Wave Motion

CHAPTER

16

Lifeguards in New South Wales, Australia, practice taking their boat over large water waves breaking near the shore. A wave moving over the surface of water is one example of a mechanical wave. *(Travel Ink/Gallo Images/Getty Images)*

Many of us experienced waves as children when we dropped a pebble into a pond. At the point the pebble hits the water's surface, circular waves are created. These waves move outward from the creation point in expanding circles until they reach the shore. If you were to examine carefully the motion of a small object floating on the disturbed water, you would see that the object moves vertically and horizontally about its original position but does not undergo any net displacement away from or toward the point at which the pebble hit the water. The small elements of water in contact with the object, as well as all the other water elements on the pond's surface, behave in the same way. That is, the water *wave* moves from the point of origin to the shore, but the water is not carried with it.

The world is full of waves, the two main types being *mechanical* waves and *electromagnetic* waves. In the case of mechanical waves, some physical medium is being disturbed; in our pebble example, elements of water are disturbed. Electromagnetic waves do not require a medium to propagate; some examples of electromagnetic waves are visible light, radio waves, television signals, and x-rays. Here, in this part of the book, we study only mechanical waves.

Consider again the small object floating on the water. We have caused the object to move at one point in the water by dropping a pebble at another location. The object has gained kinetic energy from our action, so energy must have transferred from the point at which the pebble is dropped to the position of the object. This feature is central to wave motion: *energy* is transferred over a distance, but *matter* is not.

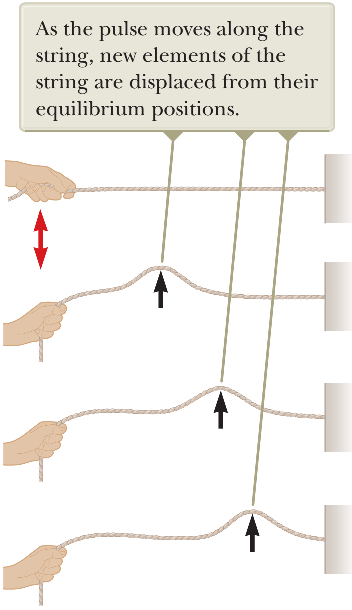


Figure 16.1 A hand moves the end of a stretched string up and down once (red arrow), causing a pulse to travel along the string.

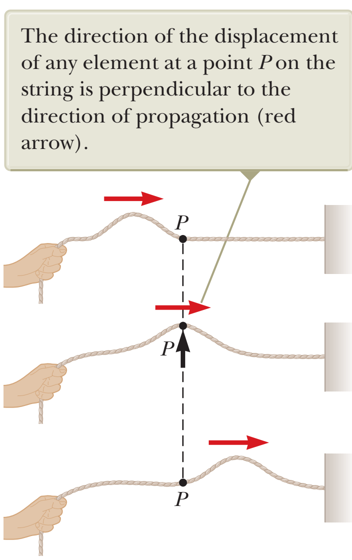


Figure 16.2 The displacement of a particular string element for a transverse pulse traveling on a stretched string.

16.1 Propagation of a Disturbance

The introduction to this chapter alluded to the essence of wave motion: the transfer of energy through space without the accompanying transfer of matter. In the list of energy transfer mechanisms in Chapter 8, two mechanisms—mechanical waves and electromagnetic radiation—depend on waves. By contrast, in another mechanism, matter transfer, the energy transfer is accompanied by a movement of matter through space with no wave character in the process.

All mechanical waves require (1) some source of disturbance, (2) a medium containing elements that can be disturbed, and (3) some physical mechanism through which elements of the medium can influence each other. One way to demonstrate wave motion is to flick one end of a long string that is under tension and has its opposite end fixed as shown in Figure 16.1. In this manner, a single bump (called a *pulse*) is formed and travels along the string with a definite speed. Figure 16.1 represents four consecutive "snapshots" of the creation and propagation of the traveling pulse. The hand is the source of the disturbance. The string is the medium through which the pulse travels—individual elements of the string are disturbed from their equilibrium position. Furthermore, the elements of the string are connected together so they influence each other. The pulse has a definite height and a definite speed of propagation along the medium. The shape of the pulse changes very little as it travels along the string.[1]

We shall first focus on a pulse traveling through a medium. Once we have explored the behavior of a pulse, we will then turn our attention to a *wave,* which is a *periodic* disturbance traveling through a medium. We create a pulse on our string by flicking the end of the string once as in Figure 16.1. If we were to move the end of the string up and down repeatedly, we would create a traveling wave, which has characteristics a pulse does not have. We shall explore these characteristics in Section 16.2.

As the pulse in Figure 16.1 travels, each disturbed element of the string moves in a direction *perpendicular* to the direction of propagation. Figure 16.2 illustrates this point for one particular element, labeled *P*. Notice that no part of the string ever moves in the direction of the propagation. A traveling wave or pulse that causes the elements of the disturbed medium to move perpendicular to the direction of propagation is called a **transverse wave.**

Compare this wave with another type of pulse, one moving down a long, stretched spring as shown in Figure 16.3. The left end of the spring is pushed briefly to the right and then pulled briefly to the left. This movement creates a sudden compression of a region of the coils. The compressed region travels along the spring (to the right in Fig. 16.3). Notice that the direction of the displacement of the coils is *parallel* to the direction of propagation of the compressed region. A traveling wave or pulse that causes the elements of the medium to move parallel to the direction of propagation is called a **longitudinal wave.**

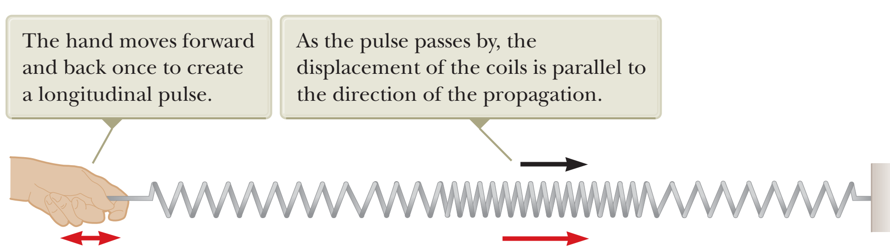


Figure 16.3 A longitudinal pulse along a stretched spring.

[1]In reality, the pulse changes shape and gradually spreads out during the motion. This effect, called *dispersion,* is common to many mechanical waves as well as to electromagnetic waves. We do not consider dispersion in this chapter.

Sound waves, which we shall discuss in Chapter 17, are another example of longitudinal waves. The disturbance in a sound wave is a series of high-pressure and low-pressure regions that travel through air.

Some waves in nature exhibit a combination of transverse and longitudinal displacements. Surface-water waves are a good example. When a water wave travels on the surface of deep water, elements of water at the surface move in nearly circular paths as shown in Figure 16.4. The disturbance has both transverse and longitudinal components. The transverse displacements seen in Figure 16.4 represent the variations in vertical position of the water elements. The longitudinal displacements represent elements of water moving back and forth in a horizontal direction.

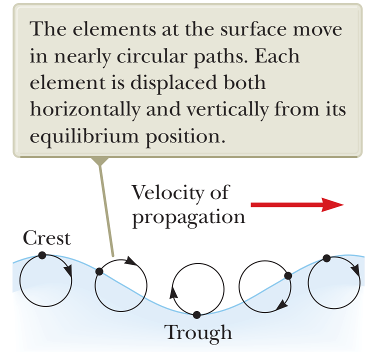


Figure 16.4 The motion of water elements on the surface of deep water in which a wave is propagating is a combination of transverse and longitudinal displacements.

The three-dimensional waves that travel out from a point under the Earth's surface at which an earthquake occurs are of both types, transverse and longitudinal. The longitudinal waves are the faster of the two, traveling at speeds in the range of 7 to 8 km/s near the surface. They are called **P waves,** with "P" standing for *primary,* because they travel faster than the transverse waves and arrive first at a seismograph (a device used to detect waves due to earthquakes). The slower transverse waves, called **S waves,** with "S" standing for *secondary,* travel through the Earth at 4 to 5 km/s near the surface. By recording the time interval between the arrivals of these two types of waves at a seismograph, the distance from the seismograph to the point of origin of the waves can be determined. This distance is the radius of an imaginary sphere centered on the seismograph. The origin of the waves is located somewhere on that sphere. The imaginary spheres from three or more monitoring stations located far apart from one another intersect at one region of the Earth, and this region is where the earthquake occurred.

Consider a pulse traveling to the right on a long string as shown in Figure 16.5. Figure 16.5a represents the shape and position of the pulse at time $t = 0$. At this time, the shape of the pulse, whatever it may be, can be represented by some mathematical function that we will write as $y(x, 0) = f(x)$. This function describes the transverse position y of the element of the string located at each value of x at time $t = 0$. Because the speed of the pulse is v, the pulse has traveled to the right a distance vt at the time t (Fig. 16.5b). We assume the shape of the pulse does not change with time. Therefore, at time t, the shape of the pulse is the same as it was at time $t = 0$ as in Figure 16.5a. Consequently, an element of the string at x at this time has the same y position as an element located at $x - vt$ had at time $t = 0$:

$$y(x, t) = y(x - vt, 0)$$

In general, then, we can represent the transverse position y for all positions and times, measured in a stationary frame with the origin at O, as

$$y(x, t) = f(x - vt) \tag{16.1}$$

Similarly, if the pulse travels to the left, the transverse positions of elements of the string are described by

$$y(x, t) = f(x + vt) \tag{16.2}$$

The function y, sometimes called the **wave function,** depends on the two variables x and t. For this reason, it is often written $y(x, t)$, which is read "y as a function of x and t."

It is important to understand the meaning of y. Consider an element of the string at point P in Figure 16.5, identified by a particular value of its x coordinate. As the pulse passes through P, the y coordinate of this element increases, reaches a maximum, and then decreases to zero. The wave function $y(x, t)$ represents the y coordinate—the transverse position—of any element located at position x at any time t. Furthermore, if t is fixed (as, for example, in the case of taking a snapshot of the pulse), the wave function $y(x)$, sometimes called the **waveform,** defines a curve representing the geometric shape of the pulse at that time.

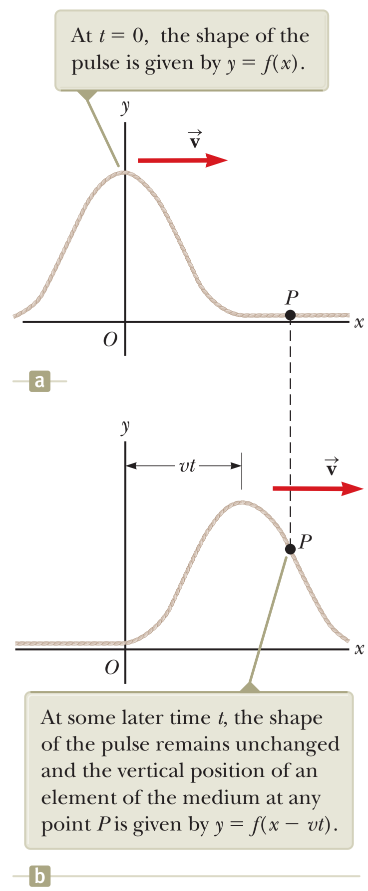


Figure 16.5 A one-dimensional pulse traveling to the right on a string with a speed v.

Quick Quiz 16.1 **(i)** In a long line of people waiting to buy tickets, the first person leaves and a pulse of motion occurs as people step forward to fill the gap. As each person steps forward, the gap moves through the line. Is the propagation of this gap (a) transverse or (b) longitudinal? **(ii)** Consider "the wave" at a baseball game: people stand up and raise their arms as the wave arrives at their location, and the resultant pulse moves around the stadium. Is this wave (a) transverse or (b) longitudinal?

Example 16.1 A Pulse Moving to the Right

A pulse moving to the right along the x axis is represented by the wave function

$$y(x, t) = \frac{2}{(x - 3.0t)^2 + 1}$$

where x and y are measured in centimeters and t is measured in seconds. Find expressions for the wave function at $t = 0$, $t = 1.0$ s, and $t = 2.0$ s.

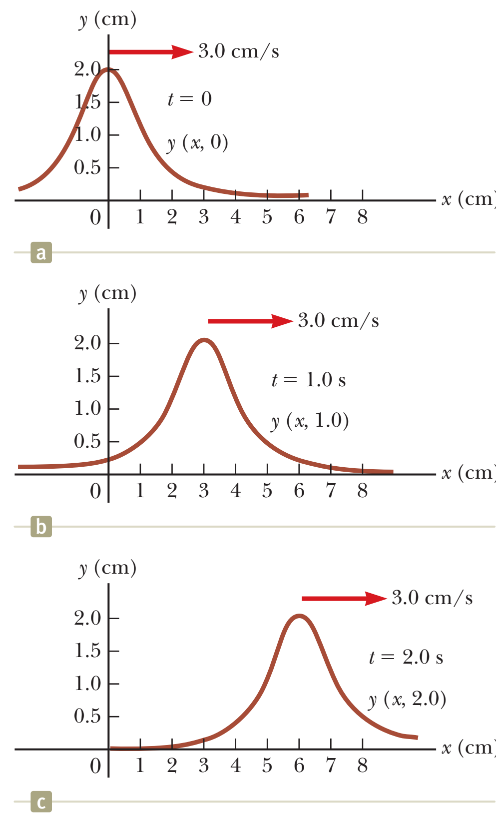


Figure 16.6 (Example 16.1) Graphs of the function $y(x, t) = 2/[(x - 3.0t)^2 + 1]$ at (a) $t = 0$, (b) $t = 1.0$ s, and (c) $t = 2.0$ s.

SOLUTION

Conceptualize Figure 16.6a shows the pulse represented by this wave function at $t = 0$. Imagine this pulse moving to the right at a speed of 3.0 cm/s and maintaining its shape as suggested by Figures 16.6b and 16.6c.

Categorize We categorize this example as a relatively simple analysis problem in which we interpret the mathematical representation of a pulse.

Analyze The wave function is of the form $y = f(x - vt)$. Inspection of the expression for $y(x, t)$ and comparison to Equation 16.1 reveal that the wave speed is $v = 3.0$ cm/s. Furthermore, by letting $x - 3.0t = 0$, we find that the maximum value of y is given by $A = 2.0$ cm.

Write the wave function expression at $t = 0$: $$y(x, 0) = \frac{2}{x^2 + 1}$$

Write the wave function expression at $t = 1.0$ s: $$y(x, 1.0) = \frac{2}{(x - 3.0)^2 + 1}$$

Write the wave function expression at $t = 2.0$ s: $$y(x, 2.0) = \frac{2}{(x - 6.0)^2 + 1}$$

For each of these expressions, we can substitute various values of x and plot the wave function. This procedure yields the wave functions shown in the three parts of Figure 16.6.

Finalize These snapshots show that the pulse moves to the right without changing its shape and that it has a constant speed of 3.0 cm/s.

WHAT IF? What if the wave function were

$$y(x, t) = \frac{4}{(x + 3.0t)^2 + 1}$$

How would that change the situation?

Answer One new feature in this expression is the plus sign in the denominator rather than the minus sign. The new expression represents a pulse with a similar shape as that in Figure 16.6, but moving to the left as time progresses.

16.1 continued

Another new feature here is the numerator of 4 rather than 2. Therefore, the new expression represents a pulse with twice the height of that in Figure 16.6.

16.2 Analysis Model: Traveling Wave

In this section, we introduce an important wave function whose shape is shown in Figure 16.7. The wave represented by this curve is called a **sinusoidal wave** because the curve is the same as that of the function sin θ plotted against θ. A sinusoidal wave could be established on the rope in Figure 16.1 by shaking the end of the rope up and down in simple harmonic motion.

The sinusoidal wave is the simplest example of a periodic continuous wave and can be used to build more complex waves (see Section 18.8). The brown curve in Figure 16.7 represents a snapshot of a traveling sinusoidal wave at $t = 0$, and the blue curve represents a snapshot of the wave at some later time t. Imagine two types of motion that can occur. First, the entire waveform in Figure 16.7 moves to the right so that the brown curve moves toward the right and eventually reaches the position of the blue curve. This movement is the motion of the *wave*. If we focus on one element of the medium, such as the element at $x = 0$, we see that each element moves up and down along the y axis in simple harmonic motion. This movement is the motion of the *elements of the medium*. It is important to differentiate between the motion of the wave and the motion of the elements of the medium.

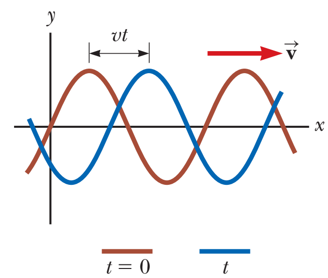


Figure 16.7 A one-dimensional sinusoidal wave traveling to the right with a speed v. The brown curve represents a snapshot of the wave at $t = 0$, and the blue curve represents a snapshot at some later time t.

In the early chapters of this book, we developed several analysis models based on three simplification models: the particle, the system, and the rigid object. With our introduction to waves, we can develop a new simplification model, the **wave,** that will allow us to explore more analysis models for solving problems. An ideal particle has zero size. We can build physical objects with nonzero size as combinations of particles. Therefore, the particle can be considered a basic building block. An ideal wave has a single frequency and is infinitely long; that is, the wave exists throughout the Universe. (A wave of finite length must necessarily have a mixture of frequencies.) When this concept is explored in Section 18.8, we will find that ideal waves can be combined to build complex waves, just as we combined particles.

In what follows, we will develop the principal features and mathematical representations of the analysis model of a **traveling wave.** This model is used in situations in which a wave moves through space without interacting with other waves or particles.

Figure 16.8a shows a snapshot of a traveling wave moving through a medium. Figure 16.8b shows a graph of the position of one element of the medium as a function of time. A point in Figure 16.8a at which the displacement of the element from its normal position is highest is called the **crest** of the wave. The lowest point is called the **trough.** The distance from one crest to the next is called the **wavelength** λ (Greek letter lambda). More generally, the wavelength is the minimum distance between any two identical points on adjacent waves as shown in Figure 16.8a.

If you count the number of seconds between the arrivals of two adjacent crests at a given point in space, you measure the **period** T of the waves. In general, the period is the time interval required for two identical points of adjacent waves to pass by a point as shown in Figure 16.8b. The period of the wave is the same as the period of the simple harmonic oscillation of one element of the medium.

The same information is more often given by the inverse of the period, which is called the **frequency** f. In general, the frequency of a periodic wave is the number of crests (or troughs, or any other point on the wave) that pass a given point in a unit time interval. The frequency of a sinusoidal wave is related to the period by the expression

$$f = \frac{1}{T} \qquad \textbf{(16.3)}$$

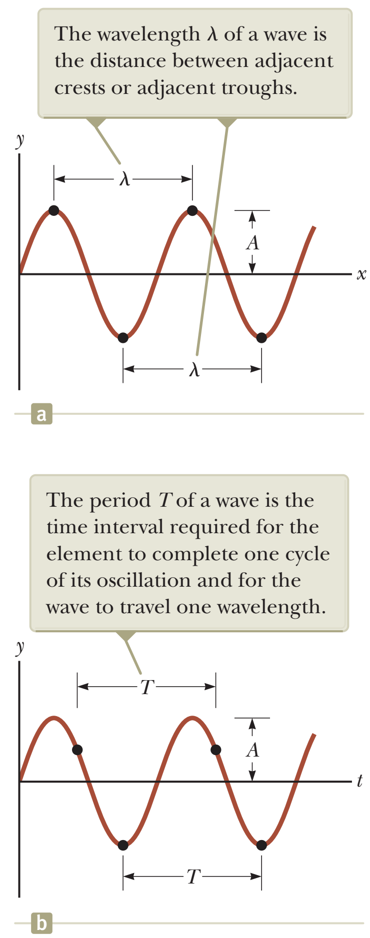


Figure 16.8 (a) A snapshot of a sinusoidal wave. (b) The position of one element of the medium as a function of time.

Pitfall Prevention 16.1

What's the Difference Between Figures 16.8a and 16.8b? Notice the visual similarity between Figures 16.8a and 16.8b. The shapes are the same, but (a) is a graph of vertical position versus horizontal position, whereas (b) is vertical position versus time. Figure 16.8a is a pictorial representation of the wave *for a series of elements of the medium;* it is what you would see at an instant of time. Figure 16.8b is a graphical representation of the position of *one element of the medium* as a function of time. That both figures have the identical shape represents Equation 16.1: a wave is the *same* function of both x and t.

The frequency of the wave is the same as the frequency of the simple harmonic oscillation of one element of the medium. The most common unit for frequency, as we learned in Chapter 15, is s^{-1}, or **hertz** (Hz). The corresponding unit for T is seconds.

The maximum position of an element of the medium relative to its equilibrium position is called the **amplitude** A of the wave as indicated in Figure 16.8.

Waves travel with a specific speed, and this speed depends on the properties of the medium being disturbed. For instance, sound waves travel through room-temperature air with a speed of about 343 m/s (781 mi/h), whereas they travel through most solids with a speed greater than 343 m/s.

Consider the sinusoidal wave in Figure 16.8a, which shows the position of the wave at $t = 0$. Because the wave is sinusoidal, we expect the wave function at this instant to be expressed as $y(x, 0) = A \sin ax$, where A is the amplitude and a is a constant to be determined. At $x = 0$, we see that $y(0, 0) = A \sin a(0) = 0$, consistent with Figure 16.8a. The next value of x for which y is zero is $x = \lambda/2$. Therefore,

$$y\left(\frac{\lambda}{2}, 0\right) = A \sin\left(a\frac{\lambda}{2}\right) = 0$$

For this equation to be true, we must have $a\lambda/2 = \pi$, or $a = 2\pi/\lambda$. Therefore, the function describing the positions of the elements of the medium through which the sinusoidal wave is traveling can be written

$$y(x, 0) = A \sin\left(\frac{2\pi}{\lambda}x\right) \tag{16.4}$$

where the constant A represents the wave amplitude and the constant λ is the wavelength. Notice that the vertical position of an element of the medium is the same whenever x is increased by an integral multiple of λ. Based on our discussion of Equation 16.1, if the wave moves to the right with a speed v, the wave function at some later time t is

$$y(x, t) = A \sin\left[\frac{2\pi}{\lambda}(x - vt)\right] \tag{16.5}$$

If the wave were traveling to the left, the quantity $x - vt$ would be replaced by $x + vt$ as we learned when we developed Equations 16.1 and 16.2.

By definition, the wave travels through a displacement Δx equal to one wavelength λ in a time interval Δt of one period T. Therefore, the wave speed, wavelength, and period are related by the expression

$$v = \frac{\Delta x}{\Delta t} = \frac{\lambda}{T} \tag{16.6}$$

Substituting this expression for v into Equation 16.5 gives

$$y = A \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right] \tag{16.7}$$

This form of the wave function shows the *periodic* nature of y. Note that we will often use y rather than $y(x, t)$ as a shorthand notation. At any given time t, y has the *same* value at the positions x, $x + \lambda$, $x + 2\lambda$, and so on. Furthermore, at any given position x, the value of y is the same at times t, $t + T$, $t + 2T$, and so on.

We can express the wave function in a convenient form by defining two other quantities, the **angular wave number** k (usually called simply the **wave number**) and the **angular frequency** ω:

Angular wave number ▶

$$k \equiv \frac{2\pi}{\lambda} \tag{16.8}$$

Angular frequency ▶

$$\omega \equiv \frac{2\pi}{T} = 2\pi f \tag{16.9}$$

Using these definitions, Equation 16.7 can be written in the more compact form

$$y = A \sin (kx - \omega t) \qquad (16.10)$$

◀ **Wave function for a sinusoidal wave**

Using Equations 16.3, 16.8, and 16.9, the wave speed v originally given in Equation 16.6 can be expressed in the following alternative forms:

$$v = \frac{\omega}{k} \qquad (16.11)$$

$$v = \lambda f \qquad (16.12)$$

◀ **Speed of a sinusoidal wave**

The wave function given by Equation 16.10 assumes the vertical position y of an element of the medium is zero at $x = 0$ and $t = 0$. That need not be the case. If it is not, we generally express the wave function in the form

$$y = A \sin (kx - \omega t + \phi) \qquad (16.13)$$

◀ **General expression for a sinusoidal wave**

where ϕ is the **phase constant,** just as we learned in our study of periodic motion in Chapter 15. This constant can be determined from the initial conditions. The primary equations in the mathematical representation of the traveling wave analysis model are Equations 16.3, 16.10, and 16.12.

Quick Quiz 16.2 A sinusoidal wave of frequency f is traveling along a stretched string. The string is brought to rest, and a second traveling wave of frequency $2f$ is established on the string. **(i)** What is the wave speed of the second wave? (a) twice that of the first wave (b) half that of the first wave (c) the same as that of the first wave (d) impossible to determine **(ii)** From the same choices, describe the wavelength of the second wave. **(iii)** From the same choices, describe the amplitude of the second wave.

Example 16.2 A Traveling Sinusoidal Wave AM

A sinusoidal wave traveling in the positive x direction has an amplitude of 15.0 cm, a wavelength of 40.0 cm, and a frequency of 8.00 Hz. The vertical position of an element of the medium at $t = 0$ and $x = 0$ is also 15.0 cm as shown in Figure 16.9.

(A) Find the wave number k, period T, angular frequency ω, and speed v of the wave.

SOLUTION

Conceptualize Figure 16.9 shows the wave at $t = 0$. Imagine this wave moving to the right and maintaining its shape.

Categorize From the description in the problem statement, we see that we are analyzing a mechanical wave moving through a medium, so we categorize the problem with the *traveling wave* model.

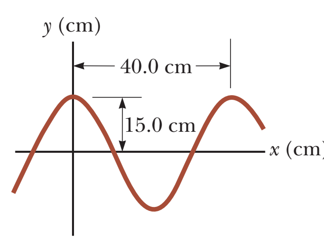


Figure 16.9 (Example 16.2) A sinusoidal wave of wavelength $\lambda = 40.0$ cm and amplitude $A = 15.0$ cm.

Analyze

Evaluate the wave number from Equation 16.8:

$$k = \frac{2\pi}{\lambda} = \frac{2\pi \text{ rad}}{40.0 \text{ cm}} = \boxed{15.7 \text{ rad/m}}$$

Evaluate the period of the wave from Equation 16.3:

$$T = \frac{1}{f} = \frac{1}{8.00 \text{ s}^{-1}} = \boxed{0.125 \text{ s}}$$

Evaluate the angular frequency of the wave from Equation 16.9:

$$\omega = 2\pi f = 2\pi(8.00 \text{ s}^{-1}) = \boxed{50.3 \text{ rad/s}}$$

Evaluate the wave speed from Equation 16.12:

$$v = \lambda f = (40.0 \text{ cm})(8.00 \text{ s}^{-1}) = \boxed{3.20 \text{ m/s}}$$

continued

16.2 continued

(B) Determine the phase constant ϕ and write a general expression for the wave function.

SOLUTION

Substitute $A = 15.0$ cm, $y = 15.0$ cm, $x = 0$, and $t = 0$ into Equation 16.13:

$$15.0 = (15.0)\sin\phi \rightarrow \sin\phi = 1 \rightarrow \phi = \frac{\pi}{2}\text{ rad}$$

Write the wave function:

$$y = A\sin\left(kx - \omega t + \frac{\pi}{2}\right) = A\cos(kx - \omega t)$$

Substitute the values for A, k, and ω in SI units into this expression:

$$y = 0.150\cos(15.7x - 50.3t)$$

Finalize Review the results carefully and make sure you understand them. How would the graph in Figure 16.9 change if the phase angle were zero? How would the graph change if the amplitude were 30.0 cm? How would the graph change if the wavelength were 10.0 cm?

Sinusoidal Waves on Strings

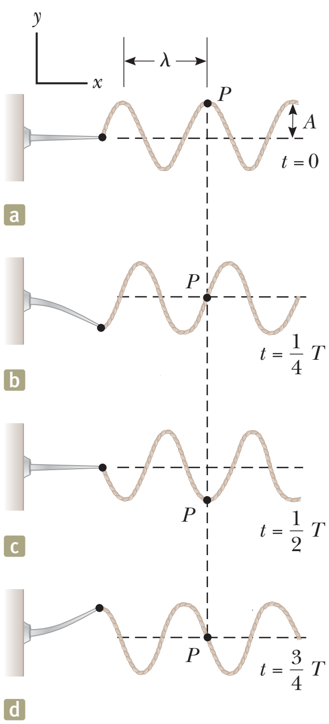


Figure 16.10 One method for producing a sinusoidal wave on a string. The left end of the string is connected to a blade that is set into oscillation. Every element of the string, such as that at point P, oscillates with simple harmonic motion in the vertical direction.

In Figure 16.1, we demonstrated how to create a pulse by jerking a taut string up and down once. To create a series of such pulses—a wave—let's replace the hand with an oscillating blade vibrating in simple harmonic motion. Figure 16.10 represents snapshots of the wave created in this way at intervals of $T/4$. Because the end of the blade oscillates in simple harmonic motion, each element of the string, such as that at P, also oscillates vertically with simple harmonic motion. Therefore, every element of the string can be treated as a simple harmonic oscillator vibrating with a frequency equal to the frequency of oscillation of the blade.[2] Notice that while each element oscillates in the y direction, the wave travels to the right in the $+x$ direction with a speed v. Of course, that is the definition of a transverse wave.

If we define $t = 0$ as the time for which the configuration of the string is as shown in Figure 16.10a, the wave function can be written as

$$y = A\sin(kx - \omega t)$$

We can use this expression to describe the motion of any element of the string. An element at point P (or any other element of the string) moves only vertically, and so its x coordinate remains constant. Therefore, the **transverse speed** v_y (not to be confused with the wave speed v) and the **transverse acceleration** a_y of elements of the string are

$$v_y = \left.\frac{dy}{dt}\right]_{x=\text{constant}} = \frac{\partial y}{\partial t} = -\omega A\cos(kx - \omega t) \qquad \textbf{(16.14)}$$

$$a_y = \left.\frac{dv_y}{dt}\right]_{x=\text{constant}} = \frac{\partial v_y}{\partial t} = -\omega^2 A\sin(kx - \omega t) \qquad \textbf{(16.15)}$$

These expressions incorporate partial derivatives because y depends on both x and t. In the operation $\partial y/\partial t$, for example, we take a derivative with respect to t while holding x constant. The maximum magnitudes of the transverse speed and transverse acceleration are simply the absolute values of the coefficients of the cosine and sine functions:

$$v_{y,\max} = \omega A \qquad \textbf{(16.16)}$$

$$a_{y,\max} = \omega^2 A \qquad \textbf{(16.17)}$$

The transverse speed and transverse acceleration of elements of the string do not reach their maximum values simultaneously. The transverse speed reaches its maximum value (ωA) when $y = 0$, whereas the magnitude of the transverse acceleration

[2]In this arrangement, we are assuming that a string element always oscillates in a vertical line. The tension in the string would vary if an element were allowed to move sideways. Such motion would make the analysis very complex.

reaches its maximum value ($\omega^2 A$) when $y = \pm A$. Finally, Equations 16.16 and 16.17 are identical in mathematical form to the corresponding equations for simple harmonic motion, Equations 15.17 and 15.18.

