0 t}{\lambda}}{t} = \frac{\mathrm{v} + \mathrm{v}_0}{\lambda} = \frac{\mathrm{v} + \mathrm{v}_0}{\mathrm{v}/\nu} = \nu \frac{\mathrm{v} + \mathrm{v}_0}{\mathrm{v}}$$

We finally conclude that the frequency actuall heard is $\nu' = \nu\left(1 + \frac{\mathrm{v}_0}{\mathrm{v}}\right)$. So as the observer runs towards the source, she hears a higher frequency (higher pitch).

Moving source, observer at rest : As the source runs towards the observed, more waves will have to packed together. Each wavelength is reduced by $\frac{\mathrm{v}_s}{\nu}$. So the wavelength seen by the observer is $\lambda' = \frac{\mathrm{v}}{\nu} - \frac{\mathrm{v}_s}{\nu}$. From this, the frequency that she hears is $\nu' = \frac{\mathrm{v}}{\lambda} = \frac{\mathrm{v}}{(\mathrm{v} - \mathrm{v}_s)/\nu} = \nu \frac{\mathrm{v}}{(\mathrm{v} - \mathrm{v}_s)}$.

Moving source and moving observer : $\nu' = \nu \frac{\mathrm{v} + \mathrm{v}_o}{\mathrm{v} - \mathrm{v}_s}$. As you can easily see, the above two results are special cases of this.

QUESTIONS AND EXERCISES – 19

Q.1 A cubical box of side 5 metres is filled uniformly with sound of 40 db intensity. What is the total power that a loudspeaker must have to maintain this intensity?

Q.2 On the same piece of graph paper plot the following functions for θ between 0 and 2π:

a) $\sin\theta$

b) $\frac{1}{2}\sin 2\theta$

c) $\sin\theta + \frac{1}{2}\sin 2\theta$

Q.3 Why are the Doppler shift formulae different for the two cases: moving observer and static source as compared to static observer and moving source? Explain without using any formula. Why is knowing the relative velocity of source and observer not sufficient for knowing the frequency received by the observer?

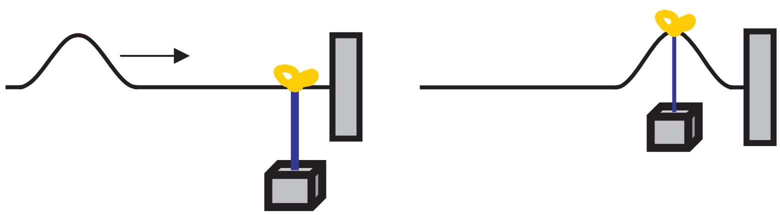
Q.4 Suppose that two trucks are racing towards each other with the same speed. One is blowing its horn with frequency 1000 Hz (cycles per second). The other truck driver hears a frequency of 1050 Hz. How fast were the trucks moving in km/hour before they collided? [The speed of sound in air is about 340 m/sec]

Q.5 In a musical instrument like the sitar, how does the player control the relative amplitude of the harmonics present?

Summary of Lecture 20 – WAVE MOTION

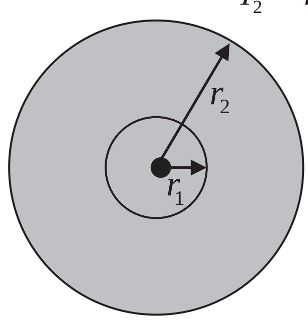
1. Wave motion is any kind of self-repeating (periodic, or oscillatory) motion that transports energy from one point to another. Waves are of two basic kinds:
   (a) **Longitudinal Waves**: the oscillation is parallel to the direction of wave travel.
   Examples: sound, spring, "P-type" earthquake waves.
   (b) **Transverse Waves**: the oscillation is perpendicular to the direction of wave travel.
   Examples: radio or light waves, string, "S-type" earthquake waves.

2. Waves transport energy, not matter. Taking the vibration of a string as an example, each segment of the string stays in the same place, but the work done on the string at one end is transmitted to the other end. Work is done in lifting the mass at the other end below.



3. The height of a wave is called the amplitude. The average power (or intensity) in a wave is proportional to the square of the amplitude. So if $a(t) = a_0 \sin(\omega t - kx)$ is a wave of some kind, then a_0 is the amplitude and $I \propto a_0^2$.

4. A sound source placed at the origin will radiate sound waves in all directions equally. These are called spherical waves. For spherical waves the amplitude $\propto \frac{1}{r}$ and so the power $\propto \frac{1}{r^2}$.
We can easily see why this is so. Consider a source of sound and draw two spheres:
Let P_1 be the total radiated power and I_1 the intensity at r_1, etc. All the power (and energy) that crosses r_1 also crosses r_2 since none is lost in between the two. We have that,
$4\pi r_1^2 I_1 = P_1$ and $4\pi r_2^2 I_2 = P_2$. But $P_1 = P_2 = P$, and so $\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$ or $I \propto \frac{1}{r^2}$.