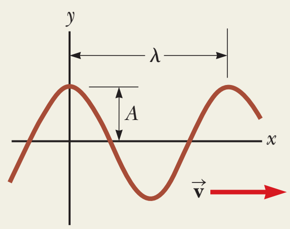
Pitfall Prevention 16.2

Two Kinds of Speed/Velocity Do not confuse v, the speed of the wave as it propagates along the string, with v_y, the transverse velocity of a point on the string. The speed v is constant for a uniform medium, whereas v_y varies sinusoidally.

Quick Quiz 16.3 The amplitude of a wave is doubled, with no other changes made to the wave. As a result of this doubling, which of the following statements is correct? **(a)** The speed of the wave changes. **(b)** The frequency of the wave changes. **(c)** The maximum transverse speed of an element of the medium changes. **(d)** Statements **(a)** through **(c)** are all true. **(e)** None of statements **(a)** through **(c)** is true.

Analysis Model Traveling Wave

Imagine a source vibrating such that it influences the medium that is in contact with the source. Such a source creates a disturbance that propagates through the medium. If the source vibrates in simple harmonic motion with period T, sinusoidal waves propagate through the medium at a speed given by



$$v = \frac{\lambda}{T} = \lambda f \qquad \textbf{(16.6, 16.12)}$$

where λ is the **wavelength** of the wave and f is its **frequency.** A sinusoidal wave can be expressed as

$$y = A \sin (kx - \omega t) \qquad \textbf{(16.10)}$$

where A is the **amplitude** of the wave, k is its **wave number,** and ω is its **angular frequency.**

Examples:

- a vibrating blade sends a sinusoidal wave down a string attached to the blade
- a loudspeaker vibrates back and forth, emitting sound waves into the air (Chapter 17)
- a guitar body vibrates, emitting sound waves into the air (Chapter 18)
- a vibrating electric charge creates an electromagnetic wave that propagates into space at the speed of light (Chapter 34)

16.3 The Speed of Waves on Strings

One aspect of the behavior of *linear* mechanical waves is that the wave speed depends only on the properties of the medium through which the wave travels. Waves for which the amplitude A is small relative to the wavelength λ can be represented as linear waves. (See Section 16.6.) In this section, we determine the speed of a transverse wave traveling on a stretched string.

Let us use a mechanical analysis to derive the expression for the speed of a pulse traveling on a stretched string under tension T. Consider a pulse moving to the right with a uniform speed v, measured relative to a stationary (with respect to the Earth) inertial reference frame as shown in Figure 16.11a. Newton's laws are valid in any inertial reference frame. Therefore, let us view this pulse from a different inertial reference frame, one that moves along with the pulse at the same speed so that the pulse appears to be at rest in the frame as in Figure 16.11b. In this reference frame, the pulse remains fixed and each element of the string moves to the left through the pulse shape.

A short element of the string, of length Δs, forms an approximate arc of a circle of radius R as shown in the magnified view in Figure 16.11b. In our moving frame of reference, the element of the string moves to the left with speed v. As it travels through the arc, we can model the element as a particle in uniform circular motion. This element has a centripetal acceleration of v^2/R, which is supplied by components of the force $\vec{\mathbf{T}}$ whose magnitude is the tension in the string. The force $\vec{\mathbf{T}}$ acts on each side of the element, tangent to the arc, as in Figure 16.11b. The horizontal components of $\vec{\mathbf{T}}$ cancel, and each vertical component $T \sin \theta$ acts downward. Hence, the magnitude of the total radial force on the element is $2T \sin \theta$.

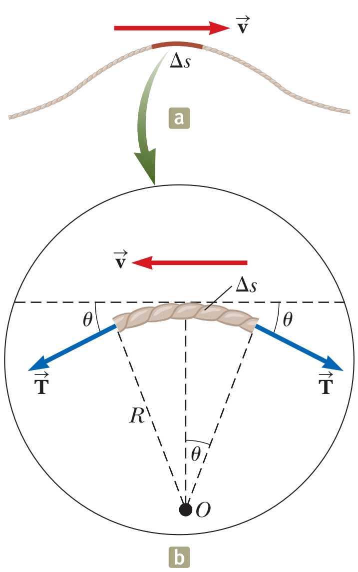


Figure 16.11 (a) In the reference frame of the Earth, a pulse moves to the right on a string with speed v. (b) In a frame of reference moving to the right with the pulse, the small element of length Δs moves to the left with speed v.

Because the element is small, θ is small and we can use the small-angle approximation $\sin\theta \approx \theta$. Therefore, the magnitude of the total radial force is

$$F_r = 2T\sin\theta \approx 2T\theta$$

The element has mass $m = \mu\Delta s$, where μ is the mass per unit length of the string. Because the element forms part of a circle and subtends an angle of 2θ at the center, $\Delta s = R(2\theta)$, and

$$m = \mu\Delta s = 2\mu R\theta$$

The element of the string is modeled as a particle under a net force. Therefore, applying Newton's second law to this element in the radial direction gives

$$F_r = \frac{mv^2}{R} \rightarrow 2T\theta = \frac{2\mu R\theta v^2}{R} \rightarrow T = \mu v^2$$

Solving for v gives

Speed of a wave on a stretched string ▶

$$v = \sqrt{\frac{T}{\mu}} \tag{16.18}$$

Pitfall Prevention 16.3

Multiple T's Do not confuse the T in Equation 16.18 for the tension with the symbol T used in this chapter for the period of a wave. The context of the equation should help you identify which quantity is meant. There simply aren't enough letters in the alphabet to assign a unique letter to each variable!

Notice that this derivation is based on the assumption that the pulse height is small relative to the length of the pulse. Using this assumption, we were able to use the approximation $\sin\theta \approx \theta$. Furthermore, the model assumes that the tension T is not affected by the presence of the pulse, so T is the same at all points on the pulse. Finally, this proof does *not* assume any particular shape for the pulse. We therefore conclude that a pulse of *any shape* will travel on the string with speed $v = \sqrt{T/\mu}$, without any change in pulse shape.

Quick Quiz 16.4 Suppose you create a pulse by moving the free end of a taut string up and down once with your hand beginning at $t = 0$. The string is attached at its other end to a distant wall. The pulse reaches the wall at time t. Which of the following actions, taken by itself, decreases the time interval required for the pulse to reach the wall? More than one choice may be correct. **(a)** moving your hand more quickly, but still only up and down once by the same amount **(b)** moving your hand more slowly, but still only up and down once by the same amount **(c)** moving your hand a greater distance up and down in the same amount of time **(d)** moving your hand a lesser distance up and down in the same amount of time **(e)** using a heavier string of the same length and under the same tension **(f)** using a lighter string of the same length and under the same tension **(g)** using a string of the same linear mass density but under decreased tension **(h)** using a string of the same linear mass density but under increased tension

Example 16.3 The Speed of a Pulse on a Cord AM

A uniform string has a mass of 0.300 kg and a length of 6.00 m (Fig. 16.12). The string passes over a pulley and supports a 2.00-kg object. Find the speed of a pulse traveling along this string.

SOLUTION

Conceptualize In Figure 16.12, the hanging block establishes a tension in the horizontal string. This tension determines the speed with which waves move on the string.

Categorize To find the tension in the string, we model the hanging block as a *particle in equilibrium*. Then we use the tension to evaluate the wave speed on the string using Equation 16.18.

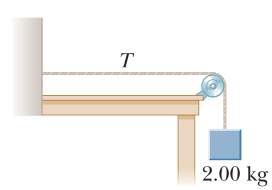


Figure 16.12 (Example 16.3) The tension T in the cord is maintained by the suspended object. The speed of any wave traveling along the cord is given by $v = \sqrt{T/\mu}$.

Analyze Apply the particle in equilibrium model to the block: $\sum F_y = T - m_{\text{block}}g = 0$

Solve for the tension in the string: $T = m_{\text{block}}g$

▶ **16.3** continued

Use Equation 16.18 to find the wave speed, using $\mu = m_{\text{string}}/\ell$ for the linear mass density of the string:

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{m_{\text{block}}\, g\ell}{m_{\text{string}}}}$$

Evaluate the wave speed:

$$v = \sqrt{\frac{(2.00 \text{ kg})(9.80 \text{ m/s}^2)(6.00 \text{ m})}{0.300 \text{ kg}}} = 19.8 \text{ m/s}$$

Finalize The calculation of the tension neglects the small mass of the string. Strictly speaking, the string can never be exactly straight; therefore, the tension is not uniform.

WHAT IF? What if the block were swinging back and forth with respect to the vertical like a pendulum? How would that affect the wave speed on the string?

Answer The swinging block is categorized as a *particle under a net force.* The magnitude of one of the forces on the block is the tension in the string, which determines the wave speed. As the block swings, the tension changes, so the wave speed changes.

When the block is at the bottom of the swing, the string is vertical and the tension is larger than the weight of the block because the net force must be upward to provide the centripetal acceleration of the block. Therefore, the wave speed must be greater than 19.8 m/s.

When the block is at its highest point at the end of a swing, it is momentarily at rest, so there is no centripetal acceleration at that instant. The block is a particle in equilibrium in the radial direction. The tension is balanced by a component of the gravitational force on the block. Therefore, the tension is smaller than the weight and the wave speed is less than 19.8 m/s. With what frequency does the speed of the wave vary? Is it the same frequency as the pendulum?

Example 16.4 Rescuing the Hiker AM

An 80.0-kg hiker is trapped on a mountain ledge following a storm. A helicopter rescues the hiker by hovering above him and lowering a cable to him. The mass of the cable is 8.00 kg, and its length is 15.0 m. A sling of mass 70.0 kg is attached to the end of the cable. The hiker attaches himself to the sling, and the helicopter then accelerates upward. Terrified by hanging from the cable in midair, the hiker tries to signal the pilot by sending transverse pulses up the cable. A pulse takes 0.250 s to travel the length of the cable. What is the acceleration of the helicopter? Assume the tension in the cable is uniform.

SOLUTION

Conceptualize Imagine the effect of the acceleration of the helicopter on the cable. The greater the upward acceleration, the larger the tension in the cable. In turn, the larger the tension, the higher the speed of pulses on the cable.

Categorize This problem is a combination of one involving the speed of pulses on a string and one in which the hiker and sling are modeled as a *particle under a net force.*

Analyze Use the time interval for the pulse to travel from the hiker to the helicopter to find the speed of the pulses on the cable:

$$v = \frac{\Delta x}{\Delta t} = \frac{15.0 \text{ m}}{0.250 \text{ s}} = 60.0 \text{ m/s}$$

Solve Equation 16.18 for the tension in the cable:

$$(1) \quad v = \sqrt{\frac{T}{\mu}} \rightarrow T = \mu v^2$$

Model the hiker and sling as a particle under a net force, noting that the acceleration of this particle of mass m is the same as the acceleration of the helicopter:

$$\sum F = ma \rightarrow T - mg = ma$$

Solve for the acceleration and substitute the tension from Equation (1):

$$a = \frac{T}{m} - g = \frac{\mu v^2}{m} - g = \frac{m_{\text{cable}}\, v^2}{\ell_{\text{cable}}\, m} - g$$

continued

16.4 continued

Substitute numerical values:

$$a = \frac{(8.00\text{ kg})(60.0\text{ m/s})^2}{(15.0\text{ m})(150.0\text{ kg})} - 9.80\text{ m/s}^2 = \boxed{3.00\text{ m/s}^2}$$

Finalize A real cable has stiffness in addition to tension. Stiffness tends to return a wire to its original straight-line shape even when it is not under tension. For example, a piano wire straightens if released from a curved shape; package-wrapping string does not.

Stiffness represents a restoring force in addition to tension and increases the wave speed. Consequently, for a real cable, the speed of 60.0 m/s that we determined is most likely associated with a smaller acceleration of the helicopter.

16.4 Reflection and Transmission

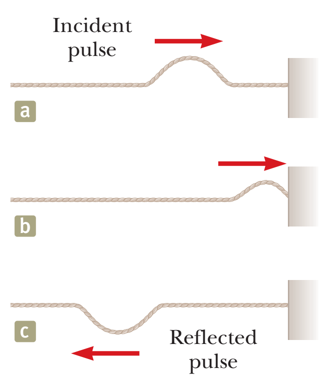


Figure 16.13 The reflection of a traveling pulse at the fixed end of a stretched string. The reflected pulse is inverted, but its shape is otherwise unchanged.

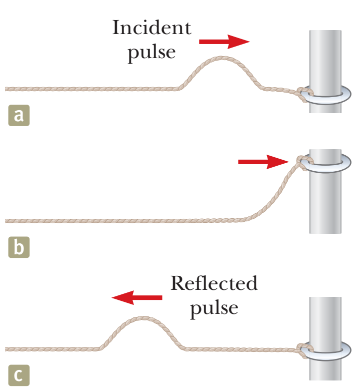


Figure 16.14 The reflection of a traveling pulse at the free end of a stretched string. The reflected pulse is not inverted.

The traveling wave model describes waves traveling through a uniform medium without interacting with anything along the way. We now consider how a traveling wave is affected when it encounters a change in the medium. For example, consider a pulse traveling on a string that is rigidly attached to a support at one end as in Figure 16.13. When the pulse reaches the support, a severe change in the medium occurs: the string ends. As a result, the pulse undergoes **reflection;** that is, the pulse moves back along the string in the opposite direction.

Notice that the reflected pulse is *inverted.* This inversion can be explained as follows. When the pulse reaches the fixed end of the string, the string produces an upward force on the support. By Newton's third law, the support must exert an equal-magnitude and oppositely directed (downward) reaction force on the string. This downward force causes the pulse to invert upon reflection.

Now consider another case. This time, the pulse arrives at the end of a string that is free to move vertically as in Figure 16.14. The tension at the free end is maintained because the string is tied to a ring of negligible mass that is free to slide vertically on a smooth post without friction. Again, the pulse is reflected, but this time it is not inverted. When it reaches the post, the pulse exerts a force on the free end of the string, causing the ring to accelerate upward. The ring rises as high as the incoming pulse, and then the downward component of the tension force pulls the ring back down. This movement of the ring produces a reflected pulse that is not inverted and that has the same amplitude as the incoming pulse.

Finally, consider a situation in which the boundary is intermediate between these two extremes. In this case, part of the energy in the incident pulse is reflected and part undergoes **transmission;** that is, some of the energy passes through the boundary. For instance, suppose a light string is attached to a heavier string as in Figure 16.15. When a pulse traveling on the light string reaches the boundary between the two strings, part of the pulse is reflected and inverted and part is transmitted to the heavier string. The reflected pulse is inverted for the same reasons described earlier in the case of the string rigidly attached to a support.

The reflected pulse has a smaller amplitude than the incident pulse. In Section 16.5, we show that the energy carried by a wave is related to its amplitude. According to the principle of conservation of energy, when the pulse breaks up into a reflected pulse and a transmitted pulse at the boundary, the sum of the energies of these two pulses must equal the energy of the incident pulse. Because the reflected pulse contains only part of the energy of the incident pulse, its amplitude must be smaller.

When a pulse traveling on a heavy string strikes the boundary between the heavy string and a lighter one as in Figure 16.16, again part is reflected and part is transmitted. In this case, the reflected pulse is not inverted.

In either case, the relative heights of the reflected and transmitted pulses depend on the relative densities of the two strings. If the strings are identical, there is no discontinuity at the boundary and no reflection takes place.

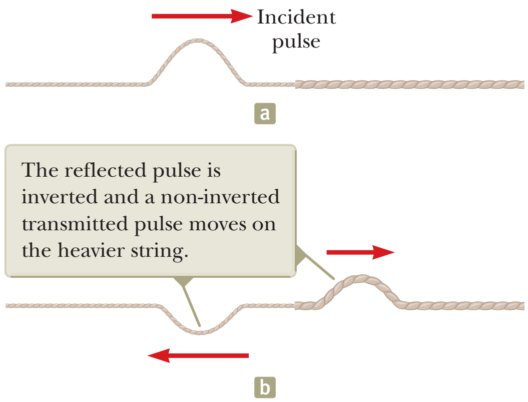


Figure 16.15 (a) A pulse traveling to the right on a light string approaches the junction with a heavier string. (b) The situation after the pulse reaches the junction.

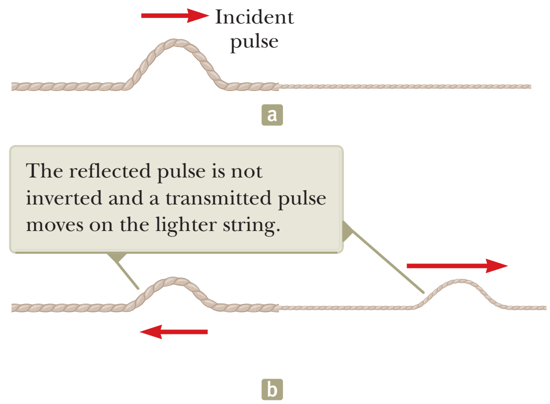


Figure 16.16 (a) A pulse traveling to the right on a heavy string approaches the junction with a lighter string. (b) The situation after the pulse reaches the junction.

According to Equation 16.18, the speed of a wave on a string increases as the mass per unit length of the string decreases. In other words, a wave travels more rapidly on a light string than on a heavy string if both are under the same tension. The following general rules apply to reflected waves: When a wave or pulse travels from medium A to medium B and $v_A > v_B$ (that is, when B is denser than A), it is inverted upon reflection. When a wave or pulse travels from medium A to medium B and $v_A < v_B$ (that is, when A is denser than B), it is not inverted upon reflection.

16.5 Rate of Energy Transfer by Sinusoidal Waves on Strings

Waves transport energy through a medium as they propagate. For example, suppose an object is hanging on a stretched string and a pulse is sent down the string as in Figure 16.17a. When the pulse meets the suspended object, the object is momentarily displaced upward as in Figure 16.17b. In the process, energy is transferred to the object and appears as an increase in the gravitational potential energy of the object–Earth system. This section examines the rate at which energy is transported along a string. We shall assume a one-dimensional sinusoidal wave in the calculation of the energy transferred.

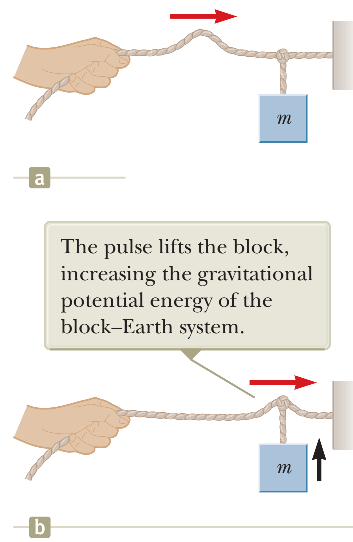


Figure 16.17 (a) A pulse travels to the right on a stretched string, carrying energy with it. (b) The energy of the pulse arrives at the hanging block.

Consider a sinusoidal wave traveling on a string (Fig. 16.18). The source of the energy is some external agent at the left end of the string. We can consider the string to be a nonisolated system. As the external agent performs work on the end of the string, moving it up and down, energy enters the system of the string and propagates along its length. Let's focus our attention on an infinitesimal element of the string of length dx and mass dm. Each such element oscillates vertically with its position described by Equation 15.6. Therefore, we can model each element of the string as a particle in simple harmonic motion, with the oscillation in the y direction. All elements have the same angular frequency ω and the same amplitude A. The kinetic energy K associated with a moving particle is $K = \frac{1}{2}mv^2$. If we apply this equation to the infinitesimal element, the kinetic energy dK associated with the up and down motion of this element is

$$dK = \tfrac{1}{2}(dm)v_y^2$$

where v_y is the transverse speed of the element. If μ is the mass per unit length of the string, the mass dm of the element of length dx is equal to $\mu\ dx$. Hence, we can express the kinetic energy of an element of the string as

$$dK = \tfrac{1}{2}(\mu\ dx)v_y^2 \qquad \textbf{(16.19)}$$

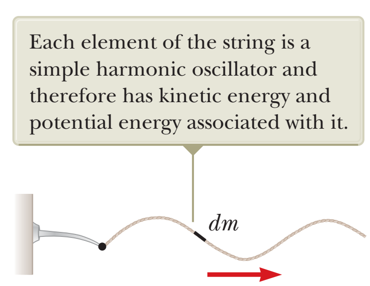


Figure 16.18 A sinusoidal wave traveling along the x axis on a stretched string.

Substituting for the general transverse speed of an element of the medium using Equation 16.14 gives

$$dK = \tfrac{1}{2}\mu[-\omega A \cos(kx - \omega t)]^2\,dx = \tfrac{1}{2}\mu\omega^2A^2\cos^2(kx - \omega t)\,dx$$

If we take a snapshot of the wave at time $t = 0$, the kinetic energy of a given element is

$$dK = \tfrac{1}{2}\mu\omega^2 A^2 \cos^2 kx\,dx$$

Integrating this expression over all the string elements in a wavelength of the wave gives the total kinetic energy K_λ in one wavelength:

$$K_\lambda = \int dK = \int_0^\lambda \tfrac{1}{2}\mu\omega^2A^2\cos^2 kx\,dx = \tfrac{1}{2}\mu\omega^2A^2\int_0^\lambda \cos^2 kx\,dx$$

$$= \tfrac{1}{2}\mu\omega^2A^2\left[\tfrac{1}{2}x + \frac{1}{4k}\sin 2kx\right]_0^\lambda = \tfrac{1}{2}\mu\omega^2A^2\left[\tfrac{1}{2}\lambda\right] = \tfrac{1}{4}\mu\omega^2A^2\lambda$$

In addition to kinetic energy, there is potential energy associated with each element of the string due to its displacement from the equilibrium position and the restoring forces from neighboring elements. A similar analysis to that above for the total potential energy U_λ in one wavelength gives exactly the same result:

$$U_\lambda = \tfrac{1}{4}\mu\omega^2A^2\lambda$$

The total energy in one wavelength of the wave is the sum of the potential and kinetic energies:

$$E_\lambda = U_\lambda + K_\lambda = \tfrac{1}{2}\mu\omega^2A^2\lambda \qquad \textbf{(16.20)}$$

As the wave moves along the string, this amount of energy passes by a given point on the string during a time interval of one period of the oscillation. Therefore, the power P, or rate of energy transfer T_{MW} associated with the mechanical wave, is

$$P = \frac{T_{MW}}{\Delta t} = \frac{E_\lambda}{T} = \frac{\tfrac{1}{2}\mu\omega^2A^2\lambda}{T} = \tfrac{1}{2}\mu\omega^2A^2\left(\frac{\lambda}{T}\right)$$

Power of a wave ▶

$$P = \tfrac{1}{2}\mu\omega^2A^2v \qquad \textbf{(16.21)}$$

Equation 16.21 shows that the rate of energy transfer by a sinusoidal wave on a string is proportional to (a) the square of the frequency, (b) the square of the amplitude, and (c) the wave speed. In fact, the rate of energy transfer in *any* sinusoidal wave is proportional to the square of the angular frequency and to the square of the amplitude.

Quick Quiz 16.5 Which of the following, taken by itself, would be most effective in increasing the rate at which energy is transferred by a wave traveling along a string? **(a)** reducing the linear mass density of the string by one half **(b)** doubling the wavelength of the wave **(c)** doubling the tension in the string **(d)** doubling the amplitude of the wave

Example 16.5 Power Supplied to a Vibrating String

A taut string for which $\mu = 5.00 \times 10^{-2}$ kg/m is under a tension of 80.0 N. How much power must be supplied to the string to generate sinusoidal waves at a frequency of 60.0 Hz and an amplitude of 6.00 cm?

SOLUTION

Conceptualize Consider Figure 16.10 again and notice that the vibrating blade supplies energy to the string at a certain rate. This energy then propagates to the right along the string.

16.5 continued

Categorize We evaluate quantities from equations developed in the chapter, so we categorize this example as a substitution problem.

Use Equation 16.21 to evaluate the power: $P = \frac{1}{2}\mu\omega^2 A^2 v$

Use Equations 16.9 and 16.18 to substitute for ω and v: $P = \frac{1}{2}\mu(2\pi f)^2 A^2\left(\sqrt{\frac{T}{\mu}}\right) = 2\pi^2 f^2 A^2\sqrt{\mu T}$

Substitute numerical values: $P = 2\pi^2(60.0 \text{ Hz})^2(0.060\,0 \text{ m})^2\sqrt{(0.050\,0 \text{ kg/m})(80.0 \text{ N})} = 512 \text{ W}$

WHAT IF? What if the string is to transfer energy at a rate of 1 000 W? What must be the required amplitude if all other parameters remain the same?

Answer Let us set up a ratio of the new and old power, reflecting only a change in the amplitude:

$$\frac{P_{\text{new}}}{P_{\text{old}}} = \frac{\frac{1}{2}\mu\omega^2 A_{\text{new}}^2\, v}{\frac{1}{2}\mu\omega^2 A_{\text{old}}^2\, v} = \frac{A_{\text{new}}^2}{A_{\text{old}}^2}$$

Solving for the new amplitude gives

$$A_{\text{new}} = A_{\text{old}}\sqrt{\frac{P_{\text{new}}}{P_{\text{old}}}} = (6.00 \text{ cm})\sqrt{\frac{1\,000 \text{ W}}{512 \text{ W}}} = 8.39 \text{ cm}$$

16.6 The Linear Wave Equation

In Section 16.1, we introduced the concept of the wave function to represent waves traveling on a string. All wave functions $y(x, t)$ represent solutions of an equation called the *linear wave equation*. This equation gives a complete description of the wave motion, and from it one can derive an expression for the wave speed. Furthermore, the linear wave equation is basic to many forms of wave motion. In this section, we derive this equation as applied to waves on strings.

Suppose a traveling wave is propagating along a string that is under a tension T. Let's consider one small string element of length Δx (Fig. 16.19). The ends of the element make small angles θ_A and θ_B with the x axis. Forces act on the string at its ends where it connects to neighboring elements. Therefore, the element is modeled as a particle under a net force. The net force acting on the element in the vertical direction is

$$\sum F_y = T\sin\theta_B - T\sin\theta_A = T(\sin\theta_B - \sin\theta_A)$$

Because the angles are small, we can use the approximation $\sin\theta \approx \tan\theta$ to express the net force as

$$\sum F_y \approx T(\tan\theta_B - \tan\theta_A) \quad \textbf{(16.22)}$$

Imagine undergoing an infinitesimal displacement outward from the right end of the rope element in Figure 16.19 along the blue line representing the force $\vec{\mathbf{T}}$. This displacement has infinitesimal x and y components and can be represented by the vector $dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}}$. The tangent of the angle with respect to the x axis for this displacement is dy/dx. Because we evaluate this tangent at a particular instant of time, we must express it in partial form as $\partial y/\partial x$. Substituting for the tangents in Equation 16.22 gives

$$\sum F_y \approx T\left[\left(\frac{\partial y}{\partial x}\right)_B - \left(\frac{\partial y}{\partial x}\right)_A\right] \quad \textbf{(16.23)}$$

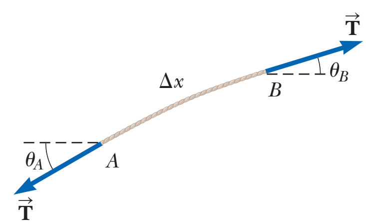


Figure 16.19 An element of a string under tension T.

Now, from the particle under a net force model, let's apply Newton's second law to the element, with the mass of the element given by $m = \mu\,\Delta x$:

$$\sum F_y = ma_y = \mu\,\Delta x\left(\frac{\partial^2 y}{\partial t^2}\right) \tag{16.24}$$

Combining Equation 16.23 with Equation 16.24 gives

$$\mu\,\Delta x\left(\frac{\partial^2 y}{\partial t^2}\right) = T\left[\left(\frac{\partial y}{\partial x}\right)_B - \left(\frac{\partial y}{\partial x}\right)_A\right]$$

$$\frac{\mu}{T}\frac{\partial^2 y}{\partial t^2} = \frac{(\partial y/\partial x)_B - (\partial y/dx)_A}{\Delta x} \tag{16.25}$$

The right side of Equation 16.25 can be expressed in a different form if we note that the partial derivative of any function is defined as

$$\frac{\partial f}{\partial x} \equiv \lim_{\Delta x \to 0}\frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Associating $f(x + \Delta x)$ with $(\partial y/\partial x)_B$ and $f(x)$ with $(\partial y/\partial x)_A$, we see that, in the limit $\Delta x \to 0$, Equation 16.25 becomes

Linear wave equation for a string ▶

$$\frac{\mu}{T}\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2} \tag{16.26}$$

This expression is the linear wave equation as it applies to waves on a string.

The linear wave equation (Eq. 16.26) is often written in the form

Linear wave equation in general ▶

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \tag{16.27}$$

Equation 16.27 applies in general to various types of traveling waves. For waves on strings, y represents the vertical position of elements of the string. For sound waves propagating through a gas, y corresponds to longitudinal position of elements of the gas from equilibrium or variations in either the pressure or the density of the gas. In the case of electromagnetic waves, y corresponds to electric or magnetic field components.

We have shown that the sinusoidal wave function (Eq. 16.10) is one solution of the linear wave equation (Eq. 16.27). Although we do not prove it here, the linear wave equation is satisfied by *any* wave function having the form $y = f(x \pm vt)$. Furthermore, we have seen that the linear wave equation is a direct consequence of the particle under a net force model applied to any element of a string carrying a traveling wave.

Summary

Definitions

A one-dimensional **sinusoidal wave** is one for which the positions of the elements of the medium vary sinusoidally. A sinusoidal wave traveling to the right can be expressed with a **wave function**

$$y(x, t) = A\sin\left[\frac{2\pi}{\lambda}(x - vt)\right] \tag{16.5}$$

where A is the **amplitude**, λ is the **wavelength**, and v is the **wave speed.**

The **angular wave number** k and **angular frequency** ω of a wave are defined as follows:

$$k \equiv \frac{2\pi}{\lambda} \tag{16.8}$$

$$\omega \equiv \frac{2\pi}{T} = 2\pi f \tag{16.9}$$

where T is the **period** of the wave and f is its **frequency.**

A **transverse wave** is one in which the elements of the medium move in a direction *perpendicular* to the direction of propagation.

A **longitudinal wave** is one in which the elements of the medium move in a direction *parallel* to the direction of propagation.

Concepts and Principles

Any one-dimensional wave traveling with a speed v in the x direction can be represented by a wave function of the form

$$y(x, t) = f(x \pm vt) \quad \textbf{(16.1, 16.2)}$$

where the positive sign applies to a wave traveling in the negative x direction and the negative sign applies to a wave traveling in the positive x direction. The shape of the wave at any instant in time (a snapshot of the wave) is obtained by holding t constant.

The speed of a wave traveling on a taut string of mass per unit length μ and tension T is

$$v = \sqrt{\frac{T}{\mu}} \quad \textbf{(16.18)}$$

A wave is totally or partially reflected when it reaches the end of the medium in which it propagates or when it reaches a boundary where its speed changes discontinuously. If a wave traveling on a string meets a fixed end, the wave is reflected and inverted. If the wave reaches a free end, it is reflected but not inverted.

The **power** transmitted by a sinusoidal wave on a stretched string is

$$P = \tfrac{1}{2}\mu\omega^2 A^2 v \quad \textbf{(16.21)}$$

Wave functions are solutions to a differential equation called the **linear wave equation:**

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \quad \textbf{(16.27)}$$

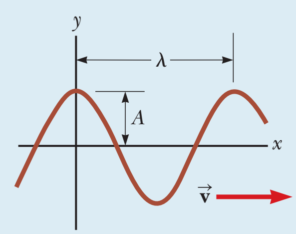
Analysis Model for Problem Solving

Traveling Wave. The wave speed of a sinusoidal wave is

$$v = \frac{\lambda}{T} = \lambda f \quad \textbf{(16.6, 16.12)}$$

A sinusoidal wave can be expressed as

$$y = A \sin(kx - \omega t) \quad \textbf{(16.10)}$$



Objective Questions

1. denotes answer available in *Student Solutions Manual/Study Guide*

1. If one end of a heavy rope is attached to one end of a lightweight rope, a wave can move from the heavy rope into the lighter one. **(i)** What happens to the speed of the wave? (a) It increases. (b) It decreases. (c) It is constant. (d) It changes unpredictably. **(ii)** What happens to the frequency? Choose from the same possibilities. **(iii)** What happens to the wavelength? Choose from the same possibilities.

2. If you stretch a rubber hose and pluck it, you can observe a pulse traveling up and down the hose. **(i)** What happens to the speed of the pulse if you stretch the hose more tightly? (a) It increases. (b) It decreases. (c) It is constant. (d) It changes unpredictably. **(ii)** What happens to the speed if you fill the hose with water? Choose from the same possibilities.

3. Rank the waves represented by the following functions from the largest to the smallest according to **(i)** their amplitudes, **(ii)** their wavelengths, **(iii)** their frequencies, **(iv)** their periods, and **(v)** their speeds. If the values of a quantity are equal for two waves, show them as having equal rank. For all functions, x and y are in meters and t is in seconds. (a) $y = 4 \sin(3x - 15t)$ (b) $y = 6 \cos(3x + 15t - 2)$ (c) $y = 8 \sin(2x + 15t)$ (d) $y = 8 \cos(4x + 20t)$ (e) $y = 7 \sin(6x - 24t)$

4. By what factor would you have to multiply the tension in a stretched string so as to double the wave speed?

Assume the string does not stretch. (a) a factor of 8 (b) a factor of 4 (c) a factor of 2 (d) a factor of 0.5 (e) You could not change the speed by a predictable factor by changing the tension.

5. When all the strings on a guitar (Fig. OQ16.5) are stretched to the same tension, will the speed of a wave along the most massive bass string be (a) faster, (b) slower, or (c) the same as the speed of a wave on the lighter strings? Alternatively, (d) is the speed on the bass string not necessarily any of these answers?

Joseph/Getty Images

Figure OQ16.5

6. Which of the following statements is not necessarily true regarding mechanical waves? (a) They are formed by some source of disturbance. (b) They are sinusoidal in nature. (c) They carry energy. (d) They require a medium through which to propagate. (e) The wave speed depends on the properties of the medium in which they travel.

7. (a) Can a wave on a string move with a wave speed that is greater than the maximum transverse speed $v_{y,\max}$ of an element of the string? (b) Can the wave speed be much greater than the maximum element speed? (c) Can the wave speed be equal to the maximum element speed? (d) Can the wave speed be less than $v_{y,\max}$?

[8.] A source vibrating at constant frequency generates a sinusoidal wave on a string under constant tension. If the power delivered to the string is doubled, by what factor does the amplitude change? (a) a factor of 4 (b) a factor of 2 (c) a factor of $\sqrt{2}$ (d) a factor of 0.707 (e) cannot be predicted

9. The distance between two successive peaks of a sinusoidal wave traveling along a string is 2 m. If the frequency of this wave is 4 Hz, what is the speed of the wave? (a) 4 m/s (b) 1 m/s (c) 8 m/s (d) 2 m/s (e) impossible to answer from the information given

Conceptual Questions

[1.] denotes answer available in *Student Solutions Manual/Study Guide*

1. Why is a solid substance able to transport both longitudinal waves and transverse waves, but a homogeneous fluid is able to transport only longitudinal waves?

[2.] (a) How would you create a longitudinal wave in a stretched spring? (b) Would it be possible to create a transverse wave in a spring?

3. When a pulse travels on a taut string, does it always invert upon reflection? Explain.

4. In mechanics, massless strings are often assumed. Why is that not a good assumption when discussing waves on strings?

5. If you steadily shake one end of a taut rope three times each second, what would be the period of the sinusoidal wave set up in the rope?

6. (a) If a long rope is hung from a ceiling and waves are sent up the rope from its lower end, why does the speed of the waves change as they ascend? (b) Does the speed of the ascending waves increase or decrease? Explain.

7. Why is a pulse on a string considered to be transverse?

8. Does the vertical speed of an element of a horizontal, taut string, through which a wave is traveling, depend on the wave speed? Explain.

9. In an earthquake, both S (transverse) and P (longitudinal) waves propagate from the focus of the earthquake. The focus is in the ground radially below the epicenter on the surface (Fig. CQ16.9). Assume the waves move in straight lines through uniform material. The S waves travel through the Earth more slowly than the P waves (at about 5 km/s versus 8 km/s). By detecting the time of arrival of the waves at a seismograph, (a) how can one determine the distance to the focus of the earthquake? (b) How many detection stations are necessary to locate the focus unambiguously?

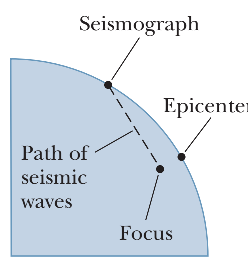


Figure CQ16.9

Problems

The problems found in this chapter may be assigned online in Enhanced WebAssign

1. straightforward; **2.** intermediate; **3.** challenging

[1.] full solution available in the *Student Solutions Manual/Study Guide*

AMT Analysis Model tutorial available in Enhanced WebAssign

GP Guided Problem

M Master It tutorial available in Enhanced WebAssign

W Watch It video solution available in Enhanced WebAssign

Section 16.1 Propagation of a Disturbance

1. W A seismographic station receives S and P waves from an earthquake, separated in time by 17.3 s. Assume the waves have traveled over the same path at speeds of 4.50 km/s and 7.80 km/s. Find the distance from the seismograph to the focus of the quake.

2. Ocean waves with a crest-to-crest distance of 10.0 m can be described by the wave function

$$y(x, t) = 0.800 \sin [0.628(x - vt)]$$

where x and y are in meters, t is in seconds, and $v = 1.20$ m/s. (a) Sketch $y(x, t)$ at $t = 0$. (b) Sketch $y(x, t)$ at $t = 2.00$ s. (c) Compare the graph in part (b) with that for part (a) and explain similarities and differences. (d) How has the wave moved between graph (a) and graph (b)?

3. At $t = 0$, a transverse pulse in a wire is described by the function

$$y = \frac{6.00}{x^2 + 3.00}$$

where x and y are in meters. If the pulse is traveling in the positive x direction with a speed of 4.50 m/s, write the function $y(x, t)$ that describes this pulse.

4. Two points A and B on the surface of the Earth are at the same longitude and 60.0° apart in latitude as shown in Figure P16.4. Suppose an earthquake at point A creates a P wave that reaches point B by traveling straight through the body of the Earth at a constant speed of 7.80 km/s. The earthquake also radiates a Rayleigh wave that travels at 4.50 km/s. In addition to P and S waves, Rayleigh waves are a third type of seismic wave that travels along the *surface* of the Earth rather than through the *bulk* of the Earth. (a) Which of these two seismic waves arrives at B first? (b) What is the time difference between the arrivals of these two waves at B?

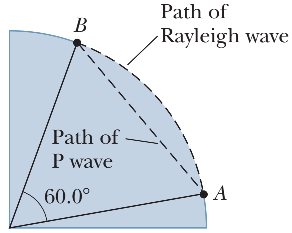


Figure P16.4

Section 16.2 Analysis Model: Traveling Wave

5. M A wave is described by $y = 0.020\,0 \sin (kx - \omega t)$, where $k = 2.11$ rad/m, $\omega = 3.62$ rad/s, x and y are in meters, and t is in seconds. Determine (a) the amplitude, (b) the wavelength, (c) the frequency, and (d) the speed of the wave.

6. A certain uniform string is held under constant tension. (a) Draw a side-view snapshot of a sinusoidal wave on a string as shown in diagrams in the text. (b) Immediately below diagram (a), draw the same wave at a moment later by one-quarter of the period of the wave. (c) Then, draw a wave with an amplitude 1.5 times larger than the wave in diagram (a). (d) Next, draw a wave differing from the one in your diagram (a) just by having a wavelength 1.5 times larger. (e) Finally, draw a wave differing from that in diagram (a) just by having a frequency 1.5 times larger.

7. M A sinusoidal wave is traveling along a rope. The oscillator that generates the wave completes 40.0 vibrations in 30.0 s. A given crest of the wave travels 425 cm along the rope in 10.0 s. What is the wavelength of the wave?

8. For a certain transverse wave, the distance between two successive crests is 1.20 m, and eight crests pass a given point along the direction of travel every 12.0 s. Calculate the wave speed.

9. M The wave function for a traveling wave on a taut string is (in SI units)

$$y(x, t) = 0.350 \sin \left(10\pi t - 3\pi x + \frac{\pi}{4}\right)$$

(a) What are the speed and direction of travel of the wave? (b) What is the vertical position of an element of the string at $t = 0$, $x = 0.100$ m? What are (c) the wavelength and (d) the frequency of the wave? (e) What is the maximum transverse speed of an element of the string?

10. W When a particular wire is vibrating with a frequency of 4.00 Hz, a transverse wave of wavelength 60.0 cm is produced. Determine the speed of waves along the wire.

11. W The string shown in Figure P16.11 is driven at a frequency of 5.00 Hz. The amplitude of the motion is $A = 12.0$ cm, and the wave speed is $v = 20.0$ m/s. Furthermore, the wave is such that $y = 0$ at $x = 0$ and $t = 0$. Determine (a) the angular frequency and (b) the wave number for this wave. (c) Write an expression for the wave function. Calculate (d) the maximum transverse speed and (e) the maximum transverse acceleration of an element of the string.

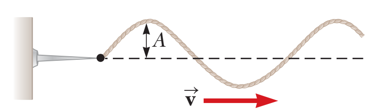


Figure P16.11

12. Consider the sinusoidal wave of Example 16.2 with the wave function

$$y = 0.150 \cos (15.7x - 50.3t)$$

where x and y are in meters and t is in seconds. At a certain instant, let point A be at the origin and point B be the closest point to A along the x axis where the wave is 60.0° out of phase with A. What is the coordinate of B?

13. A sinusoidal wave of wavelength 2.00 m and amplitude 0.100 m travels on a string with a speed of 1.00 m/s to the right. At $t = 0$, the left end of the string is at the origin. For this wave, find (a) the frequency, (b) the angular frequency, (c) the angular wave number, and (d) the wave function in SI units. Determine the equation of motion in SI units for (e) the left end of the string and (f) the point on the string at $x = 1.50$ m to the right of the left end. (g) What is the maximum speed of any element of the string?

14. (a) Plot y versus t at $x = 0$ for a sinusoidal wave of the form $y = 0.150 \cos (15.7x - 50.3t)$, where x and y are in meters and t is in seconds. (b) Determine the period of vibration. (c) State how your result compares with the value found in Example 16.2.

15. W A transverse wave on a string is described by the wave function

$$y = 0.120 \sin\left(\frac{\pi}{8}x + 4\pi t\right)$$

where x and y are in meters and t is in seconds. Determine (a) the transverse speed and (b) the transverse acceleration at $t = 0.200$ s for an element of the string located at $x = 1.60$ m. What are (c) the wavelength, (d) the period, and (e) the speed of propagation of this wave?

16. A wave on a string is described by the wave function $y = 0.100 \sin (0.50x - 20t)$, where x and y are in meters and t is in seconds. (a) Show that an element of the string at $x = 2.00$ m executes harmonic motion. (b) Determine the frequency of oscillation of this particular element.

17. W A sinusoidal wave is described by the wave function $y = 0.25 \sin (0.30x - 40t)$ where x and y are in meters and t is in seconds. Determine for this wave (a) the amplitude, (b) the angular frequency, (c) the angular wave number, (d) the wavelength, (e) the wave speed, and (f) the direction of motion.

18. GP A sinusoidal wave traveling in the negative x direction (to the left) has an amplitude of 20.0 cm, a wavelength of 35.0 cm, and a frequency of 12.0 Hz. The transverse position of an element of the medium at $t = 0$, $x = 0$ is $y = -3.00$ cm, and the element has a positive velocity here. We wish to find an expression for the wave function describing this wave. (a) Sketch the wave at $t = 0$. (b) Find the angular wave number k from the wavelength. (c) Find the period T from the frequency. Find (d) the angular frequency ω and (e) the wave speed v. (f) From the information about $t = 0$, find the phase constant ϕ. (g) Write an expression for the wave function $y(x, t)$.

19. (a) Write the expression for y as a function of x and t in SI units for a sinusoidal wave traveling along a rope in the negative x direction with the following characteristics: $A = 8.00$ cm, $\lambda = 80.0$ cm, $f = 3.00$ Hz, and $y(0, t) = 0$ at $t = 0$. (b) **What If?** Write the expression for y as a function of x and t for the wave in part (a) assuming $y(x, 0) = 0$ at the point $x = 10.0$ cm.

20. A transverse sinusoidal wave on a string has a period $T = 25.0$ ms and travels in the negative x direction with a speed of 30.0 m/s. At $t = 0$, an element of the string at $x = 0$ has a transverse position of 2.00 cm and is traveling downward with a speed of 2.00 m/s. (a) What is the amplitude of the wave? (b) What is the initial phase angle? (c) What is the maximum transverse speed of an element of the string? (d) Write the wave function for the wave.

Section 16.3 The Speed of Waves on Strings

21. Review. The elastic limit of a steel wire is 2.70×10^8 Pa. What is the maximum speed at which transverse wave pulses can propagate along this wire without exceeding this stress? (The density of steel is 7.86×10^3 kg/m^3.)

22. W A piano string having a mass per unit length equal to 5.00×10^{-3} kg/m is under a tension of 1 350 N. Find the speed with which a wave travels on this string.

23. M Transverse waves travel with a speed of 20.0 m/s on a string under a tension of 6.00 N. What tension is required for a wave speed of 30.0 m/s on the same string?

24. A student taking a quiz finds on a reference sheet the two equations

$$f = \frac{1}{T} \quad \text{and} \quad v = \sqrt{\frac{T}{\mu}}$$

She has forgotten what T represents in each equation. (a) Use dimensional analysis to determine the units required for T in each equation. (b) Explain how you can identify the physical quantity each T represents from the units.

25. W An Ethernet cable is 4.00 m long. The cable has a mass of 0.200 kg. A transverse pulse is produced by plucking one end of the taut cable. The pulse makes four trips down and back along the cable in 0.800 s. What is the tension in the cable?

26. A transverse traveling wave on a taut wire has an amplitude of 0.200 mm and a frequency of 500 Hz. It travels with a speed of 196 m/s. (a) Write an equation in SI units of the form $y = A \sin (kx - \omega t)$ for this wave. (b) The mass per unit length of this wire is 4.10 g/m. Find the tension in the wire.

27. AMT M A steel wire of length 30.0 m and a copper wire of length 20.0 m, both with 1.00-mm diameters, are connected end to end and stretched to a tension of 150 N. During what time interval will a transverse wave travel the entire length of the two wires?

28. *Why is the following situation impossible?* An astronaut on the Moon is studying wave motion using the apparatus discussed in Example 16.3 and shown in Figure 16.12. He measures the time interval for pulses to travel along the horizontal wire. Assume the horizontal wire has a mass of 4.00 g and a length of 1.60 m and assume a 3.00-kg object is suspended from its extension around the pulley. The astronaut finds that a pulse requires 26.1 ms to traverse the length of the wire.

29. AMT Tension is maintained in a string as in Figure P16.29. The observed wave speed is $v = 24.0$ m/s when the suspended mass is $m = 3.00$ kg. (a) What is the mass per unit length of the string? (b) What is the wave speed when the suspended mass is $m = 2.00$ kg?

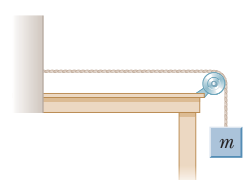


Figure P16.29 Problems 29 and 47.

30. Review. A light string with a mass per unit length of 8.00 g/m has its ends tied to two walls separated by a distance equal to three-fourths the length of the string (Fig. P16.30, p. 503). An object of mass m is suspended from the center of the string, putting a tension in the string. (a) Find an expression for the transverse wave speed in the string as a function of the mass of the hanging object. (b) What should be the mass of the object suspended from the string if the wave speed is to be 60.0 m/s?

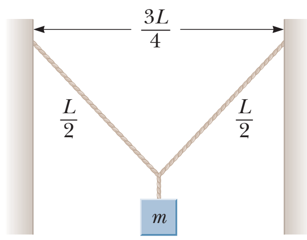


Figure P16.30

31. W Transverse pulses travel with a speed of 200 m/s along a taut copper wire whose diameter is 1.50 mm. What is the tension in the wire? (The density of copper is 8.92 g/cm^3.)

Section 16.5 Rate of Energy Transfer by Sinusoidal Waves on Strings

32. In a region far from the epicenter of an earthquake, a seismic wave can be modeled as transporting energy in a single direction without absorption, just as a string wave does. Suppose the seismic wave moves from granite into mudfill with similar density but with a much smaller bulk modulus. Assume the speed of the wave gradually drops by a factor of 25.0, with negligible reflection of the wave. (a) Explain whether the amplitude of the ground shaking will increase or decrease. (b) Does it change by a predictable factor? (This phenomenon led to the collapse of part of the Nimitz Freeway in Oakland, California, during the Loma Prieta earthquake of 1989.)

33. Transverse waves are being generated on a rope under constant tension. By what factor is the required power increased or decreased if (a) the length of the rope is doubled and the angular frequency remains constant, (b) the amplitude is doubled and the angular frequency is halved, (c) both the wavelength and the amplitude are doubled, and (d) both the length of the rope and the wavelength are halved?

34. M Sinusoidal waves 5.00 cm in amplitude are to be transmitted along a string that has a linear mass density of 4.00×10^{-2} kg/m. The source can deliver a maximum power of 300 W, and the string is under a tension of 100 N. What is the highest frequency f at which the source can operate?

35. M A sinusoidal wave on a string is described by the wave function

$$y = 0.15 \sin (0.80x - 50t)$$

where x and y are in meters and t is in seconds. The mass per unit length of this string is 12.0 g/m. Determine (a) the speed of the wave, (b) the wavelength, (c) the frequency, and (d) the power transmitted by the wave.

36. W A taut rope has a mass of 0.180 kg and a length of 3.60 m. What power must be supplied to the rope so as to generate sinusoidal waves having an amplitude of 0.100 m and a wavelength of 0.500 m and traveling with a speed of 30.0 m/s?

37. AMT A long string carries a wave; a 6.00-m segment of the string contains four complete wavelengths and has a mass of 180 g. The string vibrates sinusoidally with a frequency of 50.0 Hz and a peak-to-valley displacement of 15.0 cm. (The "peak-to-valley" distance is the vertical distance from the farthest positive position to the farthest negative position.) (a) Write the function that describes this wave traveling in the positive x direction. (b) Determine the power being supplied to the string.

38. A horizontal string can transmit a maximum power P_0 (without breaking) if a wave with amplitude A and angular frequency ω is traveling along it. To increase this maximum power, a student folds the string and uses this "double string" as a medium. Assuming the tension in the two strands together is the same as the original tension in the single string and the angular frequency of the wave remains the same, determine the maximum power that can be transmitted along the "double string."

39. The wave function for a wave on a taut string is

$$y(x, t) = 0.350 \sin \left(10\pi t - 3\pi x + \frac{\pi}{4}\right)$$

where x and y are in meters and t is in seconds. If the linear mass density of the string is 75.0 g/m, (a) what is the average rate at which energy is transmitted along the string? (b) What is the energy contained in each cycle of the wave?

40. A two-dimensional water wave spreads in circular ripples. Show that the amplitude A at a distance r from the initial disturbance is proportional to $1/\sqrt{r}$. *Suggestion:* Consider the energy carried by one outward-moving ripple.

Section 16.6 The Linear Wave Equation

41. Show that the wave function $y = \ln [b(x - vt)]$ is a solution to Equation 16.27, where b is a constant.

42. (a) Evaluate A in the scalar equality $4\ (7 + 3) = A$. (b) Evaluate A, B, and C in the vector equality $700\,\hat{\mathbf{i}} + 3.00\,\hat{\mathbf{k}} = A\hat{\mathbf{i}} + B\hat{\mathbf{j}} + C\hat{\mathbf{k}}$. (c) Explain how you arrive at the answers to convince a student who thinks that you cannot solve a single equation for three different unknowns. (d) **What If?** The functional equality or identity

$$A + B \cos (Cx + Dt + E) = 7.00 \cos (3x + 4t + 2)$$

is true for all values of the variables x and t, measured in meters and in seconds, respectively. Evaluate the constants A, B, C, D, and E. (e) Explain how you arrive at your answers to part (d).

43. Show that the wave function $y = e^{b(x-vt)}$ is a solution of the linear wave equation (Eq. 16.27), where b is a constant.

44. (a) Show that the function $y(x, t) = x^2 + v^2t^2$ is a solution to the wave equation. (b) Show that the function in part (a) can be written as $f(x + vt) + g(x - vt)$ and determine the functional forms for f and g. (c) **What If?** Repeat parts (a) and (b) for the function $y(x, t) = \sin (x) \cos (vt)$.

Additional Problems

45. Motion-picture film is projected at a frequency of 24.0 frames per second. Each photograph on the film is the same height of 19.0 mm, just like each oscillation in a wave is the same length. Model the height of a frame as the wavelength of a wave. At what constant speed does the film pass into the projector?

46. "The wave" is a particular type of pulse that can propagate through a large crowd gathered at a sports arena (Fig. P16.46). The elements of the medium are the spectators, with zero position corresponding to their being seated and maximum position corresponding to their standing and raising their arms. When a large fraction of the spectators participates in the wave motion, a somewhat stable pulse shape can develop. The wave speed depends on people's reaction time, which is typically on the order of 0.1 s. Estimate the order of magnitude, in minutes, of the time interval required for such a pulse to make one circuit around a large sports stadium. State the quantities you measure or estimate and their values.

Joe Klamar/AFP/Getty Images

Figure P16.46

47. A sinusoidal wave in a rope is described by the wave function

$$y = 0.20 \sin (0.75\pi x + 18\pi t)$$

where x and y are in meters and t is in seconds. The rope has a linear mass density of 0.250 kg/m. The tension in the rope is provided by an arrangement like the one illustrated in Figure P16.29. What is the mass of the suspended object?

48. The ocean floor is underlain by a layer of basalt that constitutes the crust, or uppermost layer, of the Earth in that region. Below this crust is found denser periodotite rock that forms the Earth's mantle. The boundary between these two layers is called the Mohorovicic discontinuity ("Moho" for short). If an explosive charge is set off at the surface of the basalt, it generates a seismic wave that is reflected back out at the Moho. If the speed of this wave in basalt is 6.50 km/s and the two-way travel time is 1.85 s, what is the thickness of this oceanic crust?

49. Review. A 2.00-kg block hangs from a rubber cord, being supported so that the cord is not stretched. The unstretched length of the cord is 0.500 m, and its mass is 5.00 g. The "spring constant" for the cord is 100 N/m. The block is released and stops momentarily at the lowest point. (a) Determine the tension in the cord when the block is at this lowest point. (b) What is the length of the cord in this "stretched" position? (c) If the block is held in this lowest position, find the speed of a transverse wave in the cord.

50. Review. A block of mass M hangs from a rubber cord. The block is supported so that the cord is not stretched. The unstretched length of the cord is L_0, and its mass is m, much less than M. The "spring constant" for the cord is k. The block is released and stops momentarily at the lowest point. (a) Determine the tension in the string when the block is at this lowest point. (b) What is the length of the cord in this "stretched" position? (c) If the block is held in this lowest position, find the speed of a transverse wave in the cord.

51. A transverse wave on a string is described by the wave function

$$y(x, t) = 0.350 \sin (1.25x + 99.6t)$$

where x and y are in meters and t is in seconds. Consider the element of the string at $x = 0$. (a) What is the time interval between the first two instants when this element has a position of $y = 0.175$ m? (b) What distance does the wave travel during the time interval found in part (a)?

52. A sinusoidal wave in a string is described by the wave function

$$y = 0.150 \sin (0.800x - 50.0t)$$

where x and y are in meters and t is in seconds. The mass per length of the string is 12.0 g/m. (a) Find the maximum transverse acceleration of an element of this string. (b) Determine the maximum transverse force on a 1.00-cm segment of the string. (c) State how the force found in part (b) compares with the tension in the string.

53. Review. A block of mass M, supported by a string, rests on a frictionless incline making an angle θ with the horizontal (Fig. P16.53). The length of the string is L, and its mass is $m << M$. Derive an expression for the time interval required for a transverse wave to travel from one end of the string to the other.

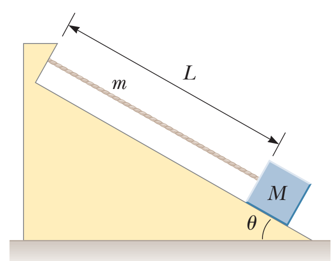


Figure P16.53

54. An undersea earthquake or a landslide can produce an ocean wave of short duration carrying great energy, called a tsunami. When its wavelength is large compared to the ocean depth d, the speed of a water wave is given approximately by $v = \sqrt{gd}$. Assume an earthquake occurs all along a tectonic plate boundary running north to south and produces a straight tsunami wave crest moving everywhere to the west. (a) What physical quantity can you consider to be constant in the motion of any one wave crest? (b) Explain why the amplitude of the wave increases as the wave approaches shore. (c) If the wave has amplitude 1.80 m when its speed is 200 m/s, what will be its amplitude where the water is 9.00 m deep? (d) Explain why the amplitude at the shore should be expected to be still greater, but cannot be meaningfully predicted by your model.

55. AMT **Review.** A block of mass $M = 0.450$ kg is attached to one end of a cord of mass 0.003 20 kg; the other end of the cord is attached to a fixed point. The block rotates with constant angular speed in a circle on a frictionless, horizontal table as shown in Figure P16.55. Through what angle does the block rotate in the time interval during which a transverse wave travels along the string from the center of the circle to the block?

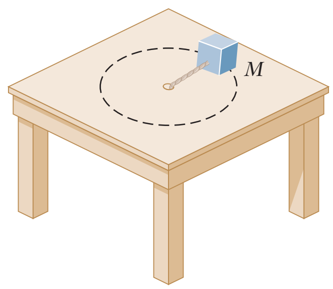


Figure P16.55 Problems 55, 56, and 57.

56. **Review.** A block of mass $M = 0.450$ kg is attached to one end of a cord of mass $m = 0.003\ 20$ kg; the other end of the cord is attached to a fixed point. The block rotates with constant angular speed $\omega = 10.0$ rad/s in a circle on a frictionless, horizontal table as shown in Figure P16.55. What time interval is required for a transverse wave to travel along the string from the center of the circle to the block?

57. **Review.** A block of mass M is attached to one end of a cord of mass m; the other end of the cord is attached to a fixed point. The block rotates with constant angular speed ω in a circle on a frictionless, horizontal table as shown in Figure P16.55. What time interval is required for a transverse wave to travel along the string from the center of the circle to the block?

58. A string with linear density 0.500 g/m is held under tension 20.0 N. As a transverse sinusoidal wave propagates on the string, elements of the string move with maximum speed $v_{y,\max}$. (a) Determine the power transmitted by the wave as a function of $v_{y,\max}$. (b) State in words the proportionality between power and $v_{y,\max}$. (c) Find the energy contained in a section of string 3.00 m long as a function of $v_{y,\max}$. (d) Express the answer to part (c) in terms of the mass m of this section. (e) Find the energy that the wave carries past a point in 6.00 s.

59. A wire of density ρ is tapered so that its cross-sectional area varies with x according to

$$A = 1.00 \times 10^{-5}\ x + 1.00 \times 10^{-6}$$

where A is in meters squared and x is in meters. The tension in the wire is T. (a) Derive a relationship for the speed of a wave as a function of position. (b) **What If?** Assume the wire is aluminum and is under a tension $T = 24.0$ N. Determine the wave speed at the origin and at $x = 10.0$ m.

60. A rope of total mass m and length L is suspended vertically. Analysis shows that for short transverse pulses, the waves above a short distance from the free end of the rope can be represented to a good approximation by the linear wave equation discussed in Section 16.6. Show that a transverse pulse travels the length of the rope in a time interval that is given approximately by $\Delta t \approx 2\sqrt{L/g}$. *Suggestion:* First find an expression for the wave speed at any point a distance x from the lower end by considering the rope's tension as resulting from the weight of the segment below that point.

61. A pulse traveling along a string of linear mass density μ is described by the wave function

$$y = [A_0 e^{-bx}] \sin (kx - \omega t)$$

where the factor in brackets is said to be the amplitude. (a) What is the power $P(x)$ carried by this wave at a point x? (b) What is the power $P(0)$ carried by this wave at the origin? (c) Compute the ratio $P(x)/P(0)$.

62. *Why is the following situation impossible?* Tsunamis are ocean surface waves that have enormous wavelengths (100 to 200 km), and the propagation speed for these waves is $v \approx \sqrt{g d_{\text{avg}}}$, where d_{avg} is the average depth of the water. An earthquake on the ocean floor in the Gulf of Alaska produces a tsunami that reaches Hilo, Hawaii, 4 450 km away, in a time interval of 5.88 h. (This method was used in 1856 to estimate the average depth of the Pacific Ocean long before soundings were made to give a direct determination.)

63. M **Review.** An aluminum wire is held between two clamps under zero tension at room temperature. Reducing the temperature, which results in a decrease in the wire's equilibrium length, increases the tension in the wire. Taking the cross-sectional area of the wire to be $5.00 \times 10^{-6}\ \text{m}^2$, the density to be $2.70 \times 10^3\ \text{kg/m}^3$, and Young's modulus to be $7.00 \times 10^{10}\ \text{N/m}^2$, what strain $(\Delta L/L)$ results in a transverse wave speed of 100 m/s?