5. We have encountered waves of the kind $y(x,t) = y_0 \sin(kx - \omega t)$ in the previous lecture. Obviously $y(0,0) = 0$. But what if the wave is not zero at $x = 0,\ t = 0$? Then it could be represented by $y(x,t) = y_m \sin(kx - \omega t - \phi)$, where $kx - \omega t - \phi$ is called the phase and ϕ is called the phase constant. Note that you can rewrite $y(x,t)$ either as,

a) $y(x,t) = y_m \sin\left[k\left(x - \frac{\phi}{k}\right) - \omega t\right]$,

or as, b) $y(x,t) = y_m \sin\left[kx - \omega\left(t + \frac{\phi}{\omega}\right)\right]$.

The two different ways of writing the same expression can be interpreted differently. In (a) x has effectively been shifted to $x - \frac{\phi}{k}$ whereas in (b) t has been shifted to $t + \frac{\phi}{\omega}$. So the phase constant only moves the wave forward or backward in space or time.

6. When two sources are present the total amplitude at any point is the sum of the two separate amplitudes, $y(x,t) = y_1(x,t) + y_2(x,t)$. Now you remember that the power is proportional to the *square* of the amplitude, so $P \propto (y_1 + y_2)^2$. This is why *interference* happens. In the following we shall see why. Just to make things easier, suppose the two waves have equal amplitude. So lets take the two waves to be :

$$y_1(x,t) = y_m \sin(kx - \omega t - \phi_1) \quad \text{and} \quad y_2(x,t) = y_m \sin(kx - \omega t - \phi_2)$$

The total amplitudes is:
$$y(x,t) = y_1(x,t) + y_2(x,t)$$
$$= y_m\left[\sin(kx - \omega t - \phi_1) + \sin(kx - \omega t - \phi_2)\right]$$

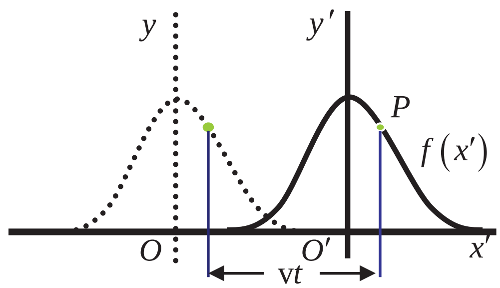
Now use the trigonometric formula, $\sin B + \sin C = 2\sin\frac{1}{2}(B + C) \times \cos(B - C)$ to get,

$$y(x,t) = y_m\left[\sin(kx - \omega t - \phi_1) + \sin(kx - \omega t - \phi_2)\right]$$
$$= \left[2y_m \cos\left(\frac{\Delta\phi}{2}\right)\right] \times \sin(kx - \omega t - \phi').$$

Here $\Delta\phi = \phi_2 - \phi_1$ is the difference of phases, and $\phi' = \frac{(\phi_1 + \phi_2)}{2}$ is the sum. So what do we learn from this? That if $\phi_2 = \phi_1$, then the two waves are in phase and the resultant amplitude is maximum (because $\cos 0 = 1$). And that if $\phi_2 = \phi + \pi$, then the two waves are out of phase and the resultant amplitude is minimum (because $\cos \pi / 2 = 0$). The two waves have interfered with each other and have increased/decreased their amplitude in these two extreme cases. In general $\cos\left(\frac{\Delta\phi}{2}\right)$ will be some number that lies between -1 and +1.

7. There was no time in the lecture to prove it, but you can look up any good book to find a proof for the formaula that the speed of sound in a medium is: $v = \sqrt{\frac{B}{\rho}}$ where B is the bulk modulus and ρ is the density of the medium.

8. **The speed of a pulse.** A pulse is a burst of energy (sound, electromagnetic, heat,...) and could have any shape. Mathematically any pulse has the form $y(x,t) = f(x - vt)$. Here f is any function (e.g. sin, cos, exp,....). Note that at time $t = 0$, $y(x,0) = f(x)$ and the shape would look as on the left in the diagram below. At a late time t, it will look just the same, but shifted to the right. In other words at time t, $y(x,t) = f(x')$ where $x' = x - vt$. Fix your attention on any one point of the curve and follow it as the pulse moves to the right. From $x - vt =$ constant it follows that $\frac{dx}{dt} - v = 0$, or $v = \frac{dx}{dt}$. This is called the *phase velocity* because we derived it using the constancy of phase.



QUESTIONS AND EXERCISES – 20

Q.1 Give a qualitative explanation (no formulas) for why sound travels faster through iron than through air. Can you think of an experiment that would demonstrate this fact?

Q.2 A stone is dropped into a lake and the waves spread out uniformly. Using the same reasoning as in point 4, find:

a) The rate at which the wave intensity decreases with distance r from the centre.

b) The rate of decrease of amplitude.

Q.3 Make a plot of the function $y(x,t) = e^{-\frac{1}{2}(x-2t)^2}$ as a function of x for two different values of t, $t = 0$ and $t = 1$.