Challenge Problems

64. Assume an object of mass M is suspended from the bottom of the rope of mass m and length L in Problem 60. (a) Show that the time interval for a transverse pulse to travel the length of the rope is

$$\Delta t = 2\sqrt{\frac{L}{mg}}\left(\sqrt{M + m} - \sqrt{M}\right)$$

(b) **What If?** Show that the expression in part (a) reduces to the result of Problem 60 when $M = 0$. (c) Show that for $m << M$, the expression in part (a) reduces to

$$\Delta t = \sqrt{\frac{mL}{Mg}}$$

65. A rope of total mass m and length L is suspended vertically. As shown in Problem 60, a pulse travels from the bottom to the top of the rope in an approximate time interval $\Delta t = 2\sqrt{L/g}$ with a speed that varies with position x measured from the bottom of the rope as $v = \sqrt{gx}$. Assume the linear wave equation in Section 16.6 describes waves at all locations on the rope. (a) Over what time interval does a pulse travel halfway up the rope? Give your answer as a fraction of the quantity $2\sqrt{L/g}$. (b) A pulse starts traveling up the rope. How far has it traveled after a time interval $\sqrt{L/g}$?

66. A string on a musical instrument is held under tension T and extends from the point $x = 0$ to the point $x = L$. The string is overwound with wire in such a way that its mass per unit length $\mu(x)$ increases uniformly from μ_0 at $x = 0$ to μ_L at $x = L$. (a) Find an expression for $\mu(x)$ as a function of x over the range $0 \le x \le L$. (b) Find an expression for the time interval required for a transverse pulse to travel the length of the string.

67. If a loop of chain is spun at high speed, it can roll along the ground like a circular hoop without collapsing. Consider a chain of uniform linear mass density μ whose center of mass travels to the right at a high speed v_0 as shown in Figure P16.67. (a) Determine the tension in the chain in terms of μ and v_0. Assume the weight of an individual link is negligible compared to the tension. (b) If the loop rolls over a small bump, the resulting deformation of the chain causes two transverse pulses to propagate along the chain, one moving clockwise and one moving counterclockwise. What is the speed of the pulses traveling along the chain? (c) Through what angle does each pulse travel during the time interval over which the loop makes one revolution?

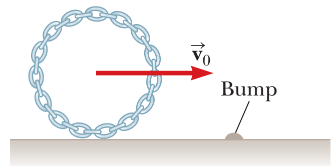


Figure P16.67

Sound Waves

CHAPTER

17

Three musicians play the alpenhorn in Valais, Switzerland. In this chapter, we explore the behavior of sound waves such as those coming from these large musical instruments. *(Stefano Cellai/AGE fotostock)*

Most of the waves we studied in Chapter 16 are constrained to move along a one-dimensional medium. For example, the wave in Figure 16.7 is a purely mathematical construct moving along the x axis. The wave in Figure 16.10 is constrained to move along the length of the string. We have also seen waves moving through a two-dimensional medium, such as the ripples on the water surface in the introduction to Part 2 on page 449 and the waves moving over the surface of the ocean in Figure 16.4. In this chapter, we investigate mechanical waves that move through three-dimensional bulk media. For example, seismic waves leaving the focus of an earthquake travel through the three-dimensional interior of the Earth.

We will focus our attention on **sound waves,** which travel through any material, but are most commonly experienced as the mechanical waves traveling through air that result in the human perception of hearing. As sound waves travel through air, elements of air are disturbed from their equilibrium positions. Accompanying these movements are changes in density and pressure of the air along the direction of wave motion. If the source of the sound waves vibrates sinusoidally, the density and pressure variations are also sinusoidal. The mathematical description of sinusoidal sound waves is very similar to that of sinusoidal waves on strings, as discussed in Chapter 16.

Sound waves are divided into three categories that cover different frequency ranges.
(1) *Audible waves* lie within the range of sensitivity of the human ear. They can be generated in a variety of ways, such as by musical instruments, human voices, or loudspeakers.
(2) *Infrasonic waves* have frequencies below the audible range. Elephants can use infrasonic waves to communicate with one another, even when separated by many kilometers.
(3) *Ultrasonic waves* have frequencies above the audible range. You may have used a "silent" whistle to retrieve your dog. Dogs easily hear the ultrasonic sound this whistle emits, although humans cannot detect it at all. Ultrasonic waves are also used in medical imaging.

This chapter begins with a discussion of the pressure variations in a sound wave, the speed of sound waves, and wave intensity, which is a function of wave amplitude. We then provide an alternative description of the intensity of sound waves that compresses the wide range of intensities to which the ear is sensitive into a smaller range for convenience. The effects of the motion of sources and listeners on the frequency of a sound are also investigated.

17.1 Pressure Variations in Sound Waves

In Chapter 16, we began our investigation of waves by imagining the creation of a single pulse that traveled down a string (Figure 16.1) or a spring (Figure 16.3). Let's do something similar for sound. We describe pictorially the motion of a one-dimensional longitudinal sound pulse moving through a long tube containing a compressible gas as shown in Figure 17.1. A piston at the left end can be quickly moved to the right to compress the gas and create the pulse. Before the piston is moved, the gas is undisturbed and of uniform density as represented by the uniformly shaded region in Figure 17.1a. When the piston is pushed to the right (Fig. 17.1b), the gas just in front of it is compressed (as represented by the more heavily shaded region); the pressure and density in this region are now higher than they were before the piston moved. When the piston comes to rest (Fig. 17.1c), the compressed region of the gas continues to move to the right, corresponding to a longitudinal pulse traveling through the tube with speed v.

One can produce a one-dimensional *periodic* sound wave in the tube of gas in Figure 17.1 by causing the piston to move in simple harmonic motion. The results are shown in Figure 17.2. The darker parts of the colored areas in this figure represent regions in which the gas is compressed and the density and pressure are above their equilibrium values. A compressed region is formed whenever the pis-

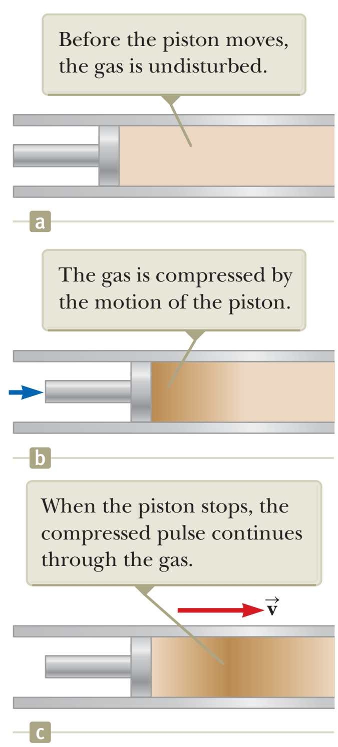


Figure 17.1 Motion of a longitudinal pulse through a compressible gas. The compression (darker region) is produced by the moving piston.

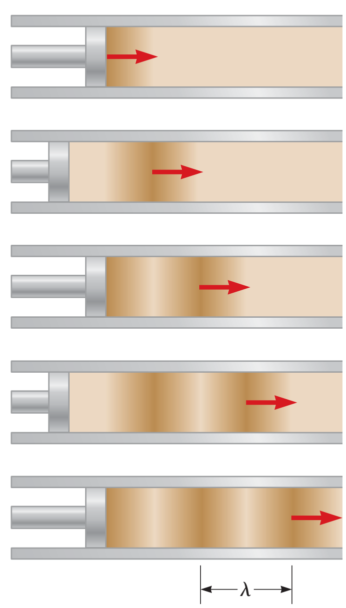


Figure 17.2 A longitudinal wave propagating through a gas-filled tube. The source of the wave is an oscillating piston at the left.

ton is pushed into the tube. This compressed region, called a **compression,** moves through the tube, continuously compressing the region just in front of itself. When the piston is pulled back, the gas in front of it expands and the pressure and density in this region fall below their equilibrium values (represented by the lighter parts of the colored areas in Fig. 17.2). These low-pressure regions, called **rarefactions,** also propagate along the tube, following the compressions. Both regions move at the speed of sound in the medium.

As the piston oscillates sinusoidally, regions of compression and rarefaction are continuously set up. The distance between two successive compressions (or two successive rarefactions) equals the wavelength λ of the sound wave. Because the sound wave is longitudinal, as the compressions and rarefactions travel through the tube, any small element of the gas moves with simple harmonic motion parallel to the direction of the wave. If $s(x, t)$ is the position of a small element relative to its equilibrium position,[1] we can express this harmonic position function as

$$s(x, t) = s_{max} \cos (kx - \omega t) \tag{17.1}$$

where s_{max} is the maximum position of the element relative to equilibrium. This parameter is often called the **displacement amplitude** of the wave. The parameter k is the wave number, and ω is the angular frequency of the wave. Notice that the displacement of the element is along x, in the direction of propagation of the sound wave.

The variation in the gas pressure ΔP measured from the equilibrium value is also periodic with the same wave number and angular frequency as for the displacement in Equation 17.1. Therefore, we can write

$$\Delta P = \Delta P_{max} \sin (kx - \omega t) \tag{17.2}$$

where **the pressure amplitude** $\boldsymbol{\Delta P_{max}}$ is the maximum change in pressure from the equilibrium value.

Notice that we have expressed the displacement by means of a cosine function and the pressure by means of a sine function. We will justify this choice in the procedure that follows and relate the pressure amplitude P_{max} to the displacement amplitude s_{max}. Consider the piston–tube arrangement of Figure 17.1 once again. In Figure 17.3a, we focus our attention on a small cylindrical element of undisturbed gas of length Δx and area A. The volume of this element is $V_i = A\,\Delta x$.

Figure 17.3b shows this element of gas after a sound wave has moved it to a new position. The cylinder's two flat faces move through different distances s_1 and s_2. The change in volume ΔV of the element in the new position is equal to $A\,\Delta s$, where $\Delta s = s_1 - s_2$.

From the definition of bulk modulus (see Eq. 12.8), we express the pressure variation in the element of gas as a function of its change in volume:

$$\Delta P = -B \frac{\Delta V}{V_i}$$

Let's substitute for the initial volume and the change in volume of the element:

$$\Delta P = -B \frac{A\,\Delta s}{A\,\Delta x}$$

Let the length Δx of the cylinder approach zero so that the ratio $\Delta s/\Delta x$ becomes a partial derivative:

$$\Delta P = -B \frac{\partial s}{\partial x} \tag{17.3}$$

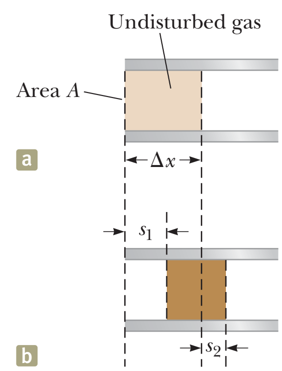


Figure 17.3 (a) An undisturbed element of gas of length Δx in a tube of cross-sectional area A. (b) When a sound wave propagates through the gas, the element is moved to a new position and has a different length. The parameters s_1 and s_2 describe the displacements of the ends of the element from their equilibrium positions.

[1]We use $s(x, t)$ here instead of $y(x, t)$ because the displacement of elements of the medium is not perpendicular to the x direction.

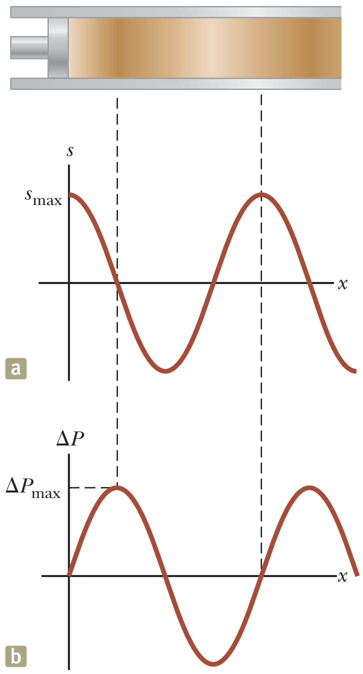


Figure 17.4 (a) Displacement amplitude and (b) pressure amplitude versus position for a sinusoidal longitudinal wave.

Substitute the position function given by Equation 17.1:

$$\Delta P = -B\frac{\partial}{\partial x}[s_{max}\cos(kx - \omega t)] = Bs_{max}k\,\sin(kx - \omega t)$$

From this result, we see that a displacement described by a cosine function leads to a pressure described by a sine function. We also see that the displacement and pressure amplitudes are related by

$$\Delta P_{max} = Bs_{max}k \qquad \textbf{(17.4)}$$

This relationship depends on the bulk modulus of the gas, which is not as readily available as is the density of the gas. Once we determine the speed of sound in a gas in Section 17.2, we will be able to provide an expression that relates ΔP_{max} and s_{max} in terms of the density of the gas.

This discussion shows that a sound wave may be described equally well in terms of either pressure or displacement. A comparison of Equations 17.1 and 17.2 shows that the pressure wave is 90° out of phase with the displacement wave. Graphs of these functions are shown in Figure 17.4. The pressure variation is a maximum when the displacement from equilibrium is zero, and the displacement from equilibrium is a maximum when the pressure variation is zero.

Quick Quiz 17.1 If you blow across the top of an empty soft-drink bottle, a pulse of sound travels down through the air in the bottle. At the moment the pulse reaches the bottom of the bottle, what is the correct description of the displacement of elements of air from their equilibrium positions and the pressure of the air at this point? **(a)** The displacement and pressure are both at a maximum. **(b)** The displacement and pressure are both at a minimum. **(c)** The displacement is zero, and the pressure is a maximum. **(d)** The displacement is zero, and the pressure is a minimum.

17.2 Speed of Sound Waves

We now extend the discussion begun in Section 17.1 to evaluate the speed of sound in a gas. In Figure 17.5a, consider the cylindrical element of gas between the piston and the dashed line. This element of gas is in equilibrium under the influence of forces of equal magnitude, from the piston on the left and from the rest of the gas on the right. The magnitude of these forces is PA, where P is the pressure in the gas and A is the cross-sectional area of the tube.

Figure 17.5b shows the situation after a time interval Δt during which the piston moves to the right at a constant speed v_x due to a force from the left on the piston that has increased in magnitude to $(P + \Delta P)A$. By the end of the time interval Δt,

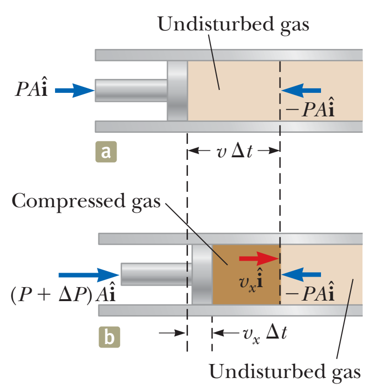


Figure 17.5 (a) An undisturbed element of gas of length $v\ \Delta t$ in a tube of cross-sectional area A. The element is in equilibrium between forces on either end. (b) When the piston moves inward at constant velocity v_x due to an increased force on the left, the element also moves with the same velocity.

every bit of gas in the element is moving with speed v_x. That will not be true in general for a macroscopic element of gas, but it will become true if we shrink the length of the element to an infinitesimal value.

The length of the undisturbed element of gas is chosen to be $v\,\Delta t$, where v is the speed of sound in the gas and Δt is the time interval between the configurations in Figures 17.5a and 17.5b. Therefore, at the end of the time interval Δt, the sound wave will just reach the right end of the cylindrical element of gas. The gas to the right of the element is undisturbed because the sound wave has not reached it yet.

The element of gas is modeled as a nonisolated system in terms of momentum. The force from the piston has provided an impulse to the element, which in turn exhibits a change in momentum. Therefore, we evaluate both sides of the impulse–momentum theorem:

$$\Delta \vec{\mathbf{p}} = \vec{\mathbf{I}} \tag{17.5}$$

On the right, the impulse is provided by the constant force due to the increased pressure on the piston:

$$\vec{\mathbf{I}} = \sum \vec{\mathbf{F}}\,\Delta t = (A\,\Delta P\,\Delta t)\,\hat{\mathbf{i}}$$

The pressure change ΔP can be related to the volume change and then to the speeds v and v_x through the bulk modulus:

$$\Delta P = -B\frac{\Delta V}{V_i} = -B\frac{(-v_x A\,\Delta t)}{vA\,\Delta t} = B\frac{v_x}{v}$$

Therefore, the impulse becomes

$$\vec{\mathbf{I}} = \left(AB\frac{v_x}{v}\,\Delta t\right)\hat{\mathbf{i}} \tag{17.6}$$

On the left-hand side of the impulse–momentum theorem, Equation 17.5, the change in momentum of the element of gas of mass m is as follows:

$$\Delta \vec{\mathbf{p}} = m\,\Delta\vec{\mathbf{v}} = (\rho V_i)(v_x\hat{\mathbf{i}} - 0) = (\rho v v_x A\,\Delta t)\,\hat{\mathbf{i}} \tag{17.7}$$

Substituting Equations 17.6 and 17.7 into Equation 17.5, we find

$$\rho v v_x A\,\Delta t = AB\frac{v_x}{v}\,\Delta t$$

which reduces to an expression for the speed of sound in a gas:

$$v = \sqrt{\frac{B}{\rho}} \tag{17.8}$$

It is interesting to compare this expression with Equation 16.18 for the speed of transverse waves on a string, $v = \sqrt{T/\mu}$. In both cases, the wave speed depends on an elastic property of the medium (bulk modulus B or string tension T) and on an inertial property of the medium (volume density ρ or linear density μ). In fact, the speed of all mechanical waves follows an expression of the general form

$$v = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}}$$

For longitudinal sound waves in a solid rod of material, for example, the speed of sound depends on Young's modulus Y and the density ρ. Table 17.1 (page 512) provides the speed of sound in several different materials.

The speed of sound also depends on the temperature of the medium. For sound traveling through air, the relationship between wave speed and air temperature is

$$v = 331\sqrt{1 + \frac{T_C}{273}} \tag{17.9}$$

Table 17.1 Speed of Sound in Various Media

Medium	v (m/s)	Medium	v (m/s)	Medium	v (m/s)
Gases		**Liquids at 25°C**		**Solids**[a]	
Hydrogen (0°C)	1 286	Glycerol	1 904	Pyrex glass	5 640
Helium (0°C)	972	Seawater	1 533	Iron	5 950
Air (20°C)	343	Water	1 493	Aluminum	6 420
Air (0°C)	331	Mercury	1 450	Brass	4 700
Oxygen (0°C)	317	Kerosene	1 324	Copper	5 010
		Methyl alcohol	1 143	Gold	3 240
		Carbon tetrachloride	926	Lucite	2 680
				Lead	1 960
				Rubber	1 600

[a]Values given are for propagation of longitudinal waves in bulk media. Speeds for longitudinal waves in thin rods are smaller, and speeds of transverse waves in bulk are smaller yet.

where v is in meters/second, 331 m/s is the speed of sound in air at 0°C, and T_C is the air temperature in degrees Celsius. Using this equation, one finds that at 20°C, the speed of sound in air is approximately 343 m/s.

This information provides a convenient way to estimate the distance to a thunderstorm. First count the number of seconds between seeing the flash of lightning and hearing the thunder. Dividing this time interval by 3 gives the approximate distance to the lightning in kilometers because 343 m/s is approximately $\frac{1}{3}$ km/s. Dividing the time interval in seconds by 5 gives the approximate distance to the lightning in miles because the speed of sound is approximately $\frac{1}{5}$ mi/s.

Having an expression (Eq. 17.8) for the speed of sound, we can now express the relationship between pressure amplitude and displacement amplitude for a sound wave (Eq. 17.4) as

$$\Delta P_{\text{max}} = Bs_{\text{max}}k = (\rho v^2)s_{\text{max}}\left(\frac{\omega}{v}\right) = \rho v \omega s_{\text{max}} \qquad \textbf{(17.10)}$$

This expression is a bit more useful than Equation 17.4 because the density of a gas is more readily available than is the bulk modulus.

17.3 Intensity of Periodic Sound Waves

In Chapter 16, we showed that a wave traveling on a taut string transports energy, consistent with the notion of energy transfer by mechanical waves in Equation 8.2. Naturally, we would expect sound waves to also represent a transfer of energy. Consider the element of gas acted on by the piston in Figure 17.5. Imagine that the piston is moving back and forth in simple harmonic motion at angular frequency ω. Imagine also that the length of the element becomes very small so that the entire element moves with the same velocity as the piston. Then we can model the element as a particle on which the piston is doing work. The rate at which the piston is doing work on the element at any instant of time is given by Equation 8.19:

$$Power = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}_x$$

where we have used *Power* rather than P so that we don't confuse power P with pressure P! The force $\vec{\mathbf{F}}$ on the element of gas is related to the pressure and the velocity $\vec{\mathbf{v}}_x$ of the element is the derivative of the displacement function, so we find

$$Power = [\Delta P(x, t)A]\hat{\mathbf{i}} \cdot \frac{\partial}{\partial t}[s(x, t)\,\hat{\mathbf{i}}]$$

$$= [\rho v \omega A s_{\text{max}} \sin(kx - \omega t)]\left\{\frac{\partial}{\partial t}[s_{\text{max}} \cos(kx - \omega t)]\right\}$$

$$= \rho v \omega A s_{\max} \sin(kx - \omega t)][\omega s_{\max} \sin(kx - \omega t)]$$

$$= \rho v \omega^2 A s_{\max}^2 \sin^2(kx - \omega t)$$

We now find the time average power over one period of the oscillation. For any given value of x, which we can choose to be $x = 0$, the average value of $\sin^2(kx - \omega t)$ over one period T is

$$\frac{1}{T}\int_0^T \sin^2(0 - \omega t)\, dt = \frac{1}{T}\int_0^T \sin^2 \omega t\, dt = \frac{1}{T}\left(\frac{t}{2} + \frac{\sin 2\omega t}{2\omega}\right)\Bigg|_0^T = \tfrac{1}{2}$$

Therefore,

$$(Power)_{\text{avg}} = \tfrac{1}{2}\rho v \omega^2 A s_{\max}^2$$

We define the **intensity** I of a wave, or the power per unit area, as the rate at which the energy transported by the wave transfers through a unit area A perpendicular to the direction of travel of the wave:

$$I \equiv \frac{(Power)_{\text{avg}}}{A} \quad \textbf{(17.11)}$$

◀ **Intensity of a sound wave**

In this case, the intensity is therefore

$$I = \tfrac{1}{2}\rho v(\omega s_{\max})^2$$

Hence, the intensity of a periodic sound wave is proportional to the square of the displacement amplitude and to the square of the angular frequency. This expression can also be written in terms of the pressure amplitude $\Delta P_{\max}$; in this case, we use Equation 17.10 to obtain

$$I = \frac{(\Delta P_{\max})^2}{2\rho v} \quad \textbf{(17.12)}$$

The string waves we studied in Chapter 16 are constrained to move along the one-dimensional string, as discussed in the introduction to this chapter. The sound waves we have studied with regard to Figures 17.1 through 17.3 and 17.5 are constrained to move in one dimension along the length of the tube. As we mentioned in the introduction, however, sound waves can move through three-dimensional bulk media, so let's place a sound source in the open air and study the results.

Consider the special case of a point source emitting sound waves equally in all directions. If the air around the source is perfectly uniform, the sound power radiated in all directions is the same, and the speed of sound in all directions is the same. The result in this situation is called a **spherical wave.** Figure 17.6 shows these spherical waves as a series of circular arcs concentric with the source. Each arc represents a surface over which the phase of the wave is constant. We call such a surface of constant phase a **wave front.** The radial distance between adjacent wave fronts that have the same phase is the wavelength λ of the wave. The radial lines pointing outward from the source, representing the direction of propagation of the waves, are called **rays.**

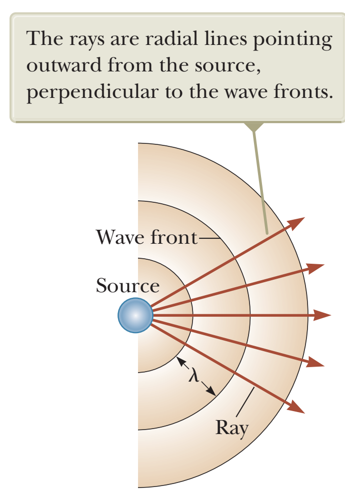


Figure 17.6 Spherical waves emitted by a point source. The circular arcs represent the spherical wave fronts that are concentric with the source.

The average power emitted by the source must be distributed uniformly over each spherical wave front of area $4\pi r^2$. Hence, the wave intensity at a distance r from the source is

$$I = \frac{(Power)_{\text{avg}}}{A} = \frac{(Power)_{\text{avg}}}{4\pi r^2} \quad \textbf{(17.13)}$$

The intensity decreases as the square of the distance from the source. This inverse-square law is reminiscent of the behavior of gravity in Chapter 13.

Quick Quiz 17.2 A vibrating guitar string makes very little sound if it is not mounted on the guitar body. Why does the sound have greater intensity if the string is attached to the guitar body? **(a)** The string vibrates with more energy. **(b)** The energy leaves the guitar at a greater rate. **(c)** The sound power is spread over a larger area at the listener's position. **(d)** The sound power is concentrated over a smaller area at the listener's position. **(e)** The speed of sound is higher in the material of the guitar body. **(f)** None of these answers is correct.

Example 17.1 Hearing Limits

The faintest sounds the human ear can detect at a frequency of 1 000 Hz correspond to an intensity of about 1.00×10^{-12} W/m^2, which is called *threshold of hearing*. The loudest sounds the ear can tolerate at this frequency correspond to an intensity of about 1.00 W/m^2, the *threshold of pain*. Determine the pressure amplitude and displacement amplitude associated with these two limits.

SOLUTION

Conceptualize Think about the quietest environment you have ever experienced. It is likely that the intensity of sound in even this quietest environment is higher than the threshold of hearing.

Categorize Because we are given intensities and asked to calculate pressure and displacement amplitudes, this problem is an analysis problem requiring the concepts discussed in this section.

Analyze To find the amplitude of the pressure variation at the threshold of hearing, use Equation 17.12, taking the speed of sound waves in air to be v = 343 m/s and the density of air to be ρ = 1.20 kg/m^3:

$$\Delta P_{\max} = \sqrt{2\rho v I}$$

$$= \sqrt{2(1.20 \text{ kg/m}^3)(343 \text{ m/s})(1.00 \times 10^{-12} \text{ W/m}^2)}$$

$$= \boxed{2.87 \times 10^{-5} \text{ N/m}^2}$$

Calculate the corresponding displacement amplitude using Equation 17.10, recalling that $\omega = 2\pi f$ (Eq. 16.9):

$$s_{\max} = \frac{\Delta P_{\max}}{\rho v \omega} = \frac{2.87 \times 10^{-5} \text{ N/m}^2}{(1.20 \text{ kg/m}^3)(343 \text{ m/s})(2\pi \times 1\,000 \text{ Hz})}$$

$$= \boxed{1.11 \times 10^{-11} \text{ m}}$$

In a similar manner, one finds that the loudest sounds the human ear can tolerate (the threshold of pain) correspond to a pressure amplitude of $\boxed{28.7 \text{ N/m}^2}$ and a displacement amplitude equal to $\boxed{1.11 \times 10^{-5} \text{ m}}$.

Finalize Because atmospheric pressure is about 10^5 N/m^2, the result for the pressure amplitude tells us that the ear is sensitive to pressure fluctuations as small as 3 parts in 10^{10}! The displacement amplitude is also a remarkably small number! If we compare this result for $s_{\max}$ to the size of an atom (about 10^{-10} m), we see that the ear is an extremely sensitive detector of sound waves.

Example 17.2 Intensity Variations of a Point Source

A point source emits sound waves with an average power output of 80.0 W.

(A) Find the intensity 3.00 m from the source.

SOLUTION

Conceptualize Imagine a small loudspeaker sending sound out at an average rate of 80.0 W uniformly in all directions. You are standing 3.00 m away from the speakers. As the sound propagates, the energy of the sound waves is spread out over an ever-expanding sphere, so the intensity of the sound falls off with distance.

Categorize We evaluate the intensity from an equation generated in this section, so we categorize this example as a substitution problem.

17.2 continued

Because a point source emits energy in the form of spherical waves, use Equation 17.13 to find the intensity:

$$I = \frac{(Power)_{avg}}{4\pi r^2} = \frac{80.0\text{ W}}{4\pi(3.00\text{ m})^2} = 0.707\text{ W/m}^2$$

This intensity is close to the threshold of pain.

(B) Find the distance at which the intensity of the sound is 1.00×10^{-8} W/m^2.

SOLUTION

Solve for r in Equation 17.13 and use the given value for I:

$$r = \sqrt{\frac{(Power)_{avg}}{4\pi I}} = \sqrt{\frac{80.0\text{ W}}{4\pi(1.00 \times 10^{-8}\text{ W/m}^2)}}$$

$$= 2.52 \times 10^4\text{ m}$$

Sound Level in Decibels

Example 17.1 illustrates the wide range of intensities the human ear can detect. Because this range is so wide, it is convenient to use a logarithmic scale, where the **sound level** β (Greek letter beta) is defined by the equation

$$\beta \equiv 10\log\left(\frac{I}{I_0}\right) \quad (17.14)$$

The constant I_0 is the *reference intensity,* taken to be at the threshold of hearing ($I_0 = 1.00 \times 10^{-12}$ W/m^2), and I is the intensity in watts per square meter to which the sound level β corresponds, where β is measured[2] in **decibels** (dB). On this scale, the threshold of pain ($I = 1.00$ W/m^2) corresponds to a sound level of $\beta = 10\log[(1\text{ W/m}^2)/(10^{-12}\text{ W/m}^2)] = 10\log(10^{12}) = 120$ dB, and the threshold of hearing corresponds to $\beta = 10\log[(10^{-12}\text{ W/m}^2)/(10^{-12}\text{ W/m}^2)] = 0$ dB.

Prolonged exposure to high sound levels may seriously damage the human ear. Ear plugs are recommended whenever sound levels exceed 90 dB. Recent evidence suggests that "noise pollution" may be a contributing factor to high blood pressure, anxiety, and nervousness. Table 17.2 gives some typical sound levels.

Table 17.2

Sound Levels

Source of Sound	β (dB)
Nearby jet airplane	150
Jackhammer; machine gun	130
Siren; rock concert	120
Subway; power lawn mower	100
Busy traffic	80
Vacuum cleaner	70
Normal conversation	60
Mosquito buzzing	40
Whisper	30
Rustling leaves	10
Threshold of hearing	0

Quick Quiz 17.3 Increasing the intensity of a sound by a factor of 100 causes the sound level to increase by what amount? **(a)** 100 dB **(b)** 20 dB **(c)** 10 dB **(d)** 2 dB

Example 17.3 Sound Levels

Two identical machines are positioned the same distance from a worker. The intensity of sound delivered by each operating machine at the worker's location is 2.0×10^{-7} W/m^2.

(A) Find the sound level heard by the worker when one machine is operating.

SOLUTION

Conceptualize Imagine a situation in which one source of sound is active and is then joined by a second identical source, such as one person speaking and then a second person speaking at the same time or one musical instrument playing and then being joined by a second instrument.

Categorize This example is a relatively simple analysis problem requiring Equation 17.14.

continued

[2]The unit *bel* is named after the inventor of the telephone, Alexander Graham Bell (1847–1922). The prefix *deci-* is the SI prefix that stands for 10^{-1}.

17.3 continued

Analyze Use Equation 17.14 to calculate the sound level at the worker's location with one machine operating:

$$\beta_1 = 10 \log \left(\frac{2.0 \times 10^{-7} \text{ W/m}^2}{1.00 \times 10^{-12} \text{ W/m}^2} \right) = 10 \log (2.0 \times 10^5) = \boxed{53 \text{ dB}}$$

(B) Find the sound level heard by the worker when two machines are operating.

SOLUTION

Use Equation 17.14 to calculate the sound level at the worker's location with double the intensity:

$$\beta_2 = 10 \log \left(\frac{4.0 \times 10^{-7} \text{ W/m}^2}{1.00 \times 10^{-12} \text{ W/m}^2} \right) = 10 \log (4.0 \times 10^5) = \boxed{56 \text{ dB}}$$

Finalize These results show that when the intensity is doubled, the sound level increases by only 3 dB. This 3-dB increase is independent of the original sound level. (Prove this to yourself!)

WHAT IF? *Loudness* is a psychological response to a sound. It depends on both the intensity and the frequency of the sound. As a rule of thumb, a doubling in loudness is approximately associated with an increase in sound level of 10 dB. (This rule of thumb is relatively inaccurate at very low or very high frequencies.) If the loudness of the machines in this example is to be doubled, how many machines at the same distance from the worker must be running?

Answer Using the rule of thumb, a doubling of loudness corresponds to a sound level increase of 10 dB. Therefore,

$$\beta_2 - \beta_1 = 10 \text{ dB} = 10 \log \left(\frac{I_2}{I_0} \right) - 10 \log \left(\frac{I_1}{I_0} \right) = 10 \log \left(\frac{I_2}{I_1} \right)$$

$$\log \left(\frac{I_2}{I_1} \right) = 1 \rightarrow I_2 = 10 I_1$$

Therefore, ten machines must be operating to double the loudness.

Loudness and Frequency

The discussion of sound level in decibels relates to a *physical* measurement of the strength of a sound. Let us now extend our discussion from the What If? section of Example 17.3 concerning the *psychological* "measurement" of the strength of a sound.

Of course, we don't have instruments in our bodies that can display numerical values of our reactions to stimuli. We have to "calibrate" our reactions somehow by comparing different sounds to a reference sound, but that is not easy to accomplish. For example, earlier we mentioned that the threshold intensity is 10^{-12} W/m^2, corresponding to an intensity level of 0 dB. In reality, this value is the threshold only for a sound of frequency 1 000 Hz, which is a standard reference frequency in acoustics. If we perform an experiment to measure the threshold intensity at other frequencies, we find a distinct variation of this threshold as a function of frequency. For example, at 100 Hz, a barely audible sound must have an intensity level of about 30 dB! Unfortunately, there is no simple relationship between physical measurements and psychological "measurements." The 100-Hz, 30-dB sound is psychologically "equal" in loudness to the 1 000-Hz, 0-dB sound (both are just barely audible), but they are not physically equal in sound level (30 dB $\neq$ 0 dB).

By using test subjects, the human response to sound has been studied, and the results are shown in the white area of Figure 17.7 along with the approximate frequency and sound-level ranges of other sound sources. The lower curve of the white area corresponds to the threshold of hearing. Its variation with frequency is clear from this diagram. Notice that humans are sensitive to frequencies ranging from about 20 Hz to about 20 000 Hz. The upper bound of the white area is the thresh-

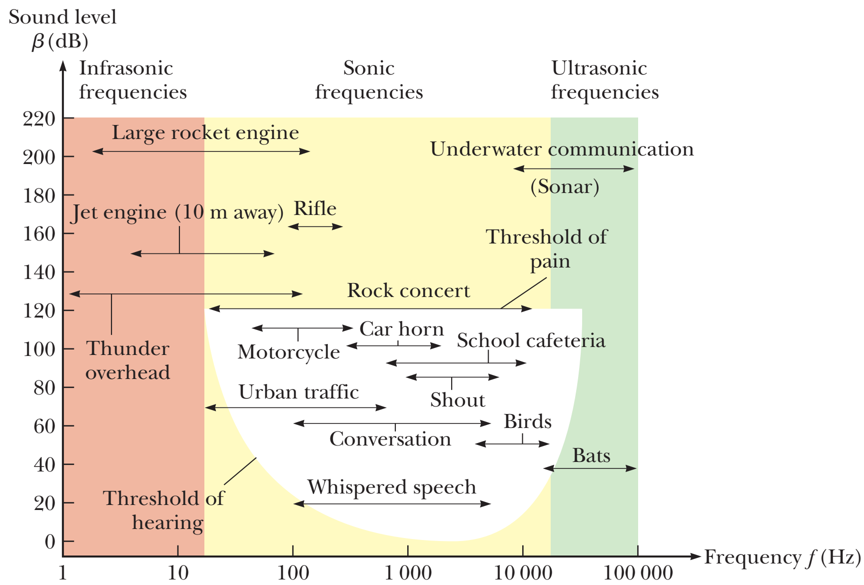


Figure 17.7 Approximate ranges of frequency and sound level of various sources and that of normal human hearing, shown by the white area. (From R. L. Reese, *University Physics*, Pacific Grove, Brooks/Cole, 2000.)

old of pain. Here the boundary of the white area appears straight because the psychological response is relatively independent of frequency at this high sound level.

The most dramatic change with frequency is in the lower left region of the white area, for low frequencies and low intensity levels. Our ears are particularly insensitive in this region. If you are listening to your home entertainment system and the bass (low frequencies) and treble (high frequencies) sound balanced at a high volume, try turning the volume down and listening again. You will probably notice that the bass seems weak, which is due to the insensitivity of the ear to low frequencies at low sound levels as shown in Figure 17.7.

17.4 The Doppler Effect

Perhaps you have noticed how the sound of a vehicle's horn changes as the vehicle moves past you. The frequency of the sound you hear as the vehicle approaches you is higher than the frequency you hear as it moves away from you. This experience is one example of the **Doppler effect.**[3]

To see what causes this apparent frequency change, imagine you are in a boat that is lying at anchor on a gentle sea where the waves have a period of $T = 3.0$ s. Hence, every 3.0 s a crest hits your boat. Figure 17.8a shows this situation, with the water waves moving toward the left. If you set your watch to $t = 0$ just as one crest hits, the watch reads 3.0 s when the next crest hits, 6.0 s when the third crest

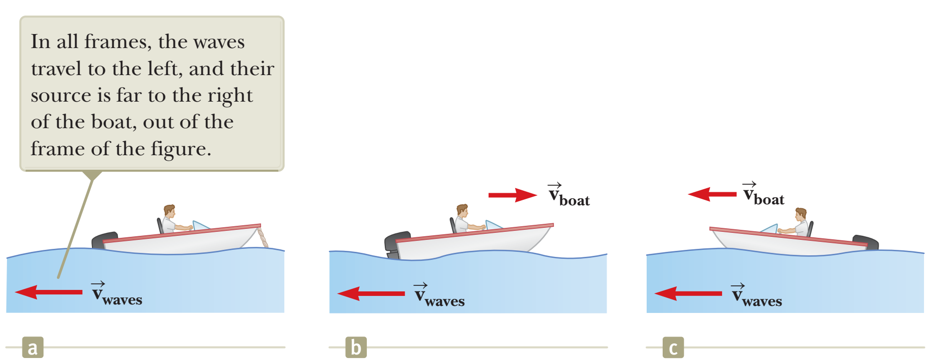


Figure 17.8 (a) Waves moving toward a stationary boat. (b) The boat moving toward the wave source. (c) The boat moving away from the wave source.

[3]Named after Austrian physicist Christian Johann Doppler (1803–1853), who in 1842 predicted the effect for both sound waves and light waves.

hits, and so on. From these observations, you conclude that the wave frequency is $f = 1/T = 1/(3.0 \text{ s}) = 0.33$ Hz. Now suppose you start your motor and head directly into the oncoming waves as in Figure 17.8b. Again you set your watch to $t = 0$ as a crest hits the front (the bow) of your boat. Now, however, because you are moving toward the next wave crest as it moves toward you, it hits you less than 3.0 s after the first hit. In other words, the period you observe is shorter than the 3.0-s period you observed when you were stationary. Because $f = 1/T$, you observe a higher wave frequency than when you were at rest.

If you turn around and move in the same direction as the waves (Fig. 17.8c), you observe the opposite effect. You set your watch to $t = 0$ as a crest hits the back (the stern) of the boat. Because you are now moving away from the next crest, more than 3.0 s has elapsed on your watch by the time that crest catches you. Therefore, you observe a lower frequency than when you were at rest.

These effects occur because the *relative* speed between your boat and the waves depends on the direction of travel and on the speed of your boat. (See Section 4.6.) When you are moving toward the right in Figure 17.8b, this relative speed is higher than that of the wave speed, which leads to the observation of an increased frequency. When you turn around and move to the left, the relative speed is lower, as is the observed frequency of the water waves.

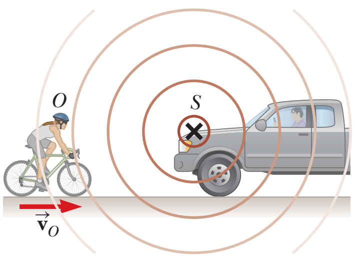


Figure 17.9 An observer O (the cyclist) moves with a speed v_O toward a stationary point source S, the horn of a parked truck. The observer hears a frequency f' that is greater than the source frequency.

Let's now examine an analogous situation with sound waves in which the water waves become sound waves, the water becomes the air, and the person on the boat becomes an observer listening to the sound. In this case, an observer O is moving and a sound source S is stationary. For simplicity, we assume the air is also stationary and the observer moves directly toward the source (Fig. 17.9). The observer moves with a speed v_O toward a stationary point source ($v_S = 0$), where *stationary* means at rest with respect to the medium, air.

If a point source emits sound waves and the medium is uniform, the waves move at the same speed in all directions radially away from the source; the result is a spherical wave as mentioned in Section 17.3. The distance between adjacent wave fronts equals the wavelength λ. In Figure 17.9, the circles are the intersections of these three-dimensional wave fronts with the two-dimensional paper.

We take the frequency of the source in Figure 17.9 to be f, the wavelength to be λ, and the speed of sound to be v. If the observer were also stationary, he would detect wave fronts at a frequency f. (That is, when $v_O = 0$ and $v_S = 0$, the observed frequency equals the source frequency.) When the observer moves toward the source, the speed of the waves relative to the observer is $v' = v + v_O$, as in the case of the boat in Figure 17.8, but the wavelength λ is unchanged. Hence, using Equation 16.12, $v = \lambda f$, we can say that the frequency f' heard by the observer is *increased* and is given by

$$f' = \frac{v'}{\lambda} = \frac{v + v_O}{\lambda}$$

Because $\lambda = v/f$, we can express f' as

$$f' = \left(\frac{v + v_O}{v}\right) f \quad \text{(observer moving toward source)} \qquad \textbf{(17.15)}$$

If the observer is moving away from the source, the speed of the wave relative to the observer is $v' = v - v_O$. The frequency heard by the observer in this case is *decreased* and is given by

$$f' = \left(\frac{v - v_O}{v}\right) f \quad \text{(observer moving away from source)} \qquad \textbf{(17.16)}$$

These last two equations can be reduced to a single equation by adopting a sign convention. Whenever an observer moves with a speed v_O relative to a stationary source, the frequency heard by the observer is given by Equation 17.15, with v_O interpreted as follows: a positive value is substituted for v_O when the observer moves

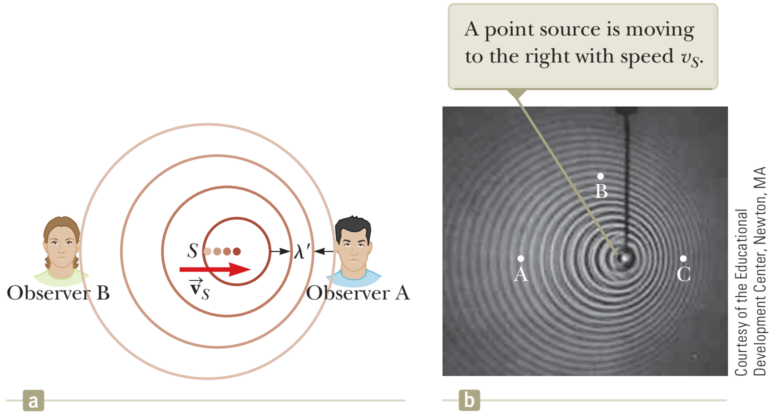


Figure 17.10 (a) A source S moving with a speed v_S toward a stationary observer A and away from a stationary observer B. Observer A hears an increased frequency, and observer B hears a decreased frequency. (b) The Doppler effect in water, observed in a ripple tank. Letters shown in the photo refer to Quick Quiz 17.4.

toward the source, and a negative value is substituted when the observer moves away from the source.

Now suppose the source is in motion and the observer is at rest. If the source moves directly toward observer A in Figure 17.10a, each new wave is emitted from a position to the right of the origin of the previous wave. As a result, the wave fronts heard by the observer are closer together than they would be if the source were not moving. (Fig. 17.10b shows this effect for waves moving on the surface of water.) As a result, the wavelength λ' measured by observer A is shorter than the wavelength λ of the source. During each vibration, which lasts for a time interval T (the period), the source moves a distance $v_S T = v_S/f$ and the wavelength is *shortened* by this amount. Therefore, the observed wavelength λ' is

$$\lambda' = \lambda - \Delta\lambda = \lambda - \frac{v_S}{f}$$

Because $\lambda = v/f$, the frequency f' heard by observer A is

$$f' = \frac{v}{\lambda'} = \frac{v}{\lambda - (v_S/f)} = \frac{v}{(v/f) - (v_S/f)}$$

$$f' = \left(\frac{v}{v - v_S}\right) f \quad \text{(source moving toward observer)} \qquad \textbf{(17.17)}$$

That is, the observed frequency is *increased* whenever the source is moving toward the observer.

When the source moves away from a stationary observer, as is the case for observer B in Figure 17.10a, the observer measures a wavelength λ' that is *greater* than λ and hears a *decreased* frequency:

$$f' = \left(\frac{v}{v + v_S}\right) f \quad \text{(source moving away from observer)} \qquad \textbf{(17.18)}$$

We can express the general relationship for the observed frequency when a source is moving and an observer is at rest as Equation 17.17, with the same sign convention applied to v_S as was applied to v_O: a positive value is substituted for v_S when the source moves toward the observer, and a negative value is substituted when the source moves away from the observer.

Finally, combining Equations 17.15 and 17.17 gives the following general relationship for the observed frequency that includes all four conditions described by Equations 17.15 through 17.18:

$$f' = \left(\frac{v + v_O}{v - v_S}\right) f \qquad \textbf{(17.19)}$$

◀ **General Doppler-shift expression**

Pitfall Prevention 17.1

Doppler Effect Does Not Depend on Distance Some people think that the Doppler effect depends on the distance between the source and the observer. Although the *intensity* of a sound varies as the distance changes, the apparent *frequency* depends only on the relative speed of source and observer. As you listen to an approaching source, you will detect increasing intensity but constant frequency. As the source passes, you will hear the frequency suddenly drop to a new constant value and the intensity begin to decrease.

In this expression, the signs for the values substituted for v_O and v_S depend on the direction of the velocity. A positive value is used for motion of the observer or the source *toward* the other (associated with an *increase* in observed frequency), and a negative value is used for motion of one *away from* the other (associated with a *decrease* in observed frequency).

Although the Doppler effect is most typically experienced with sound waves, it is a phenomenon common to all waves. For example, the relative motion of source and observer produces a frequency shift in light waves. The Doppler effect is used in police radar systems to measure the speeds of motor vehicles. Likewise, astronomers use the effect to determine the speeds of stars, galaxies, and other celestial objects relative to the Earth.

Quick Quiz 17.4 Consider detectors of water waves at three locations A, B, and C in Figure 17.10b. Which of the following statements is true? **(a)** The wave speed is highest at location A. **(b)** The wave speed is highest at location C. **(c)** The detected wavelength is largest at location B. **(d)** The detected wavelength is largest at location C. **(e)** The detected frequency is highest at location C. **(f)** The detected frequency is highest at location A.

Quick Quiz 17.5 You stand on a platform at a train station and listen to a train approaching the station at a constant velocity. While the train approaches, but before it arrives, what do you hear? **(a)** the intensity and the frequency of the sound both increasing **(b)** the intensity and the frequency of the sound both decreasing **(c)** the intensity increasing and the frequency decreasing **(d)** the intensity decreasing and the frequency increasing **(e)** the intensity increasing and the frequency remaining the same **(f)** the intensity decreasing and the frequency remaining the same

Example 17.4 The Broken Clock Radio AM

Your clock radio awakens you with a steady and irritating sound of frequency 600 Hz. One morning, it malfunctions and cannot be turned off. In frustration, you drop the clock radio out of your fourth-story dorm window, 15.0 m from the ground. Assume the speed of sound is 343 m/s. As you listen to the falling clock radio, what frequency do you hear just before you hear it striking the ground?

SOLUTION

Conceptualize The speed of the clock radio increases as it falls. Therefore, it is a source of sound moving away from you with an increasing speed so the frequency you hear should be less than 600 Hz.

Categorize We categorize this problem as one in which we combine the *particle under constant acceleration* model for the falling radio with our understanding of the frequency shift of sound due to the Doppler effect.

Analyze Because the clock radio is modeled as a particle under constant acceleration due to gravity, use Equation 2.13 to express the speed of the source of sound:

$$(1)\quad v_S = v_{yi} + a_y t = 0 - gt = -gt$$

From Equation 2.16, find the time at which the clock radio strikes the ground:

$$y_f = y_i + v_{yi}t - \tfrac{1}{2}gt^2 = 0 + 0 - \tfrac{1}{2}gt^2 \rightarrow t = \sqrt{-\frac{2y_f}{g}}$$

Substitute into Equation (1):

$$v_S = (-g)\sqrt{-\frac{2y_f}{g}} = -\sqrt{-2gy_f}$$

Use Equation 17.19 to determine the Doppler-shifted frequency heard from the falling clock radio:

$$f' = \left[\frac{v + 0}{v - (-\sqrt{-2gy_f})}\right] f = \left(\frac{v}{v + \sqrt{-2gy_f}}\right) f$$

17.4 continued

Substitute numerical values:

$$f' = \left[\frac{343\text{ m/s}}{343\text{ m/s} + \sqrt{-2(9.80\text{ m/s}^2)(-15.0\text{ m})}}\right](600\text{ Hz})$$

$$= 571\text{ Hz}$$

Finalize The frequency is lower than the actual frequency of 600 Hz because the clock radio is moving away from you. If it were to fall from a higher floor so that it passes below $y = -15.0$ m, the clock radio would continue to accelerate and the frequency would continue to drop.

Example 17.5 Doppler Submarines

A submarine (sub A) travels through water at a speed of 8.00 m/s, emitting a sonar wave at a frequency of 1 400 Hz. The speed of sound in the water is 1 533 m/s. A second submarine (sub B) is located such that both submarines are traveling directly toward each other. The second submarine is moving at 9.00 m/s.

(A) What frequency is detected by an observer riding on sub B as the subs approach each other?

SOLUTION

Conceptualize Even though the problem involves subs moving in water, there is a Doppler effect just like there is when you are in a moving car and listening to a sound moving through the air from another car.

Categorize Because both subs are moving, we categorize this problem as one involving the Doppler effect for both a moving source and a moving observer.

Analyze Use Equation 17.19 to find the Doppler-shifted frequency heard by the observer in sub B, being careful with the signs assigned to the source and observer speeds:

$$f' = \left(\frac{v + v_O}{v - v_S}\right)f$$

$$f' = \left[\frac{1\,533\text{ m/s} + (+9.00\text{ m/s})}{1\,533\text{ m/s} - (+8.00\text{ m/s})}\right](1\,400\text{ Hz}) = 1\,416\text{ Hz}$$

(B) The subs barely miss each other and pass. What frequency is detected by an observer riding on sub B as the subs recede from each other?

SOLUTION

Use Equation 17.19 to find the Doppler-shifted frequency heard by the observer in sub B, again being careful with the signs assigned to the source and observer speeds:

$$f' = \left(\frac{v + v_O}{v - v_S}\right)f$$

$$f' = \left[\frac{1\,533\text{ m/s} + (-9.00\text{ m/s})}{1\,533\text{ m/s} - (-8.00\text{ m/s})}\right](1\,400\text{ Hz}) = 1\,385\text{ Hz}$$

Notice that the frequency drops from 1 416 Hz to 1 385 Hz as the subs pass. This effect is similar to the drop in frequency you hear when a car passes by you while blowing its horn.

(C) While the subs are approaching each other, some of the sound from sub A reflects from sub B and returns to sub A. If this sound were to be detected by an observer on sub A, what is its frequency?

SOLUTION

The sound of apparent frequency 1 416 Hz found in part (A) is reflected from a moving source (sub B) and then detected by a moving observer (sub A). Find the frequency detected by sub A:

$$f'' = \left(\frac{v + v_O}{v - v_S}\right)f'$$

$$= \left[\frac{1\,533\text{ m/s} + (+8.00\text{ m/s})}{1\,533\text{ m/s} - (+9.00\text{ m/s})}\right](1\,416\text{ Hz}) = 1\,432\text{ Hz}$$

continued

▶ 17.5 continued

Finalize This technique is used by police officers to measure the speed of a moving car. Microwaves are emitted from the police car and reflected by the moving car. By detecting the Doppler-shifted frequency of the reflected microwaves, the police officer can determine the speed of the moving car.

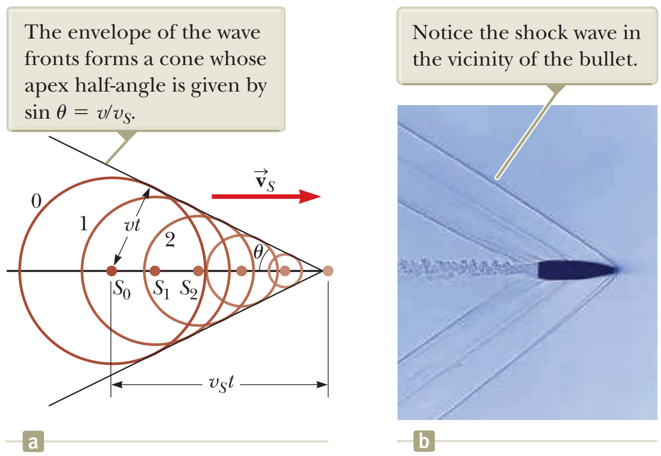


Figure 17.11 (a) A representation of a shock wave produced when a source moves from S_0 to the right with a speed v_S that is greater than the wave speed v in the medium. (b) A stroboscopic photograph of a bullet moving at supersonic speed through the hot air above a candle.

Shock Waves

Now consider what happens when the speed v_S of a source *exceeds* the wave speed v. This situation is depicted graphically in Figure 17.11a. The circles represent spherical wave fronts emitted by the source at various times during its motion. At $t = 0$, the source is at S_0 and moving toward the right. At later times, the source is at S_1, and then S_2, and so on. At the time t, the wave front centered at S_0 reaches a radius of vt. In this same time interval, the source travels a distance $v_S t$. Notice in Figure 17.11a that a straight line can be drawn tangent to all the wave fronts generated at various times. Therefore, the envelope of these wave fronts is a cone whose apex half-angle θ (the "Mach angle") is given by

$$\sin\theta = \frac{vt}{v_S t} = \frac{v}{v_S}$$

The ratio v_S/v is referred to as the *Mach number,* and the conical wave front produced when $v_S > v$ (supersonic speeds) is known as a *shock wave.* An interesting analogy to shock waves is the V-shaped wave fronts produced by a boat (the bow wave) when the boat's speed exceeds the speed of the surface-water waves (Fig. 17.12).

Jet airplanes traveling at supersonic speeds produce shock waves, which are responsible for the loud "sonic boom" one hears. The shock wave carries a great deal of energy concentrated on the surface of the cone, with correspondingly great pressure variations. Such shock waves are unpleasant to hear and can cause damage to buildings when aircraft fly supersonically at low altitudes. In fact, an airplane flying at supersonic speeds produces a double boom because two shock waves are formed, one from the nose of the plane and one from the tail. People near the path of a space shuttle as it glides toward its landing point have reported hearing what sounds like two very closely spaced cracks of thunder.

Figure 17.12 The V-shaped bow wave of a boat is formed because the boat speed is greater than the speed of the water waves it generates. A bow wave is analogous to a shock wave formed by an airplane traveling faster than sound.

Quick Quiz 17.6 An airplane flying with a constant velocity moves from a cold air mass into a warm air mass. Does the Mach number **(a)** increase, **(b)** decrease, or **(c)** stay the same?

Summary

Definitions

The **intensity** of a periodic sound wave, which is the power per unit area, is

$$I \equiv \frac{(Power)_{avg}}{A} = \frac{(\Delta P_{max})^2}{2\rho v} \quad \textbf{(17.11, 17.12)}$$

The **sound level** of a sound wave in decibels is

$$\beta \equiv 10 \log \left(\frac{I}{I_0}\right) \quad \textbf{(17.14)}$$

The constant I_0 is a reference intensity, usually taken to be at the threshold of hearing (1.00×10^{-12} W/m^2), and I is the intensity of the sound wave in watts per square meter.

Concepts and Principles

Sound waves are longitudinal and travel through a compressible medium with a speed that depends on the elastic and inertial properties of that medium. The speed of sound in a gas having a bulk modulus B and density ρ is

$$v = \sqrt{\frac{B}{\rho}} \quad \textbf{(17.8)}$$

For sinusoidal sound waves, the variation in the position of an element of the medium is

$$s(x, t) = s_{max} \cos (kx - \omega t) \quad \textbf{(17.1)}$$

and the variation in pressure from the equilibrium value is

$$\Delta P = \Delta P_{max} \sin (kx - \omega t) \quad \textbf{(17.2)}$$

where ΔP_{max} is the **pressure amplitude.** The pressure wave is 90° out of phase with the displacement wave. The relationship between s_{max} and ΔP_{max} is

$$\Delta P_{max} = \rho v \omega s_{max} \quad \textbf{(17.10)}$$

The change in frequency heard by an observer whenever there is relative motion between a source of sound waves and the observer is called the **Doppler effect.** The observed frequency is

$$f' = \left(\frac{v + v_O}{v - v_S}\right) f \quad \textbf{(17.19)}$$

In this expression, the signs for the values substituted for v_O and v_S depend on the direction of the velocity. A positive value for the speed of the observer or source is substituted if the velocity of one is toward the other, whereas a negative value represents a velocity of one away from the other.

Objective Questions

1. denotes answer available in *Student Solutions Manual/Study Guide*

1. Table 17.1 shows the speed of sound is typically an order of magnitude larger in solids than in gases. To what can this higher value be most directly attributed? (a) the difference in density between solids and gases (b) the difference in compressibility between solids and gases (c) the limited size of a solid object compared to a free gas (d) the impossibility of holding a gas under significant tension

2. Two sirens A and B are sounding so that the frequency from A is twice the frequency from B. Compared with the speed of sound from A, is the speed of sound from B (a) twice as fast, (b) half as fast, (c) four times as fast, (d) one-fourth as fast, or (e) the same?

3. As you travel down the highway in your car, an ambulance approaches you from the rear at a high speed (Fig. OQ17.3) sounding its siren at a frequency of 500 Hz. Which statement is correct? (a) You hear a frequency less than 500 Hz. (b) You hear a frequency equal to 500 Hz. (c) You hear a frequency greater

Anthony Redpath/Corbis

Figure OQ17.3

than 500 Hz. (d) You hear a frequency greater than 500 Hz, whereas the ambulance driver hears a frequency lower than 500 Hz. (e) You hear a frequency less than 500 Hz, whereas the ambulance driver hears a frequency of 500 Hz.

4. What happens to a sound wave as it travels from air into water? (a) Its intensity increases. (b) Its wavelength decreases. (c) Its frequency increases. (d) Its frequency remains the same. (e) Its velocity decreases.

5. A church bell in a steeple rings once. At 300 m in front of the church, the maximum sound intensity is 2 $\mu W/m^2$. At 950 m behind the church, the maximum intensity is 0.2 $\mu W/m^2$. What is the main reason for the difference in the intensity? (a) Most of the sound is absorbed by the air before it gets far away from the source. (b) Most of the sound is absorbed by the ground as it travels away from the source. (c) The bell broadcasts the sound mostly toward the front. (d) At a larger distance, the power is spread over a larger area.

6. If a 1.00-kHz sound source moves at a speed of 50.0 m/s toward a listener who moves at a speed of 30.0 m/s in a direction away from the source, what is the apparent frequency heard by the listener? (a) 796 Hz (b) 949 Hz (c) 1 000 Hz (d) 1 068 Hz (e) 1 273 Hz

7. A sound wave can be characterized as (a) a transverse wave, (b) a longitudinal wave, (c) a transverse wave or a longitudinal wave, depending on the nature of its source, (d) one that carries no energy, or (e) a wave that does not require a medium to be transmitted from one place to the other.

8. Assume a change at the source of sound reduces the wavelength of a sound wave in air by a factor of 2. **(i)** What happens to its frequency? (a) It increases by a factor of 4. (b) It increases by a factor of 2. (c) It is unchanged. (d) It decreases by a factor of 2. (e) It changes by an unpredictable factor. **(ii)** What happens to its speed? Choose from the same possibilities as in part (i).

9. A point source broadcasts sound into a uniform medium. If the distance from the source is tripled, how does the intensity change? (a) It becomes one-ninth as large. (b) It becomes one-third as large. (c) It is unchanged. (d) It becomes three times larger. (e) It becomes nine times larger.

10. Suppose an observer and a source of sound are both at rest relative to the ground and a strong wind is blowing away from the source toward the observer. **(i)** What effect does the wind have on the observed frequency? (a) It causes an increase. (b) It causes a decrease. (c) It causes no change. **(ii)** What effect does the wind have on the observed wavelength? Choose from the same possibilities as in part (i). **(iii)** What effect does the wind have on the observed speed of the wave? Choose from the same possibilities as in part (i).

11. A source of sound vibrates with constant frequency. Rank the frequency of sound observed in the following cases from highest to the lowest. If two frequencies are equal, show their equality in your ranking. All the motions mentioned have the same speed, 25 m/s. (a) The source and observer are stationary. (b) The source is moving toward a stationary observer. (c) The source is moving away from a stationary observer. (d) The observer is moving toward a stationary source. (e) The observer is moving away from a stationary source.

12. With a sensitive sound-level meter, you measure the sound of a running spider as −10 dB. What does the negative sign imply? (a) The spider is moving away from you. (b) The frequency of the sound is too low to be audible to humans. (c) The intensity of the sound is too faint to be audible to humans. (d) You have made a mistake; negative signs do not fit with logarithms.

13. Doubling the power output from a sound source emitting a single frequency will result in what increase in decibel level? (a) 0.50 dB (b) 2.0 dB (c) 3.0 dB (d) 4.0 dB (e) above 20 dB

14. Of the following sounds, which one is most likely to have a sound level of 60 dB? (a) a rock concert (b) the turning of a page in this textbook (c) dinner-table conversation (d) a cheering crowd at a football game

Conceptual Questions

1. denotes answer available in *Student Solutions Manual/Study Guide*

1. How can an object move with respect to an observer so that the sound from it is not shifted in frequency?

2. Older auto-focus cameras sent out a pulse of sound and measured the time interval required for the pulse to reach an object, reflect off of it, and return to be detected. Can air temperature affect the camera's focus? New cameras use a more reliable infrared system.

3. A friend sitting in her car far down the road waves to you and beeps her horn at the same moment. How far away must she be for you to calculate the speed of sound to two significant figures by measuring the time interval required for the sound to reach you?

4. How can you determine that the speed of sound is the same for all frequencies by listening to a band or orchestra?

5. Explain how the distance to a lightning bolt (Fig. CQ17.5) can be determined by counting the seconds between the flash and the sound of thunder.

Figure CQ17.5

6. You are driving toward a cliff and honk your horn. Is there a Doppler shift of the sound when you hear the echo? If so, is it like a moving source or a moving observer? What if the reflection occurs not from a cliff, but from the forward edge of a huge alien spacecraft moving toward you as you drive?

7. The radar systems used by police to detect speeders are sensitive to the Doppler shift of a pulse of microwaves. Discuss how this sensitivity can be used to measure the speed of a car.

8. *The Tunguska event.* On June 30, 1908, a meteor burned up and exploded in the atmosphere above the Tunguska River valley in Siberia. It knocked down trees over thousands of square kilometers and started a forest fire, but produced no crater and apparently caused no human casualties. A witness sitting on his doorstep outside the zone of falling trees recalled events in the following sequence. He saw a moving light in the sky, brighter than the Sun and descending at a low angle to the horizon. He felt his face become warm. He felt the ground shake. An invisible agent picked him up and immediately dropped him about a meter from where he had been seated. He heard a very loud protracted rumbling. Suggest an explanation for these observations and for the order in which they happened.

9. A sonic ranger is a device that determines the distance to an object by sending out an ultrasonic sound pulse and measuring the time interval required for the wave to return by reflection from the object. Typically, these devices cannot reliably detect an object that is less than half a meter from the sensor. Why is that?

Problems

ENHANCED WebAssign The problems found in this chapter may be assigned online in Enhanced WebAssign

1. straightforward; **2.** intermediate; **3.** challenging

1. full solution available in the *Student Solutions Manual/Study Guide*

AMT Analysis Model tutorial available in Enhanced WebAssign

GP Guided Problem

M Master It tutorial available in Enhanced WebAssign

W Watch It video solution available in Enhanced WebAssign

Note: Throughout this chapter, pressure variations ΔP are measured relative to atmospheric pressure, 1.013×10^5 Pa.

Section 17.1 Pressure Variations in Sound Waves

1. W A sinusoidal sound wave moves through a medium and is described by the displacement wave function

$$s(x, t) = 2.00 \cos (15.7x - 858t)$$

where s is in micrometers, x is in meters, and t is in seconds. Find (a) the amplitude, (b) the wavelength, and (c) the speed of this wave. (d) Determine the instantaneous displacement from equilibrium of the elements of the medium at the position $x = 0.050\ 0$ m at $t = 3.00$ ms. (e) Determine the maximum speed of the element's oscillatory motion.

2. As a certain sound wave travels through the air, it produces pressure variations (above and below atmospheric pressure) given by $\Delta P = 1.27 \sin (\pi x - 340\pi t)$ in SI units. Find (a) the amplitude of the pressure variations, (b) the frequency, (c) the wavelength in air, and (d) the speed of the sound wave.

3. Write an expression that describes the pressure variation as a function of position and time for a sinusoidal sound wave in air. Assume the speed of sound is 343 m/s, $\lambda = 0.100$ m, and $\Delta P_{max} = 0.200$ Pa.

Section 17.2 Speed of Sound Waves

Problem 85 in Chapter 2 can also be assigned with this section.

Note: In the rest of this chapter, unless otherwise specified, the equilibrium density of air is $\rho = 1.20$ kg/m^3 and the speed of sound in air is $v = 343$ m/s. Use Table 17.1 to find speeds of sound in other media.

4. M An experimenter wishes to generate in air a sound wave that has a displacement amplitude of 5.50×10^{-6} m. The pressure amplitude is to be limited to 0.840 Pa. What is the minimum wavelength the sound wave can have?

5. Calculate the pressure amplitude of a 2.00-kHz sound wave in air, assuming that the displacement amplitude is equal to 2.00×10^{-8} m.

6. Earthquakes at fault lines in the Earth's crust create seismic waves, which are longitudinal (P waves) or transverse (S waves). The P waves have a speed of about 7 km/s. Estimate the average bulk modulus of the Earth's crust given that the density of rock is about 2 500 kg/m^3.

7. A dolphin (Fig. P17.7) in seawater at a temperature of 25°C emits a sound wave directed toward the ocean floor 150 m below. How much time passes before it hears an echo?

Stephen Frink/Photographer's Choice/Getty Images

Figure P17.7

8. A sound wave propagates in air at 27°C with frequency 4.00 kHz. It passes through a region where the temperature gradually changes and then moves through air at 0°C. Give numerical answers to the following questions to the extent possible and state your reasoning about what happens to the wave physically. (a) What happens to the speed of the wave? (b) What happens to its frequency? (c) What happens to its wavelength?

9. Ultrasound is used in medicine both for diagnostic imaging (Fig. P17.9, page 526) and for therapy. For diagnosis, short pulses of ultrasound are passed through the patient's body. An echo reflected from a structure of interest is recorded, and the distance to the structure can be determined from the time delay for the echo's return. To reveal detail, the wavelength of the reflected ultrasound must be small compared to the size of the object reflecting the wave. The speed of ultrasound in human tissue is about 1 500 m/s (nearly the same as the speed of sound in water). (a) What is the wavelength of ultrasound with a frequency of 2.40 MHz? (b) In the whole set of imaging techniques, frequencies in the range 1.00 MHz to 20.0 MHz are used. What is the range of wavelengths corresponding to this range of frequencies?

Figure P17.9 A view of a fetus in the uterus made with ultrasound imaging.

10. W A sound wave in air has a pressure amplitude equal to 4.00×10^{-3} Pa. Calculate the displacement amplitude of the wave at a frequency of 10.0 kHz.

11. W Suppose you hear a clap of thunder 16.2 s after seeing the associated lightning strike. The speed of light in air is 3.00×10^8 m/s. (a) How far are you from the lightning strike? (b) Do you need to know the value of the speed of light to answer? Explain.

12. W A rescue plane flies horizontally at a constant speed searching for a disabled boat. When the plane is directly above the boat, the boat's crew blows a loud horn. By the time the plane's sound detector receives the horn's sound, the plane has traveled a distance equal to half its altitude above the ocean. Assuming it takes the sound 2.00 s to reach the plane, determine (a) the speed of the plane and (b) its altitude.

13. AMT W A flowerpot is knocked off a window ledge from a height d = 20.0 m above the sidewalk as shown in Figure P17.13. It falls toward an unsuspecting man of height h = 1.75 m who is standing below. Assume the man requires a time interval of Δt = 0.300 s to respond to the warning. How close to the sidewalk can the flowerpot fall before it is too late for a warning shouted from the balcony to reach the man in time?

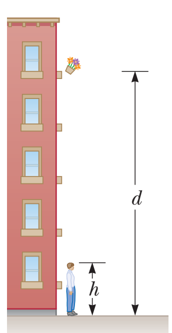


Figure P17.13 Problems 13 and 14.

14. A flowerpot is knocked off a balcony from a height d above the sidewalk as shown in Figure P17.13. It falls toward an unsuspecting man of height h who is standing below. Assume the man requires a time interval of Δt to respond to the warning. How close to the sidewalk can the flowerpot fall before it is too late for a warning shouted from the balcony to reach the man in time? Use the symbol v for the speed of sound.

15. The speed of sound in air (in meters per second) depends on temperature according to the approximate expression

$$v = 331.5 + 0.607T_C$$

where T_C is the Celsius temperature. In dry air, the temperature decreases about 1°C for every 150-m rise in altitude. (a) Assume this change is constant up to an altitude of 9 000 m. What time interval is required for the sound from an airplane flying at 9 000 m to reach the ground on a day when the ground temperature is 30°C? (b) **What If?** Compare your answer with the time interval required if the air were uniformly at 30°C. Which time interval is longer?

16. A sound wave moves down a cylinder as in Figure 17.2. Show that the pressure variation of the wave is described by $\Delta P = \pm \rho v \omega \sqrt{s_{max}^2 - s^2}$, where $s = s(x, t)$ is given by Equation 17.1.

17. A hammer strikes one end of a thick iron rail of length 8.50 m. A microphone located at the opposite end of the rail detects two pulses of sound, one that travels through the air and a longitudinal wave that travels through the rail. (a) Which pulse reaches the microphone first? (b) Find the separation in time between the arrivals of the two pulses.

18. A cowboy stands on horizontal ground between two parallel, vertical cliffs. He is not midway between the cliffs. He fires a shot and hears its echoes. The second echo arrives 1.92 s after the first and 1.47 s before the third. Consider only the sound traveling parallel to the ground and reflecting from the cliffs. (a) What is the distance between the cliffs? (b) **What If?** If he can hear a fourth echo, how long after the third echo does it arrive?

Section 17.3 Intensity of Periodic Sound Waves

19. Calculate the sound level (in decibels) of a sound wave that has an intensity of 4.00 μW/m^2.

20. The area of a typical eardrum is about 5.00×10^{-5} m^2. (a) Calculate the average sound power incident on an eardrum at the threshold of pain, which corresponds to an intensity of 1.00 W/m^2. (b) How much energy is transferred to the eardrum exposed to this sound for 1.00 min?

21. The intensity of a sound wave at a fixed distance from a speaker vibrating at 1.00 kHz is 0.600 W/m^2. (a) Determine the intensity that results if the frequency is increased to 2.50 kHz while a constant displacement amplitude is maintained. (b) Calculate the intensity if the frequency is reduced to 0.500 kHz and the displacement amplitude is doubled.

22. The intensity of a sound wave at a fixed distance from a speaker vibrating at a frequency f is I. (a) Determine the intensity that results if the frequency is increased to f' while a constant displacement amplitude is maintained. (b) Calculate the intensity if the frequency is reduced to $f/2$ and the displacement amplitude is doubled.

23. A person wears a hearing aid that uniformly increases the sound level of all audible frequencies of sound by 30.0 dB. The hearing aid picks up sound having a frequency of 250 Hz at an intensity of 3.0×10^{-11} W/m^2. What is the intensity delivered to the eardrum?

24. The sound intensity at a distance of 16 m from a noisy generator is measured to be 0.25 W/m^2. What is the sound intensity at a distance of 28 m from the generator?

25. W The power output of a certain public-address speaker is 6.00 W. Suppose it broadcasts equally in all directions. (a) Within what distance from the speaker would the sound be painful to the ear? (b) At what distance from the speaker would the sound be barely audible?

26. A sound wave from a police siren has an intensity of 100.0 W/m^2 at a certain point; a second sound wave from a nearby ambulance has an intensity level that is 10 dB greater than the police siren's sound wave at the same point. What is the sound level of the sound wave due to the ambulance?

27. M A train sounds its horn as it approaches an intersection. The horn can just be heard at a level of 50 dB by an observer 10 km away. (a) What is the average power generated by the horn? (b) What intensity level of the horn's sound is observed by someone waiting at an intersection 50 m from the train? Treat the horn as a point source and neglect any absorption of sound by the air.

28. As the people sing in church, the sound level everywhere inside is 101 dB. No sound is transmitted through the massive walls, but all the windows and doors are open on a summer morning. Their total area is 22.0 m^2. (a) How much sound energy is radiated through the windows and doors in 20.0 min? (b) Suppose the ground is a good reflector and sound radiates from the church uniformly in all horizontal and upward directions. Find the sound level 1.00 km away.

29. The most soaring vocal melody is in Johann Sebastian Bach's Mass in B Minor. In one section, the basses, tenors, altos, and sopranos carry the melody from a low D to a high A. In concert pitch, these notes are now assigned frequencies of 146.8 Hz and 880.0 Hz. Find the wavelengths of (a) the initial note and (b) the final note. Assume the chorus sings the melody with a uniform sound level of 75.0 dB. Find the pressure amplitudes of (c) the initial note and (d) the final note. Find the displacement amplitudes of (e) the initial note and (f) the final note.

30. Show that the difference between decibel levels β_1 and β_2 of a sound is related to the ratio of the distances r_1 and r_2 from the sound source by

$$\beta_2 - \beta_1 = 20 \log \left(\frac{r_1}{r_2}\right)$$

31. M A family ice show is held at an enclosed arena. The skaters perform to music with level 80.0 dB. This level is too loud for your baby, who yells at 75.0 dB. (a) What total sound intensity engulfs you? (b) What is the combined sound level?

32. W Two small speakers emit sound waves of different frequencies equally in all directions. Speaker A has an output of 1.00 mW, and speaker B has an output of 1.50 mW. Determine the sound level (in decibels) at point C in Figure P17.32 assuming (a) only speaker A emits sound, (b) only speaker B emits sound, and (c) both speakers emit sound.

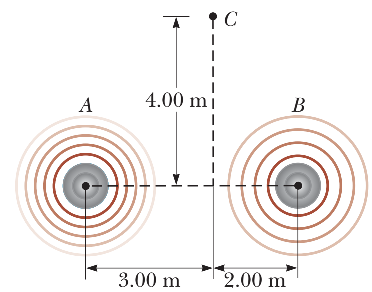


Figure P17.32

33. M A firework charge is detonated many meters above the ground. At a distance of $d_1 = 500$ m from the explosion, the acoustic pressure reaches a maximum of $\Delta P_{max} = 10.0$ Pa (Fig. P17.33). Assume the speed of sound is constant at 343 m/s throughout the atmosphere over the region considered, the ground absorbs all the sound falling on it, and the air absorbs sound energy as described by the rate 7.00 dB/km. What is the sound level (in decibels) at a distance of $d_2 = 4.00 \times 10^3$ m from the explosion?

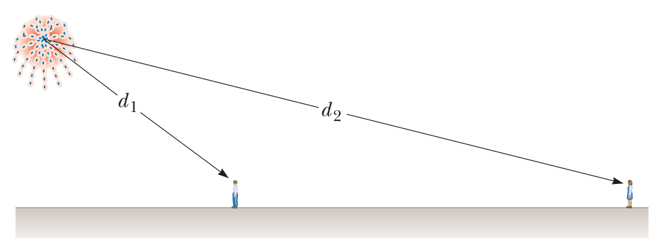


Figure P17.33

34. A fireworks rocket explodes at a height of 100 m above the ground. An observer on the ground directly under the explosion experiences an average sound intensity of 7.00×10^{-2} W/m^2 for 0.200 s. (a) What is the total amount of energy transferred away from the explosion by sound? (b) What is the sound level (in decibels) heard by the observer?

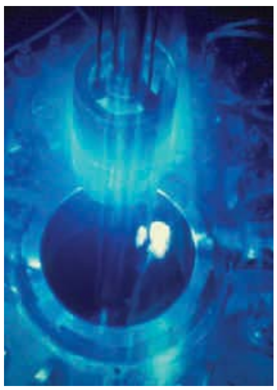
35. The sound level at a distance of 3.00 m from a source is 120 dB. At what distance is the sound level (a) 100 dB and (b) 10.0 dB?

36. *Why is the following situation impossible?* It is early on a Saturday morning, and much to your displeasure your next-door neighbor starts mowing his lawn. As you try to get back to sleep, your next-door neighbor on the other side of your house also begins to mow the lawn with an identical mower the same distance away. This situation annoys you greatly because the total sound now has twice the loudness it had when only one neighbor was mowing.

Section 17.4 The Doppler Effect

37. An ambulance moving at 42 m/s sounds its siren whose frequency is 450 Hz. A car is moving in the same direction as the ambulance at 25 m/s. What frequency does a person in the car hear (a) as the ambulance approaches the car? (b) After the ambulance passes the car?

38. When high-energy charged particles move through a transparent medium with a speed greater than the speed of light in that medium, a shock wave, or bow wave, of light is produced. This phenomenon is called the *Cerenkov effect*. When a nuclear reactor is shielded by a large pool of water, Cerenkov radiation can be seen as a blue glow in the vicinity of the reactor core due to high-speed electrons moving through the water (Fig. 17.38). In a particular case, the Cerenkov radiation produces a wave front with an apex half-angle of 53.0°. Calculate the speed of the electrons in the water. The speed of light in water is 2.25×10^8 m/s.



U.S. Department of Energy/Photo Researchers, Inc.

Figure P17.38

39. A driver travels northbound on a highway at a speed of 25.0 m/s. A police car, traveling southbound at a speed of 40.0 m/s, approaches with its siren producing sound at a frequency of 2 500 Hz. (a) What frequency does the driver observe as the police car approaches? (b) What frequency does the driver detect after the police car passes him? (c) Repeat parts (a) and (b) for the case when the police car is behind the driver and travels northbound.

40. GP Submarine A travels horizontally at 11.0 m/s through ocean water. It emits a sonar signal of frequency $f = 5.27 \times 10^3$ Hz in the forward direction. Submarine B is in front of submarine A and traveling at 3.00 m/s relative to the water in the same direction as submarine A. A crewman in submarine B uses his equipment to detect the sound waves ("pings") from submarine A. We wish to determine what is heard by the crewman in submarine B. (a) An observer on which submarine detects a frequency f' as described by Equation 17.19? (b) In Equation 17.19, should the sign of v_S be positive or negative? (c) In Equation 17.19, should the sign of v_O be positive or negative? (d) In Equation 17.19, what speed of sound should be used? (e) Find the frequency of the sound detected by the crewman on submarine B.

41. AMT **Review.** A block with a speaker bolted to it is connected to a spring having spring constant $k = 20.0$ N/m and oscillates as shown in Figure P17.41. The total mass of the block and speaker is 5.00 kg, and the amplitude of this unit's motion is 0.500 m. The speaker emits sound waves of frequency 440 Hz. Determine (a) the highest and (b) the lowest frequencies heard by the person to the right of the speaker. (c)If the maximum sound level heard by the person is 60.0 dB when the speaker is at its closest distance $d = 1.00$ m from him, what is the minimum sound level heard by the observer?

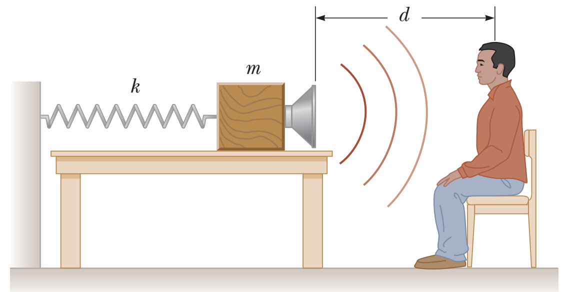


Figure P17.41 Problems 41 and 42.

42. **Review.** A block with a speaker bolted to it is connected to a spring having spring constant k and oscillates as shown in Figure P17.41. The total mass of the block and speaker is m, and the amplitude of this unit's motion is A. The speaker emits sound waves of frequency f. Determine (a) the highest and (b) the lowest frequencies heard by the person to the right of the speaker. (c) If the maximum sound level heard by the person is β when the speaker is at its closest distance d from him, what is the minimum sound level heard by the observer?

43. Expectant parents are thrilled to hear their unborn baby's heartbeat, revealed by an ultrasonic detector that produces beeps of audible sound in synchronization with the fetal heartbeat. Suppose the fetus's ventricular wall moves in simple harmonic motion with an amplitude of 1.80 mm and a frequency of 115 beats per minute. (a) Find the maximum linear speed of the heart wall. Suppose a source mounted on the detector in contact with the mother's abdomen produces sound at 2 000 000.0 Hz, which travels through tissue at 1.50 km/s. (b) Find the maximum change in frequency between the sound that arrives at the wall of the baby's heart and the sound emitted by the source. (c) Find the maximum change in frequency between the reflected sound received by the detector and that emitted by the source.

44. *Why is the following situation impossible?* At the Summer Olympics, an athlete runs at a constant speed down a straight track while a spectator near the edge of the track blows a note on a horn with a fixed frequency. When the athlete passes the horn, she hears the frequency of the horn fall by the musical interval called a minor third. That is, the frequency she hears drops to five-sixths its original value.

45. M Standing at a crosswalk, you hear a frequency of 560 Hz from the siren of an approaching ambulance. After the ambulance passes, the observed frequency of the siren is 480 Hz. Determine the ambulance's speed from these observations.

46. **Review.** A tuning fork vibrating at 512 Hz falls from rest and accelerates at 9.80 m/s^2. How far below the point of release is the tuning fork when waves of frequency 485 Hz reach the release point?

47. AMT M A supersonic jet traveling at Mach 3.00 at an altitude of $h = 20\ 000$ m is directly over a person at time $t = 0$ as shown in Figure P17.47. Assume the average speed of sound in air is 335 m/s over the path of the sound. (a) At what time will the person encounter the shock wave due to the sound emitted at $t = 0$? (b) Where will the plane be when this shock wave is heard?

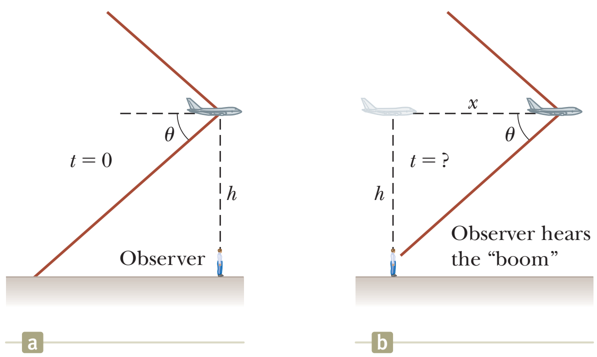


Figure P17.47

Additional Problems

48. A bat (Fig. P17.48) can detect very small objects, such as an insect whose length is approximately equal to one wavelength of the sound the bat makes. If a bat emits chirps at a frequency of 60.0 kHz and the speed of sound in air is 340 m/s, what is the smallest insect the bat can detect?

Figure P17.48 Problems 48 and 63.

49. Some studies suggest that the upper frequency limit of hearing is determined by the diameter of the eardrum. The diameter of the eardrum is approximately equal to half the wavelength of the sound wave at this upper limit. If the relationship holds exactly, what is the diameter of the eardrum of a person capable of hearing 20 000 Hz? (Assume a body temperature of 37.0°C.)

50. The highest note written for a singer in a published score was F-sharp above high C, 1.480 kHz, for Zerbinetta in the original version of Richard Strauss's opera *Ariadne auf Naxos.* (a) Find the wavelength of this sound in air. (b) Suppose people in the fourth row of seats hear this note with level 81.0 dB. Find the displacement amplitude of the sound. (c) **What If?** In response to complaints, Strauss later transposed the note down to F above high C, 1.397 kHz. By what increment did the wavelength change?

51. Trucks carrying garbage to the town dump form a nearly steady procession on a country road, all traveling at 19.7 m/s in the same direction. Two trucks arrive at the dump every 3 min. A bicyclist is also traveling toward the dump, at 4.47 m/s. (a) With what frequency do the trucks pass the cyclist? (b) **What If?** A hill does not slow down the trucks, but makes the out-of-shape cyclist's speed drop to 1.56 m/s. How often do the trucks whiz past the cyclist now?

52. If a salesman claims a loudspeaker is rated at 150 W, he is referring to the maximum electrical power input to the speaker. Assume a loudspeaker with an input power of 150 W broadcasts sound equally in all directions and produces sound with a level of 103 dB at a distance of 1.60 m from its center. (a) Find its sound power output. (b) Find the efficiency of the speaker, that is, the fraction of input power that is converted into useful output power.

53. An interstate highway has been built through a neighborhood in a city. In the afternoon, the sound level in an apartment in the neighborhood is 80.0 dB as 100 cars pass outside the window every minute. Late at night, the traffic flow is only five cars per minute. What is the average late-night sound level?

54. A train whistle ($f = 400$ Hz) sounds higher or lower in frequency depending on whether it approaches or recedes. (a) Prove that the difference in frequency between the approaching and receding train whistle is

$$\Delta f = \frac{2u/v}{1 - u^2/v^2} f$$

where u is the speed of the train and v is the speed of sound. (b) Calculate this difference for a train moving at a speed of 130 km/h. Take the speed of sound in air to be 340 m/s.

55. An ultrasonic tape measure uses frequencies above 20 MHz to determine dimensions of structures such as buildings. It does so by emitting a pulse of ultrasound into air and then measuring the time interval for an echo to return from a reflecting surface whose distance away is to be measured. The distance is displayed as a digital readout. For a tape measure that emits a pulse of ultrasound with a frequency of 22.0 MHz, (a) what is the distance to an object from which the echo pulse returns after 24.0 ms when the air temperature is 26°C? (b) What should be the duration of the emitted pulse if it is to include ten cycles of the ultrasonic wave? (c) What is the spatial length of such a pulse?

56. The tensile stress in a thick copper bar is 99.5% of its elastic breaking point of 13.0×10^{10} N/m^2. If a 500-Hz sound wave is transmitted through the material, (a) what displacement amplitude will cause the bar to break? (b) What is the maximum speed of the elements of copper at this moment? (c) What is the sound intensity in the bar?

57. **Review.** A 150-g glider moves at $v_1 = 2.30$ m/s on an air track toward an originally stationary 200-g glider as shown in Figure P17.57. The gliders undergo a completely inelastic collision and latch together over a time interval of 7.00 ms. A student suggests roughly half the decrease in mechanical energy of the two-glider system is transferred to the environment by sound. Is this suggestion reasonable? To evaluate the idea, find the implied sound level at a position 0.800 m from the gliders. If the student's idea is unreasonable, suggest a better idea.

AMT

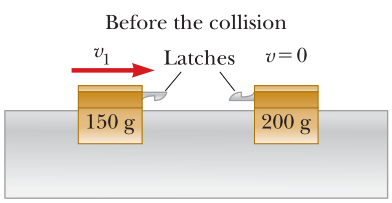


Figure P17.57

58. Consider the following wave function in SI units:

$$\Delta P(r, t) = \left(\frac{25.0}{r}\right) \sin\,(1.36r - 2\,030t)$$

Explain how this wave function can apply to a wave radiating from a small source, with r being the radial distance from the center of the source to any point outside the source. Give the most detailed description of the wave that you can. Include answers to such questions as the following and give representative values for any quantities that can be evaluated. (a) Does the wave move more toward the right or the left? (b) As it moves away from the source, what happens to its amplitude? (c) Its speed? (d) Its frequency? (e) Its wavelength? (f) Its power? (g) Its intensity?

59. **Review.** For a certain type of steel, stress is always proportional to strain with Young's modulus 20×10^{10} N/m^2. The steel has density 7.86×10^3 kg/m^3. It will fail by bending permanently if subjected to compressive stress greater than its yield strength $\sigma_y = 400$ MPa. A rod 80.0 cm long, made of this steel, is fired at 12.0 m/s straight at a very hard wall. (a) The speed of a one-dimensional compressional wave moving along the rod is given by $v = \sqrt{Y/\rho}$, where Y is Young's modulus for the rod and ρ is the density. Calculate this speed. (b) After the front end of the rod hits the wall and stops, the back end of the rod keeps moving as described by Newton's first law until it is stopped by excess pressure in a sound wave moving back through the rod. What time interval elapses before the back end of the rod receives the message that it should stop? (c) How far has the back end of the rod moved in this time interval? Find (d) the strain and (e) the stress in the rod. (f) If it is not to fail, what is the maximum impact speed a rod can have in terms of σ_y, Y, and ρ?

60. A large set of unoccupied football bleachers has solid seats and risers. You stand on the field in front of the bleachers and sharply clap two wooden boards together once. The sound pulse you produce has no definite frequency and no wavelength. The sound you hear reflected from the bleachers has an identifiable frequency and may remind you of a short toot on a trumpet, buzzer, or kazoo. (a) Explain what accounts for this sound. Compute order-of-magnitude estimates for (b) the frequency, (c) the wavelength, and (d) the duration of the sound on the basis of data you specify.

61. To measure her speed, a skydiver carries a buzzer emitting a steady tone at 1 800 Hz. A friend on the ground at the landing site directly below listens to the amplified sound he receives. Assume the air is calm and the speed of sound is independent of altitude. While the skydiver is falling at terminal speed, her friend on the ground receives waves of frequency 2 150 Hz. (a) What is the skydiver's speed of descent? (b) **What If?** Suppose the skydiver can hear the sound of the buzzer reflected from the ground. What frequency does she receive?

M

62. Spherical waves of wavelength 45.0 cm propagate outward from a point source. (a) Explain how the intensity at a distance of 240 cm compares with the intensity at a distance of 60.0 cm. (b) Explain how the amplitude at a distance of 240 cm compares with the amplitude at a distance of 60.0 cm. (c) Explain how the phase of the wave at a distance of 240 cm compares with the phase at 60.0 cm at the same moment.

63. A bat (Fig. P17.48), moving at 5.00 m/s, is chasing a flying insect. If the bat emits a 40.0-kHz chirp and receives back an echo at 40.4 kHz, (a) what is the speed of the insect? (b) Will the bat be able to catch the insect? Explain.

64. Two ships are moving along a line due east (Fig. P17.64). The trailing vessel has a speed relative to a land-based observation point of $v_1 = 64.0$ km/h, and the leading ship has a speed of $v_2 = 45.0$ km/h relative to that point. The two ships are in a region of the ocean where the current is moving uniformly due west at $v_{current} = 10.0$ km/h. The trailing ship transmits a sonar signal at a frequency of 1 200.0 Hz through the water. What frequency is monitored by the leading ship?

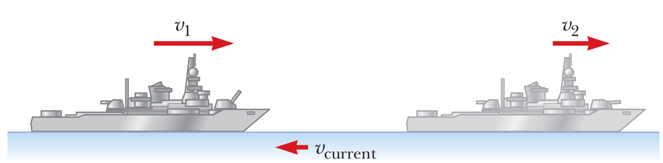


Figure P17.64

65. A police car is traveling east at 40.0 m/s along a straight road, overtaking a car ahead of it moving east at 30.0 m/s. The police car has a malfunctioning siren that is stuck at 1 000 Hz. (a) What would be the wavelength in air of the siren sound if the police car were at rest? (b) What is the wavelength in front of the police car? (c) What is it behind the police car? (d) What is the frequency heard by the driver being chased?

66. The speed of a one-dimensional compressional wave traveling along a thin copper rod is 3.56 km/s. The rod is given a sharp hammer blow at one end. A listener at the far end of the rod hears the sound twice, transmitted through the metal and through air, with a time interval Δt between the two pulses. (a) Which sound arrives first? (b) Find the length of the rod as a function of Δt. (c) Find the length of the rod if $\Delta t = 127$ ms. (d) Imagine that the copper rod is replaced by another material through which the speed of sound is v_r. What is the length of the rod in terms of t and v_r? (e) Would the answer to part (d) go to a well-defined limit as the speed of sound in the rod goes to infinity? Explain your answer.

67. A large meteoroid enters the Earth's atmosphere at a speed of 20.0 km/s and is not significantly slowed before entering the ocean. (a) What is the Mach angle of the shock wave from the meteoroid in the lower atmosphere? (b) If we assume the meteoroid survives the impact with the ocean surface, what is the (initial) Mach angle of the shock wave the meteoroid produces in the water?

68. Three metal rods are located relative to each other as shown in Figure P17.68, where $L_3 = L_1 + L_2$. The speed of sound in a rod is given by $v = \sqrt{Y/\rho}$, where Y is Young's modulus for the rod and ρ is the density. Values of density and Young's modulus for the three materials are $\rho_1 = 2.70 \times 10^3$ kg/m^3, $Y_1 = 7.00 \times 10^{10}$ N/m^2, $\rho_2 = 11.3 \times 10^3$ kg/m^3, $Y_2 = 1.60 \times 10^{10}$ N/m^2, $\rho_3 = 8.80 \times 10^3$ kg/m^3, $Y_3 = 11.0 \times 10^{10}$ N/m^2. If $L_3 = 1.50$ m, what must the ratio L_1/L_2 be if a sound wave is to travel the length of rods 1 and 2 in the same time interval required for the wave to travel the length of rod 3?

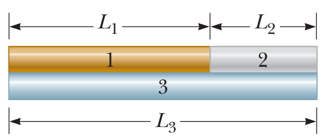


Figure P17.68

69. M With particular experimental methods, it is possible to produce and observe in a long, thin rod both a transverse wave whose speed depends primarily on tension in the rod and a longitudinal wave whose speed is determined by Young's modulus and the density of the material according to the expression $v = \sqrt{Y/\rho}$. The transverse wave can be modeled as a wave in a stretched string. A particular metal rod is 150 cm long and has a radius of 0.200 cm and a mass of 50.9 g. Young's modulus for the material is 6.80×10^{10} N/m^2. What must the tension in the rod be if the ratio of the speed of longitudinal waves to the speed of transverse waves is 8.00?

70. A siren mounted on the roof of a firehouse emits sound at a frequency of 900 Hz. A steady wind is blowing with a speed of 15.0 m/s. Taking the speed of sound in calm air to be 343 m/s, find the wavelength of the sound (a) upwind of the siren and (b) downwind of the siren. Firefighters are approaching the siren from various directions at 15.0 m/s. What frequency does a firefighter hear (c) if she is approaching from an upwind position so that she is moving in the direction in which the wind is blowing and (d) if she is approaching from a downwind position and moving against the wind?

Challenge Problems

71. The Doppler equation presented in the text is valid when the motion between the observer and the source occurs on a straight line so that the source and observer are moving either directly toward or directly away from each other. If this restriction is relaxed, one must use the more general Doppler equation

$$f' = \left(\frac{v + v_O \cos\theta_O}{v - v_S \cos\theta_S}\right) f$$

where θ_O and θ_S are defined in Figure P17.71a. Use the preceding equation to solve the following problem. A train moves at a constant speed of $v = 25.0$ m/s toward the intersection shown in Figure P17.71b. A car is stopped near the crossing, 30.0 m from the tracks. The train's horn emits a frequency of 500 Hz when the train is 40.0 m from the intersection. (a) What is the frequency heard by the passengers in the car? (b) If the train emits this sound continuously and the car is stationary at this position long before the train arrives until long after it leaves, what range of frequencies do passengers in the car hear? (c) Suppose the car is foolishly trying to beat the train to the intersection and is traveling at 40.0 m/s toward the tracks. When the car is 30.0 m from the tracks and the train is 40.0 m from the intersection, what is the frequency heard by the passengers in the car now?

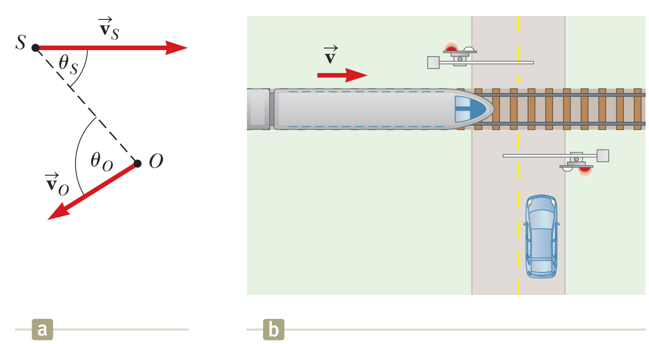


Figure P17.71

72. In Section 17.2, we derived the speed of sound in a gas using the impulse–momentum theorem applied to the cylinder of gas in Figure 17.5. Let us find the speed of sound in a gas using a different approach based on the element of gas in Figure 17.3. Proceed as follows. (a) Draw a force diagram for this element showing the forces exerted on the left and right surfaces due to the pressure of the gas on either side of the element. (b) By applying Newton's second law to the element, show that

$$-\frac{\partial(\Delta P)}{\partial x} A\,\Delta x = \rho A\,\Delta x \frac{\partial^2 s}{\partial t^2}$$

(c) By substituting $\Delta P = -(B\,\partial s/\partial x)$ (Eq. 17.3), derive the following wave equation for sound:

$$\frac{B}{\rho}\frac{\partial^2 s}{\partial x^2} = \frac{\partial^2 s}{\partial t^2}$$

(d) To a mathematical physicist, this equation demonstrates the existence of sound waves and determines their speed. As a physics student, you must take another step or two. Substitute into the wave equation the trial solution $s(x, t) = s_{\max} \cos (kx - \omega t)$. Show that this function satisfies the wave equation, provided $\omega/k = v = \sqrt{B/\rho}$.

73. Equation 17.13 states that at distance r away from a point source with power $(Power)_{avg}$, the wave intensity is

$$I = \frac{(Power)_{avg}}{4\pi r^2}$$

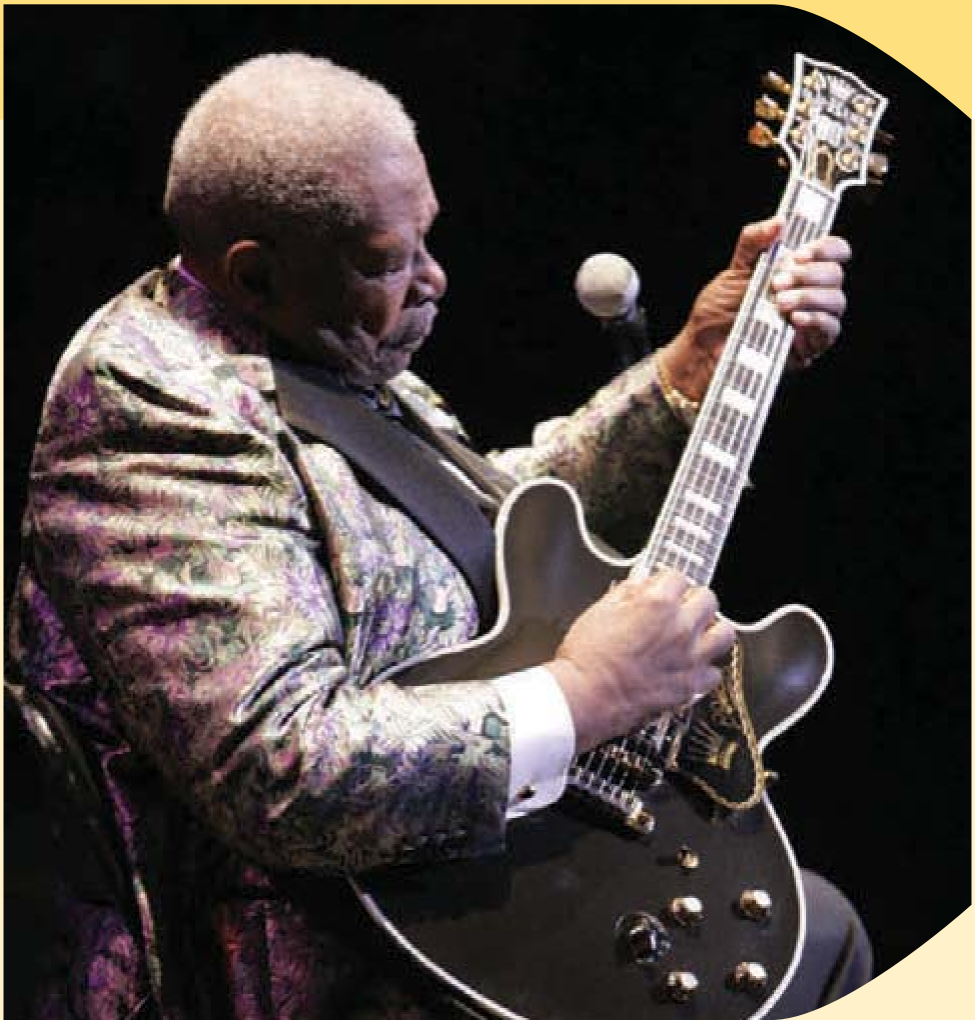
Study Figure 17.10 and prove that at distance r straight in front of a point source with power $(Power)_{avg}$ moving with constant speed v_S the wave intensity is

$$I = \frac{(Power)_{avg}}{4\pi r^2}\left(\frac{v - v_S}{v}\right)$$

CHAPTER

18

Superposition and Standing Waves



Blues master B. B. King takes advantage of standing waves on strings. He changes to higher notes on the guitar by pushing the strings against the frets on the fingerboard, shortening the lengths of the portions of the strings that vibrate. *(AP Photo/Danny Moloshok)*

The wave model was introduced in the previous two chapters. We have seen that waves are very different from particles. A particle is of zero size, whereas a wave has a characteristic size, its wavelength. Another important difference between waves and particles is that we can explore the possibility of two or more waves combining at one point in the same medium. Particles can be combined to form extended objects, but the particles must be at *different* locations. In contrast, two waves can both be present at the same location. The ramifications of this possibility are explored in this chapter.

When waves are combined in systems with boundary conditions, only certain allowed frequencies can exist and we say the frequencies are *quantized.* Quantization is a notion that is at the heart of quantum mechanics, a subject introduced formally in Chapter 40. There we show that analysis of waves under boundary conditions explains many of the quantum phenomena. In this chapter, we use quantization to understand the behavior of the wide array of musical instruments that are based on strings and air columns.

We also consider the combination of waves having different frequencies. When two sound waves having nearly the same frequency interfere, we hear variations in the loudness called *beats*. Finally, we discuss how any nonsinusoidal periodic wave can be described as a sum of sine and cosine functions.

18.1 Analysis Model: Waves in Interference

Many interesting wave phenomena in nature cannot be described by a single traveling wave. Instead, one must analyze these phenomena in terms of a combination of traveling waves. As noted in the introduction, waves have a remarkable difference from particles in that waves can be combined at the *same* location in space. To analyze such wave combinations, we make use of the **superposition principle:**

Superposition principle

> If two or more traveling waves are moving through a medium, the resultant value of the wave function at any point is the algebraic sum of the values of the wave functions of the individual waves.

Waves that obey this principle are called *linear waves.* (See Section 16.6.) In the case of mechanical waves, linear waves are generally characterized by having amplitudes much smaller than their wavelengths. Waves that violate the superposition principle are called *nonlinear waves* and are often characterized by large amplitudes. In this book, we deal only with linear waves.

One consequence of the superposition principle is that two traveling waves can pass through each other without being destroyed or even altered. For instance, when two pebbles are thrown into a pond and hit the surface at different locations, the expanding circular surface waves from the two locations simply pass through each other with no permanent effect. The resulting complex pattern can be viewed as two independent sets of expanding circles.

Pitfall Prevention 18.1

Do Waves Actually *Interfere?* In popular usage, the term *interfere* implies that an agent affects a situation in some way so as to preclude something from happening. For example, in American football, *pass interference* means that a defending player has affected the receiver so that the receiver is unable to catch the ball. This usage is very different from its use in physics, where waves pass through each other and interfere, but do not affect each other in any way. In physics, interference is similar to the notion of *combination* as described in this chapter.

Figure 18.1 is a pictorial representation of the superposition of two pulses. The wave function for the pulse moving to the right is y_1, and the wave function for the pulse moving to the left is y_2. The pulses have the same speed but different shapes, and the displacement of the elements of the medium is in the positive y direction for both pulses. When the waves overlap (Fig. 18.1b), the wave function for the resulting complex wave is given by $y_1 + y_2$. When the crests of the pulses coincide (Fig. 18.1c), the resulting wave given by $y_1 + y_2$ has a larger amplitude than that of the individual pulses. The two pulses finally separate and continue moving in their original directions (Fig. 18.1d). Notice that the pulse shapes remain unchanged after the interaction, as if the two pulses had never met!

The combination of separate waves in the same region of space to produce a resultant wave is called **interference.** For the two pulses shown in Figure 18.1, the displacement of the elements of the medium is in the positive y direction for both pulses, and the resultant pulse (created when the individual pulses overlap) exhibits an amplitude greater than that of either individual pulse. Because the displacements caused by the two pulses are in the same direction, we refer to their superposition as **constructive interference.**

Constructive interference

Now consider two pulses traveling in opposite directions on a taut string where one pulse is inverted relative to the other as illustrated in Figure 18.2. When these pulses begin to overlap, the resultant pulse is given by $y_1 + y_2$, but the values of the function y_2 are negative. Again, the two pulses pass through each other; because the displacements caused by the two pulses are in opposite directions, however, we refer to their superposition as **destructive interference.**

Destructive interference

The superposition principle is the centerpiece of the analysis model called **waves in interference.** In many situations, both in acoustics and optics, waves combine according to this principle and exhibit interesting phenomena with practical applications.

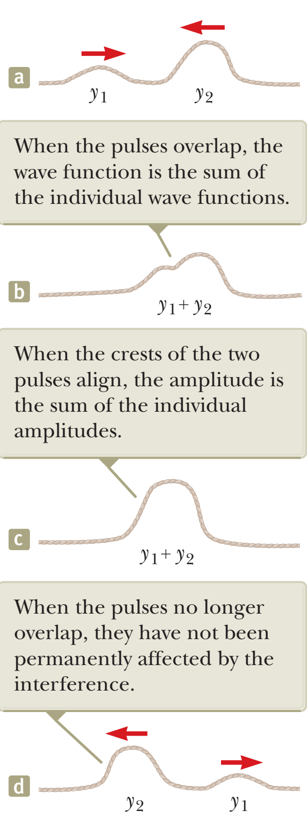


Figure 18.1 Constructive interference. Two positive pulses travel on a stretched string in opposite directions and overlap.

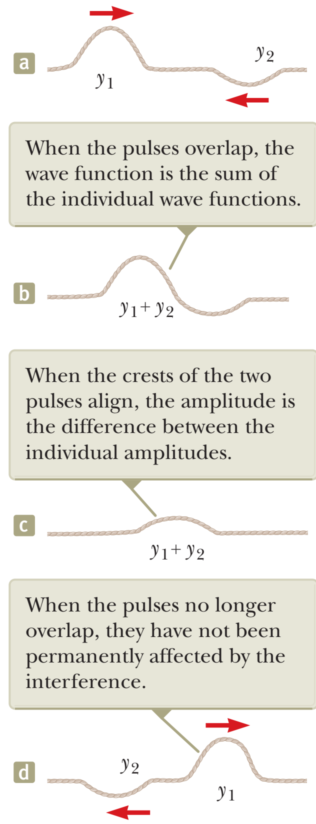


Figure 18.2 Destructive interference. Two pulses, one positive and one negative, travel on a stretched string in opposite directions and overlap.

Quick Quiz 18.1 Two pulses move in opposite directions on a string and are identical in shape except that one has positive displacements of the elements of the string and the other has negative displacements. At the moment the two pulses completely overlap on the string, what happens? **(a)** The energy associated with the pulses has disappeared. **(b)** The string is not moving. **(c)** The string forms a straight line. **(d)** The pulses have vanished and will not reappear.

Superposition of Sinusoidal Waves

Let us now apply the principle of superposition to two sinusoidal waves traveling in the same direction in a linear medium. If the two waves are traveling to the right and have the same frequency, wavelength, and amplitude but differ in phase, we can express their individual wave functions as

$$y_1 = A \sin(kx - \omega t) \qquad y_2 = A \sin(kx - \omega t + \phi)$$

where, as usual, $k = 2\pi/\lambda$, $\omega = 2\pi f$, and ϕ is the phase constant as discussed in Section 16.2. Hence, the resultant wave function y is

$$y = y_1 + y_2 = A[\sin(kx - \omega t) + \sin(kx - \omega t + \phi)]$$

To simplify this expression, we use the trigonometric identity

$$\sin a + \sin b = 2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)$$

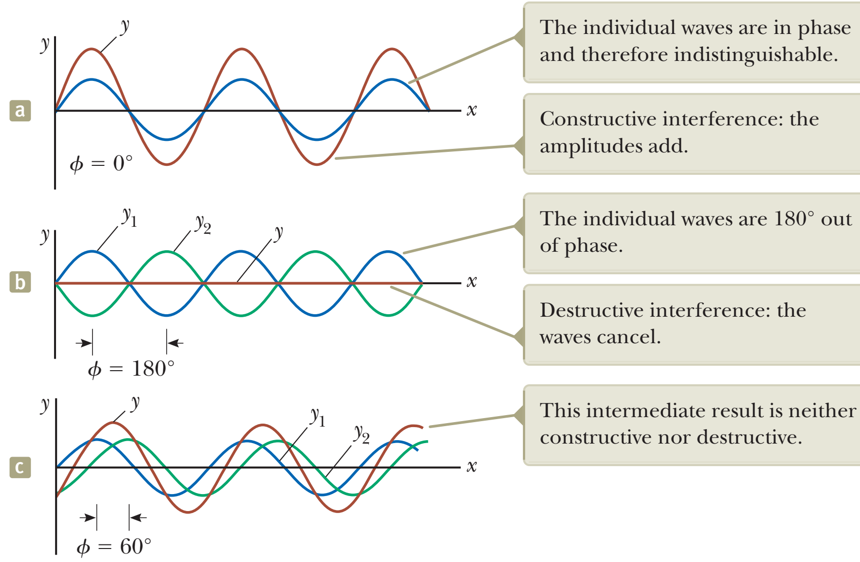


Figure 18.3 The superposition of two identical waves y_1 and y_2 (blue and green, respectively) to yield a resultant wave (red-brown).

Letting $a = kx - \omega t$ and $b = kx - \omega t + \phi$, we find that the resultant wave function y reduces to

Resultant of two traveling sinusoidal waves ▶

$$y = 2A\cos\left(\frac{\phi}{2}\right)\sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

This result has several important features. The resultant wave function y also is sinusoidal and has the same frequency and wavelength as the individual waves because the sine function incorporates the same values of k and ω that appear in the original wave functions. The amplitude of the resultant wave is $2A\cos(\phi/2)$, and its phase constant is $\phi/2$. If the phase constant ϕ of the original wave equals 0, then $\cos(\phi/2) = \cos 0 = 1$ and the amplitude of the resultant wave is $2A$, twice the amplitude of either individual wave. In this case, the crests of the two waves are at the same locations in space and the waves are said to be everywhere *in phase* and therefore interfere constructively. The individual waves y_1 and y_2 combine to form the red-brown curve y of amplitude $2A$ shown in Figure 18.3a. Because the individual waves are in phase, they are indistinguishable in Figure 18.3a, where they appear as a single blue curve. In general, constructive interference occurs when $\cos(\phi/2) = \pm 1$. That is true, for example, when $\phi = 0, 2\pi, 4\pi, \ldots$ rad, that is, when ϕ is an *even* multiple of π.

When ϕ is equal to π rad or to any *odd* multiple of π, then $\cos(\phi/2) = \cos(\pi/2) = 0$ and the crests of one wave occur at the same positions as the troughs of the second wave (Fig. 18.3b). Therefore, as a consequence of destructive interference, the resultant wave has *zero* amplitude everywhere as shown by the straight red-brown line in Figure 18.3b. Finally, when the phase constant has an arbitrary value other than 0 or an integer multiple of π rad (Fig. 18.3c), the resultant wave has an amplitude whose value is somewhere between 0 and $2A$.

In the more general case in which the waves have the same wavelength but different amplitudes, the results are similar with the following exceptions. In the in-phase case, the amplitude of the resultant wave is not twice that of a single wave, but rather is the sum of the amplitudes of the two waves. When the waves are π rad out of phase, they do not completely cancel as in Figure 18.3b. The result is a wave whose amplitude is the difference in the amplitudes of the individual waves.

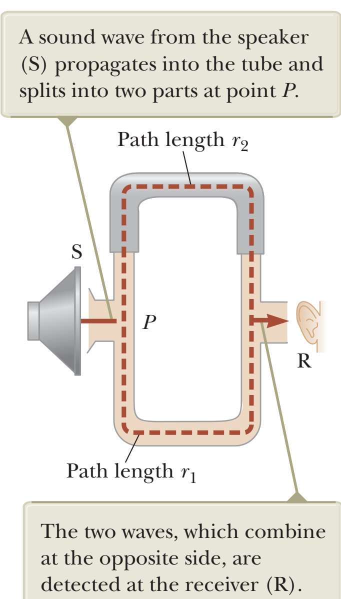


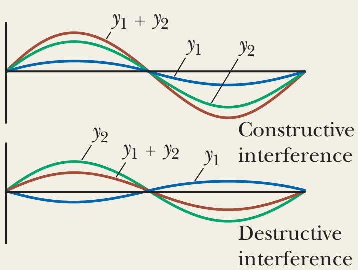
Figure 18.4 An acoustical system for demonstrating interference of sound waves. The upper path length r_2 can be varied by sliding the upper section.

Interference of Sound Waves

One simple device for demonstrating interference of sound waves is illustrated in Figure 18.4. Sound from a loudspeaker S is sent into a tube at point P, where there is a T-shaped junction. Half the sound energy travels in one direction, and half travels in the opposite direction. Therefore, the sound waves that reach the receiver R can travel along either of the two paths. The distance along any path from speaker to receiver is called the **path length** r. The lower path length r_1 is fixed, but the upper path length r_2 can be varied by sliding the U-shaped tube, which is similar to that on a slide trombone. When the difference in the path lengths $\Delta r = |r_2 - r_1|$ is either zero or some integer multiple of the wavelength λ (that is, $\Delta r = n\lambda$, where $n = 0, 1, 2, 3, \ldots$), the two waves reaching the receiver at any instant are in phase and interfere constructively as shown in Figure 18.3a. For this case, a maximum in the sound intensity is detected at the receiver. If the path length r_2 is adjusted such that the path difference $\Delta r = \lambda/2, 3\lambda/2, \ldots, n\lambda/2$ (for n odd), the two waves are exactly π rad, or 180°, out of phase at the receiver and hence cancel each other. In this case of destructive interference, no sound is detected at the receiver. This simple experiment demonstrates that a phase difference may arise between two waves generated by the same source when they travel along paths of unequal lengths. This important phenomenon will be indispensable in our investigation of the interference of light waves in Chapter 37.

Analysis Model Waves in Interference

Imagine two waves traveling in the same location through a medium. The displacement of elements of the medium is affected by both waves. According to the **principle of superposition**, the displacement is the sum of the individual displacements that would be caused by each wave. When the waves are in phase, **constructive interference** occurs and the resultant displacement is larger than the individual displacements. **Destructive interference** occurs when the waves are out of phase.



Examples:

- a piano tuner listens to a piano string and a tuning fork vibrating together and notices beats (Section 18.7)
- light waves from two coherent sources combine to form an interference pattern on a screen (Chapter 37)
- a thin film of oil on top of water shows swirls of color (Chapter 37)
- x-rays passing through a crystalline solid combine to form a Laue pattern (Chapter 38)

Example 18.1 Two Speakers Driven by the Same Source AM

Two identical loudspeakers placed 3.00 m apart are driven by the same oscillator (Fig. 18.5). A listener is originally at point O, located 8.00 m from the center of the line connecting the two speakers. The listener then moves to point P, which is a perpendicular distance 0.350 m from O, and she experiences the *first minimum* in sound intensity. What is the frequency of the oscillator?

SOLUTION

Conceptualize In Figure 18.4, a sound wave enters a tube and is then *acoustically* split into two different paths before recombining at the other end. In this example, a signal representing the sound is *electrically* split and sent to two different loudspeakers. After leaving the speakers, the sound waves recombine at the position of the listener. Despite the difference in how the splitting occurs, the path difference discussion related to Figure 18.4 can be applied here.

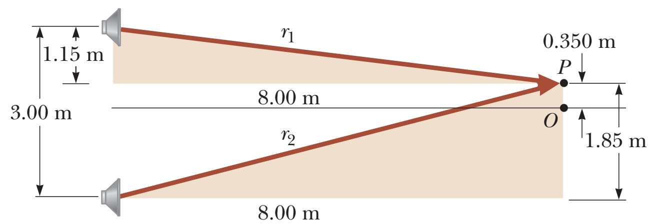


Figure 18.5 (Example 18.1) Two identical loudspeakers emit sound waves to a listener at P.

Categorize Because the sound waves from two separate sources combine, we apply the *waves in interference* analysis model.

continued

18.1 continued

Analyze Figure 18.5 shows the physical arrangement of the speakers, along with two shaded right triangles that can be drawn on the basis of the lengths described in the problem. The first minimum occurs when the two waves reaching the listener at point *P* are 180° out of phase, in other words, when their path difference Δr equals $\lambda/2$.

From the shaded triangles, find the path lengths from the speakers to the listener:

$$r_1 = \sqrt{(8.00\text{ m})^2 + (1.15\text{ m})^2} = 8.08\text{ m}$$

$$r_2 = \sqrt{(8.00\text{ m})^2 + (1.85\text{ m})^2} = 8.21\text{ m}$$

Hence, the path difference is $r_2 - r_1 = 0.13$ m. Because this path difference must equal $\lambda/2$ for the first minimum, $\lambda = 0.26$ m.

To obtain the oscillator frequency, use Equation 16.12, $v = \lambda f$, where v is the speed of sound in air, 343 m/s:

$$f = \frac{v}{\lambda} = \frac{343\text{ m/s}}{0.26\text{ m}} = \boxed{1.3\text{ kHz}}$$

Finalize This example enables us to understand why the speaker wires in a stereo system should be connected properly. When connected the wrong way—that is, when the positive (or red) wire is connected to the negative (or black) terminal on one of the speakers and the other is correctly wired—the speakers are said to be "out of phase," with one speaker moving outward while the other moves inward. As a consequence, the sound wave coming from one speaker destructively interferes with the wave coming from the other at point *O* in Figure 18.5. A rarefaction region due to one speaker is superposed on a compression region from the other speaker. Although the two sounds probably do not completely cancel each other (because the left and right stereo signals are usually not identical), a substantial loss of sound quality occurs at point *O*.

WHAT IF? What if the speakers were connected out of phase? What happens at point *P* in Figure 18.5?

Answer In this situation, the path difference of $\lambda/2$ combines with a phase difference of $\lambda/2$ due to the incorrect wiring to give a full phase difference of λ. As a result, the waves are in phase and there is a *maximum* intensity at point *P*.

Figure 18.6 Two identical loudspeakers emit sound waves toward each other. When they overlap, identical waves traveling in opposite directions will combine to form standing waves.

18.2 Standing Waves

The sound waves from the pair of loudspeakers in Example 18.1 leave the speakers in the forward direction, and we considered interference at a point in front of the speakers. Suppose we turn the speakers so that they face each other and then have them emit sound of the same frequency and amplitude. In this situation, two identical waves travel in opposite directions in the same medium as in Figure 18.6. These waves combine in accordance with the waves in interference model.

We can analyze such a situation by considering wave functions for two transverse sinusoidal waves having the same amplitude, frequency, and wavelength but traveling in opposite directions in the same medium:

$$y_1 = A \sin (kx - \omega t) \qquad y_2 = A \sin (kx + \omega t)$$

where y_1 represents a wave traveling in the positive x direction and y_2 represents one traveling in the negative x direction. Adding these two functions gives the resultant wave function y:

$$y = y_1 + y_2 = A \sin (kx - \omega t) + A \sin (kx + \omega t)$$

When we use the trigonometric identity $\sin (a \pm b) = \sin a \cos b \pm \cos a \sin b$, this expression reduces to

$$y = (2A \sin kx) \cos \omega t \qquad \textbf{(18.1)}$$

Equation 18.1 represents the wave function of a **standing wave.** A standing wave such as the one on a string shown in Figure 18.7 is an oscillation pattern *with a stationary outline* that results from the superposition of two identical waves traveling in opposite directions.

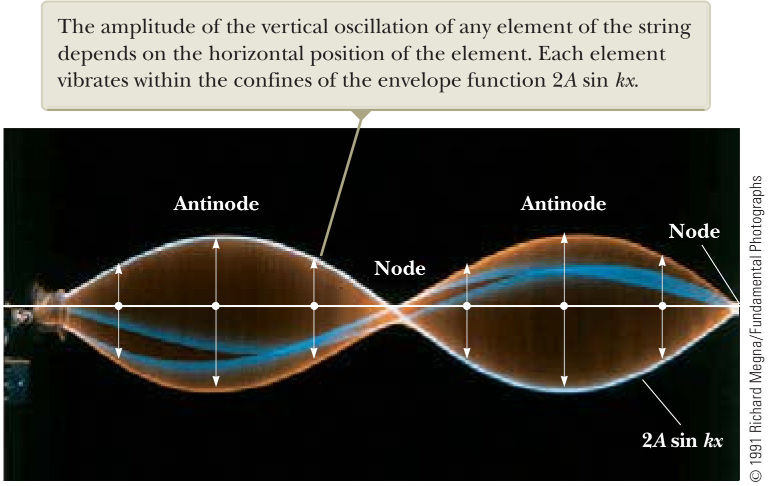


Figure 18.7 Multiflash photograph of a standing wave on a string. The time behavior of the vertical displacement from equilibrium of an individual element of the string is given by cos ωt. That is, each element vibrates at an angular frequency ω.

Notice that Equation 18.1 does not contain a function of $kx - \omega t$. Therefore, it is not an expression for a single traveling wave. When you observe a standing wave, there is no sense of motion in the direction of propagation of either original wave. Comparing Equation 18.1 with Equation 15.6, we see that it describes a special kind of simple harmonic motion. Every element of the medium oscillates in simple harmonic motion with the same angular frequency ω (according to the cos ωt factor in the equation). The amplitude of the simple harmonic motion of a given element (given by the factor $2A \sin kx$, the coefficient of the cosine function) depends on the location x of the element in the medium, however.

If you can find a noncordless telephone with a coiled cord connecting the handset to the base unit, you can see the difference between a standing wave and a traveling wave. Stretch the coiled cord out and flick it with a finger. You will see a pulse traveling along the cord. Now shake the handset up and down and adjust your shaking frequency until every coil on the cord is moving up at the same time and then down. That is a standing wave, formed from the combination of waves moving away from your hand and reflected from the base unit toward your hand. Notice that there is no sense of traveling along the cord like there was for the pulse. You only see up-and-down motion of the elements of the cord.

Pitfall Prevention 18.2

Three Types of Amplitude We need to distinguish carefully here between the **amplitude of the individual waves,** which is A, and the **amplitude of the simple harmonic motion of the elements of the medium,** which is $2A \sin kx$. A given element in a standing wave vibrates within the constraints of the *envelope* function $2A \sin kx$, where x is that element's position in the medium. Such vibration is in contrast to traveling sinusoidal waves, in which all elements oscillate with the same amplitude and the same frequency and the amplitude A of the wave is the same as the amplitude A of the simple harmonic motion of the elements. Furthermore, we can identify the **amplitude of the standing wave** as $2A$.

Equation 18.1 shows that the amplitude of the simple harmonic motion of an element of the medium has a minimum value of zero when x satisfies the condition $\sin kx = 0$, that is, when

$$kx = 0, \pi, 2\pi, 3\pi, \ldots$$

Because $k = 2\pi/\lambda$, these values for kx give

$$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots = \frac{n\lambda}{2} \quad n = 0, 1, 2, 3, \ldots \tag{18.2}$$

◀ Positions of nodes

These points of zero amplitude are called **nodes.**

The element of the medium with the *greatest* possible displacement from equilibrium has an amplitude of $2A$, which we define as the amplitude of the standing wave. The positions in the medium at which this maximum displacement occurs are called **antinodes.** The antinodes are located at positions for which the coordinate x satisfies the condition $\sin kx = \pm 1$, that is, when

$$kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

Therefore, the positions of the antinodes are given by

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots = \frac{n\lambda}{4} \quad n = 1, 3, 5, \ldots \tag{18.3}$$

◀ Positions of antinodes

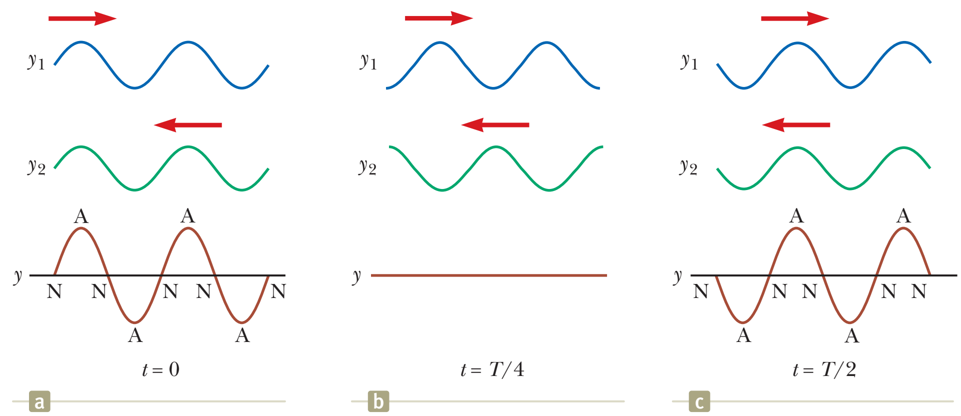


Figure 18.8 Standing-wave patterns produced at various times by two waves of equal amplitude traveling in opposite directions. For the resultant wave y, the nodes (N) are points of zero displacement and the antinodes (A) are points of maximum displacement.

Two nodes and two antinodes are labeled in the standing wave in Figure 18.7. The light blue curve labeled $2A \sin kx$ in Figure 18.7 represents one wavelength of the traveling waves that combine to form the standing wave. Figure 18.7 and Equations 18.2 and 18.3 provide the following important features of the locations of nodes and antinodes:

The distance between adjacent antinodes is equal to $\lambda/2$.
The distance between adjacent nodes is equal to $\lambda/2$.
The distance between a node and an adjacent antinode is $\lambda/4$.

Wave patterns of the elements of the medium produced at various times by two transverse traveling waves moving in opposite directions are shown in Figure 18.8. The blue and green curves are the wave patterns for the individual traveling waves, and the red-brown curves are the wave patterns for the resultant standing wave. At $t = 0$ (Fig. 18.8a), the two traveling waves are in phase, giving a wave pattern in which each element of the medium is at rest and experiencing its maximum displacement from equilibrium. One-quarter of a period later, at $t = T/4$ (Fig. 18.8b), the traveling waves have moved one-fourth of a wavelength (one to the right and the other to the left). At this time, the traveling waves are out of phase, and each element of the medium is passing through the equilibrium position in its simple harmonic motion. The result is zero displacement for elements at all values of x; that is, the wave pattern is a straight line. At $t = T/2$ (Fig. 18.8c), the traveling waves are again in phase, producing a wave pattern that is inverted relative to the $t = 0$ pattern. In the standing wave, the elements of the medium alternate in time between the extremes shown in Figures 18.8a and 18.8c.

Quick Quiz 18.2 Consider the waves in Figure 18.8 to be waves on a stretched string. Define the velocity of elements of the string as positive if they are moving upward in the figure. **(i)** At the moment the string has the shape shown by the red-brown curve in Figure 18.8a, what is the instantaneous velocity of elements along the string? (a) zero for all elements (b) positive for all elements (c) negative for all elements (d) varies with the position of the element **(ii)** From the same choices, at the moment the string has the shape shown by the red-brown curve in Figure 18.8b, what is the instantaneous velocity of elements along the string?

Example 18.2 Formation of a Standing Wave

Two waves traveling in opposite directions produce a standing wave. The individual wave functions are

$$y_1 = 4.0 \sin (3.0x - 2.0t)$$

$$y_2 = 4.0 \sin (3.0x + 2.0t)$$

where x and y are measured in centimeters and t is in seconds.

(A) Find the amplitude of the simple harmonic motion of the element of the medium located at $x = 2.3$ cm.

18.2 continued

SOLUTION

Conceptualize The waves described by the given equations are identical except for their directions of travel, so they indeed combine to form a standing wave as discussed in this section. We can represent the waves graphically by the blue and green curves in Figure 18.8.

Categorize We will substitute values into equations developed in this section, so we categorize this example as a substitution problem.

From the equations for the waves, we see that $A = 4.0$ cm, $k = 3.0$ rad/cm, and $\omega = 2.0$ rad/s. Use Equation 18.1 to write an expression for the standing wave:

$$y = (2A \sin kx) \cos \omega t = 8.0 \sin 3.0x \cos 2.0t$$

Find the amplitude of the simple harmonic motion of the element at the position $x = 2.3$ cm by evaluating the sine function at this position:

$$y_{\max} = (8.0 \text{ cm}) \sin 3.0x \big|_{x=2.3}$$
$$= (8.0 \text{ cm}) \sin (6.9 \text{ rad}) = \boxed{4.6 \text{ cm}}$$

(B) Find the positions of the nodes and antinodes if one end of the string is at $x = 0$.

SOLUTION

Find the wavelength of the traveling waves:

$$k = \frac{2\pi}{\lambda} = 3.0 \text{ rad/cm} \rightarrow \lambda = \frac{2\pi}{3.0} \text{ cm}$$

Use Equation 18.2 to find the locations of the nodes:

$$x = n\frac{\lambda}{2} = \boxed{n\left(\frac{\pi}{3.0}\right) \text{cm} \quad n = 0, 1, 2, 3, \ldots}$$

Use Equation 18.3 to find the locations of the antinodes:

$$x = n\frac{\lambda}{4} = \boxed{n\left(\frac{\pi}{6.0}\right) \text{cm} \quad n = 1, 3, 5, 7, \ldots}$$

18.3 Analysis Model: Waves Under Boundary Conditions

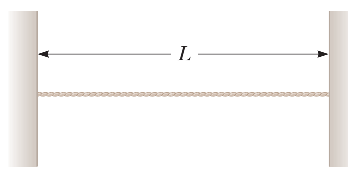


Figure 18.9 A string of length L fixed at both ends.

Consider a string of length L fixed at both ends as shown in Figure 18.9. We will use this system as a model for a guitar string or piano string. Waves can travel in both directions on the string. Therefore, standing waves can be set up in the string by a continuous superposition of waves incident on and reflected from the ends. Notice that there is a *boundary condition* for the waves on the string: because the ends of the string are fixed, they must necessarily have zero displacement and are therefore nodes by definition. The condition that both ends of the string must be nodes fixes the wavelength of the standing wave on the string according to Equation 18.2, which, in turn, determines the frequency of the wave. The boundary condition results in the string having a number of discrete natural patterns of oscillation, called **normal modes,** each of which has a characteristic frequency that is easily calculated. This situation in which only certain frequencies of oscillation are allowed is called **quantization.** Quantization is a common occurrence when waves are subject to boundary conditions and is a central feature in our discussions of quantum physics in the extended version of this text. Notice in Figure 18.8 that there are no boundary conditions, so standing waves of *any* frequency can be established; there is no quantization without boundary conditions. Because boundary conditions occur so often for waves, we identify an analysis model called **waves under boundary conditions** for the discussion that follows.

The normal modes of oscillation for the string in Figure 18.9 can be described by imposing the boundary conditions that the ends be nodes and that the nodes be separated by one-half of a wavelength with antinodes halfway between the nodes. The first normal mode that is consistent with these requirements, shown in Figure 18.10a (page 542), has nodes at its ends and one antinode in the middle. This normal

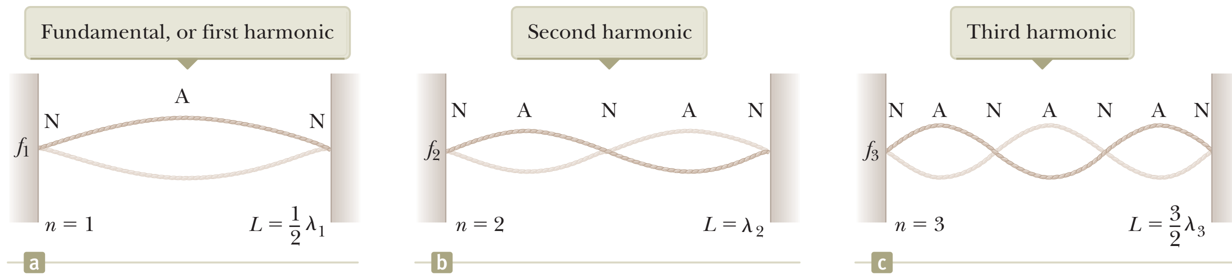


Figure 18.10 The normal modes of vibration of the string in Figure 18.9 form a harmonic series. The string vibrates between the extremes shown.

mode is the longest-wavelength mode that is consistent with our boundary conditions. The first normal mode occurs when the wavelength λ_1 is equal to twice the length of the string, or $\lambda_1 = 2L$. The section of a standing wave from one node to the next node is called a *loop*. In the first normal mode, the string is vibrating in one loop. In the second normal mode (see Fig. 18.10b), the string vibrates in two loops. When the left half of the string is moving upward, the right half is moving downward. In this case, the wavelength λ_2 is equal to the length of the string, as expressed by $\lambda_2 = L$. The third normal mode (see Fig. 18.10c) corresponds to the case in which $\lambda_3 = 2L/3$, and the string vibrates in three loops. In general, the wavelengths of the various normal modes for a string of length L fixed at both ends are

Wavelengths of normal modes ▶

$$\lambda_n = \frac{2L}{n} \quad n = 1, 2, 3, \ldots \tag{18.4}$$

where the index n refers to the nth normal mode of oscillation. These modes are *possible*. The *actual* modes that are excited on a string are discussed shortly.

The natural frequencies associated with the modes of oscillation are obtained from the relationship $f = v/\lambda$, where the wave speed v is the same for all frequencies. Using Equation 18.4, we find that the natural frequencies f_n of the normal modes are

Natural frequencies of normal modes as functions of wave speed and length of string ▶

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2L} \quad n = 1, 2, 3, \ldots \tag{18.5}$$

These natural frequencies are also called the *quantized frequencies* associated with the vibrating string fixed at both ends.

Because $v = \sqrt{T/\mu}$ (see Eq. 16.18) for waves on a string, where T is the tension in the string and μ is its linear mass density, we can also express the natural frequencies of a taut string as

Natural frequencies of normal modes as functions of string tension and linear mass density ▶

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}} \quad n = 1, 2, 3, \ldots \tag{18.6}$$

The lowest frequency f_1, which corresponds to $n = 1$, is called either the **fundamental** or the **fundamental frequency** and is given by

Fundamental frequency of a taut string ▶

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \tag{18.7}$$

The frequencies of the remaining normal modes are integer multiples of the fundamental frequency (Eq. 18.5). Frequencies of normal modes that exhibit such an integer- multiple relationship form a **harmonic series,** and the normal modes are called **harmonics.** The fundamental frequency f_1 is the frequency of the first harmonic, the frequency $f_2 = 2f_1$ is that of the second harmonic, and the frequency $f_n = nf_1$ is that of the nth harmonic. Other oscillating systems, such as a drumhead, exhibit normal modes, but the frequencies are not related as integer multiples of a fundamental (see Section 18.6). Therefore, we do not use the term *harmonic* in association with those types of systems.

Let us examine further how the various harmonics are created in a string. To excite only a single harmonic, the string would have to be distorted into a shape that corresponds to that of the desired harmonic. After being released, the string would vibrate at the frequency of that harmonic. This maneuver is difficult to perform, however, and is not how a string of a musical instrument is excited. If the string is distorted into a general, nonsinusoidal shape, the resulting vibration includes a combination of various harmonics. Such a distortion occurs in musical instruments when the string is plucked (as in a guitar), bowed (as in a cello), or struck (as in a piano). When the string is distorted into a nonsinusoidal shape, only waves that satisfy the boundary conditions can persist on the string. These waves are the harmonics.

The frequency of a string that defines the musical note that it plays is that of the fundamental, even though other harmonics are present. The string's frequency can be varied by changing the string's tension or its length. For example, the tension in guitar and violin strings is varied by a screw adjustment mechanism or by tun ing pegs located on the neck of the instrument. As the tension is increased, the frequency of the normal modes increases in accordance with Equation 18.6. Once the instrument is "tuned," players vary the frequency by moving their fingers along the neck, thereby changing the length of the oscillating portion of the string. As the length is shortened, the frequency increases because, as Equation 18.6 specifies, the normal-mode frequencies are inversely proportional to string length.

uick Quiz 18.3 When a standing wave is set up on a string fixed at both ends, which of the following statements is true? **(a)** The number of nodes is equal to the number of antinodes. **(b)** The wavelength is equal to the length of the string divided by an integer. **(c)** The frequency is equal to the number of nodes times the fundamental frequency. **(d)** The shape of the string at any instant shows a symmetry about the midpoint of the string.

Analysis Model Waves Under Boundary Conditions

Imagine a wave that is not free to travel throughout all space as in the traveling wave model. If the wave is subject to bound ary conditions, such that cer tain requirements must be met at specific locations in space, the wave is limited to a set of **normal modes** with quantized wavelengths and quantized natural frequencies.

For waves on a string fixed at both ends, the natural frequencies are

— — 1, 2, 3, **(18.6)**

where is the tension in the string and is its linear mass density.

Examples:

- waves traveling back and forth on a guitar string combine to form a standing wave
- sound waves traveling back and forth in a clarinet combine to form standing waves (Section 18.5)
- a microscopic particle confined to small region of space is modeled as a wave and exhibits quantized energies (Chapter 41)
- the Fermi energy of metal is deter mined by modeling electrons as wave-like particles in a box (Chapter 43)

Example 18.3 Give Me a C Note!

The middle C string on a piano has a fundamental frequency of 262 Hz, and the string for the first A above middle C has a fundamental frequency of 440 Hz.

Calculate the frequencies of the next two harmonics of the C string.

continued

18.3 continued

SOLUTION

Conceptualize Remember that the harmonics of a vibrating string have frequencies that are related by integer multiples of the fundamental.

Categorize This first part of the example is a simple substitution problem.

Knowing that the fundamental frequency is $f_1 = 262$ Hz, find the frequencies of the next harmonics by multiplying by integers:

$$f_2 = 2f_1 = \boxed{524 \text{ Hz}}$$
$$f_3 = 3f_1 = \boxed{786 \text{ Hz}}$$

(B) If the A and C strings have the same linear mass density μ and length L, determine the ratio of tensions in the two strings.

SOLUTION

Categorize This part of the example is more of an analysis problem than is part (A) and uses the *waves under boundary conditions* model.

Analyze Use Equation 18.7 to write expressions for the fundamental frequencies of the two strings:

$$f_{1A} = \frac{1}{2L}\sqrt{\frac{T_A}{\mu}} \quad \text{and} \quad f_{1C} = \frac{1}{2L}\sqrt{\frac{T_C}{\mu}}$$

Divide the first equation by the second and solve for the ratio of tensions:

$$\frac{f_{1A}}{f_{1C}} = \sqrt{\frac{T_A}{T_C}} \rightarrow \frac{T_A}{T_C} = \left(\frac{f_{1A}}{f_{1C}}\right)^2 = \left(\frac{440}{262}\right)^2 = \boxed{2.82}$$

Finalize If the frequencies of piano strings were determined solely by tension, this result suggests that the ratio of tensions from the lowest string to the highest string on the piano would be enormous. Such large tensions would make it difficult to design a frame to support the strings. In reality, the frequencies of piano strings vary due to additional parameters, including the mass per unit length and the length of the string. The What If? below explores a variation in length.

WHAT IF? If you look inside a real piano, you'll see that the assumption made in part (B) is only partially true. The strings are not likely to have the same length. The string densities for the given notes might be equal, but suppose the length of the A string is only 64% of the length of the C string. What is the ratio of their tensions?

Answer Using Equation 18.7 again, we set up the ratio of frequencies:

$$\frac{f_{1A}}{f_{1C}} = \frac{L_C}{L_A}\sqrt{\frac{T_A}{T_C}} \rightarrow \frac{T_A}{T_C} = \left(\frac{L_A}{L_C}\right)^2\left(\frac{f_{1A}}{f_{1C}}\right)^2$$

$$\frac{T_A}{T_C} = (0.64)^2\left(\frac{440}{262}\right)^2 = 1.16$$

Notice that this result represents only a 16% increase in tension, compared with the 182% increase in part (B).

Example 18.4 Changing String Vibration with Water AM

One end of a horizontal string is attached to a vibrating blade, and the other end passes over a pulley as in Figure 18.11a. A sphere of mass 2.00 kg hangs on the end of the string. The string is vibrating in its second harmonic. A container of water is raised under the sphere so that the sphere is completely submerged. In this configuration, the string vibrates in its fifth harmonic as shown in Figure 18.11b. What is the radius of the sphere?

SOLUTION

Conceptualize Imagine what happens when the sphere is immersed in the water. The buoyant force acts upward on the sphere, reducing the tension in the string. The change in tension causes a change in the speed of waves on the

18.4 continued

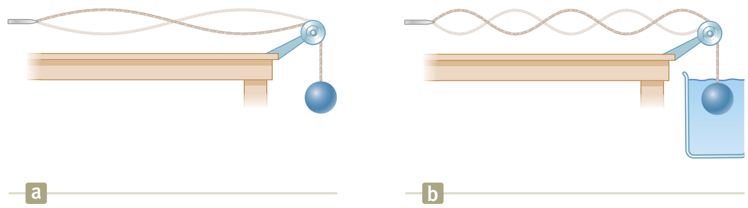


Figure 18.11 (Example 18.4) (a) When the sphere hangs in air, the string vibrates in its second harmonic. (b) When the sphere is immersed in water, the string vibrates in its fifth harmonic.

string, which in turn causes a change in the wavelength. This altered wavelength results in the string vibrating in its fifth normal mode rather than the second.

Categorize The hanging sphere is modeled as a *particle in equilibrium*. One of the forces acting on it is the buoyant force from the water. We also apply the *waves under boundary conditions* model to the string.

Analyze Apply the particle in equilibrium model to the sphere in Figure 18.11a, identifying T_1 as the tension in the string as the sphere hangs in air:

$$\sum F = T_1 - mg = 0$$

$$T_1 = mg$$

Apply the particle in equilibrium model to the sphere in Figure 18.11b, where T_2 is the tension in the string as the sphere is immersed in water:

$$T_2 + B - mg = 0$$

$$(1) \quad B = mg - T_2$$

The desired quantity, the radius of the sphere, will appear in the expression for the buoyant force B. Before proceeding in this direction, however, we must evaluate T_2 from the information about the standing wave.

Write the equation for the frequency of a standing wave on a string (Eq. 18.6) twice, once before the sphere is immersed and once after. Notice that the frequency f is the same in both cases because it is determined by the vibrating blade. In addition, the linear mass density μ and the length L of the vibrating portion of the string are the same in both cases. Divide the equations:

$$f = \frac{n_1}{2L}\sqrt{\frac{T_1}{\mu}}, \quad f = \frac{n_2}{2L}\sqrt{\frac{T_2}{\mu}} \quad \rightarrow \quad 1 = \frac{n_1}{n_2}\sqrt{\frac{T_1}{T_2}}$$

Solve for T_2:

$$T_2 = \left(\frac{n_1}{n_2}\right)^2 T_1 = \left(\frac{n_1}{n_2}\right)^2 mg$$

Substitute this result into Equation (1):

$$(2) \quad B = mg - \left(\frac{n_1}{n_2}\right)^2 mg = mg\left[1 - \left(\frac{n_1}{n_2}\right)^2\right]$$

Using Equation 14.5, express the buoyant force in terms of the radius of the sphere:

$$B = \rho_{\text{water}} g V_{\text{sphere}} = \rho_{\text{water}} g\left(\tfrac{4}{3}\pi r^3\right)$$

Solve for the radius of the sphere and substitute from Equation (2):

$$r = \left(\frac{3B}{4\pi\rho_{\text{water}} g}\right)^{1/3} = \left\{\frac{3m}{4\pi\rho_{\text{water}}}\left[1 - \left(\frac{n_1}{n_2}\right)^2\right]\right\}^{1/3}$$

Substitute numerical values:

$$r = \left\{\frac{3(2.00\text{ kg})}{4\pi(1\,000\text{ kg/m}^3)}\left[1 - \left(\frac{2}{5}\right)^2\right]\right\}^{1/3}$$

$$= 0.073\,7\text{ m} = \boxed{7.37\text{ cm}}$$

Finalize Notice that only certain radii of the sphere will result in the string vibrating in a normal mode; the speed of waves on the string must be changed to a value such that the length of the string is an integer multiple of half wavelengths. This limitation is a feature of the *quantization* that was introduced earlier in this chapter: the sphere radii that cause the string to vibrate in a normal mode are *quantized*.

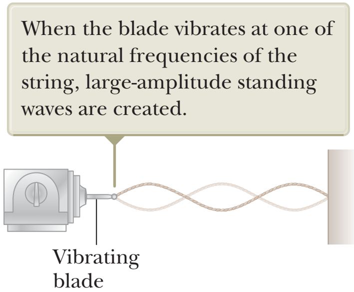


Figure 18.12 Standing waves are set up in a string when one end is connected to a vibrating blade.

18.4 Resonance

We have seen that a system such as a taut string is capable of oscillating in one or more normal modes of oscillation. Suppose we drive such a string with a vibrating blade as in Figure 18.12. We find that if a periodic force is applied to such a system, the amplitude of the resulting motion of the string is greatest when the frequency of the applied force is equal to one of the natural frequencies of the system. This phenomenon, known as *resonance,* was discussed in Section 15.7 with regard to a simple harmonic oscillator. Although a block–spring system or a simple pendulum has only one natural frequency, standing-wave systems have a whole set of natural frequencies, such as that given by Equation 18.6 for a string. Because an oscillating system exhibits a large amplitude when driven at any of its natural frequencies, these frequencies are often referred to as **resonance frequencies.**

Consider the string in Figure 18.12 again. The fixed end is a node, and the end connected to the blade is very nearly a node because the amplitude of the blade's motion is small compared with that of the elements of the string. As the blade oscillates, transverse waves sent down the string are reflected from the fixed end. As we learned in Section 18.3, the string has natural frequencies that are determined by its length, tension, and linear mass density (see Eq. 18.6). When the frequency of the blade equals one of the natural frequencies of the string, standing waves are produced and the string oscillates with a large amplitude. In this resonance case, the wave generated by the oscillating blade is in phase with the reflected wave and the string absorbs energy from the blade. If the string is driven at a frequency that is not one of its natural frequencies, the oscillations are of low amplitude and exhibit no stable pattern.

Resonance is very important in the excitation of musical instruments based on air columns. We shall discuss this application of resonance in Section 18.5.

18.5 Standing Waves in Air Columns

The waves under boundary conditions model can also be applied to sound waves in a column of air such as that inside an organ pipe or a clarinet. Standing waves in this case are the result of interference between longitudinal sound waves traveling in opposite directions.

In a pipe closed at one end, the closed end is a **displacement node** because the rigid barrier at this end does not allow longitudinal motion of the air. Because the pressure wave is 90° out of phase with the displacement wave (see Section 17.1), the closed end of an air column corresponds to a **pressure antinode** (that is, a point of maximum pressure variation).

The open end of an air column is approximately a **displacement antinode**[1] and a pressure node. We can understand why no pressure variation occurs at an open end by noting that the end of the air column is open to the atmosphere; therefore, the pressure at this end must remain constant at atmospheric pressure.

You may wonder how a sound wave can reflect from an open end because there may not appear to be a change in the medium at this point: the medium through which the sound wave moves is air both inside and outside the pipe. Sound can be represented as a pressure wave, however, and a compression region of the sound wave is constrained by the sides of the pipe as long as the region is inside the pipe. As the compression region exits at the open end of the pipe, the constraint of the pipe is removed and the compressed air is free to expand into the atmosphere. Therefore, there is a change in the *character* of the medium between the inside

[1]Strictly speaking, the open end of an air column is not exactly a displacement antinode. A compression reaching an open end does not reflect until it passes beyond the end. For a tube of circular cross section, an end correction equal to approximately $0.6R$, where R is the tube's radius, must be added to the length of the air column. Hence, the effective length of the air column is longer than the true length L. We ignore this end correction in this discussion.

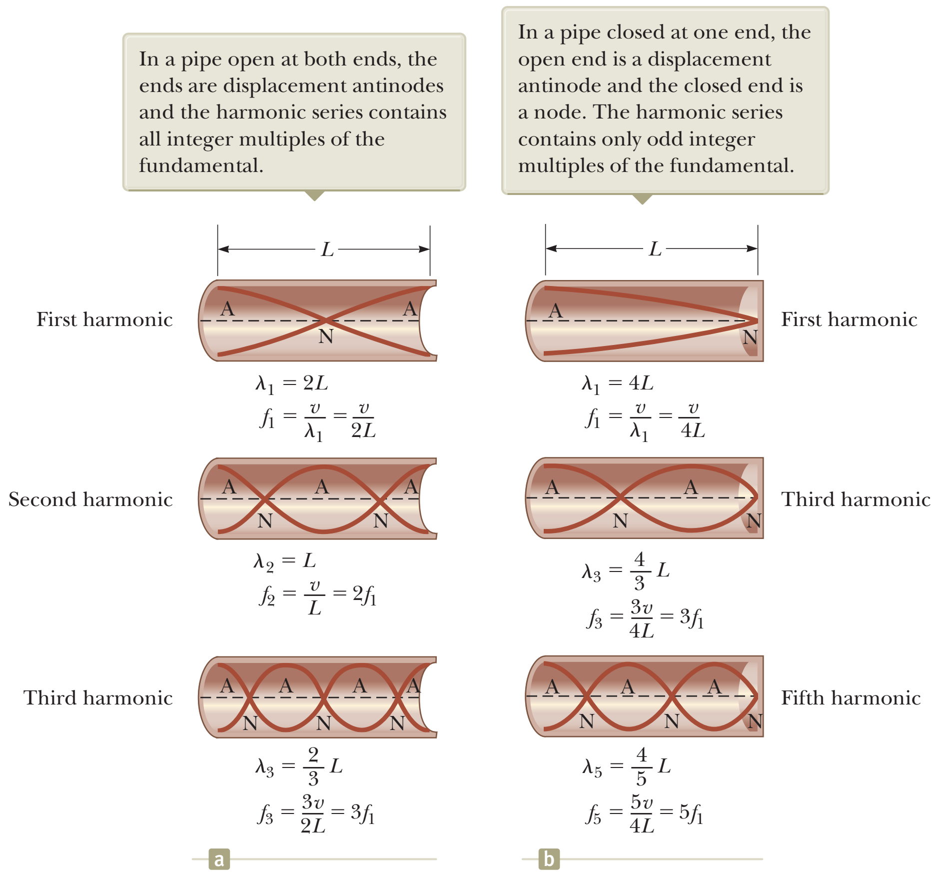


Figure 18.13 Graphical representations of the motion of elements of air in standing longitudinal waves in (a) a column open at both ends and (b) a column closed at one end.

of the pipe and the outside even though there is no change in the *material* of the medium. This change in character is sufficient to allow some reflection.

With the boundary conditions of nodes or antinodes at the ends of the air column, we have a set of normal modes of oscillation as is the case for the string fixed at both ends. Therefore, the air column has quantized frequencies.

The first three normal modes of oscillation of a pipe open at both ends are shown in Figure 18.13a. Notice that both ends are displacement antinodes (approximately). In the first normal mode, the standing wave extends between two adjacent antinodes, which is a distance of half a wavelength. Therefore, the wavelength is twice the length of the pipe, and the fundamental frequency is $f_1 = v/2L$. As Figure 18.13a shows, the frequencies of the higher harmonics are $2f_1, 3f_1, \ldots$.

> **Pitfall Prevention 18.3**
>
> **Sound Waves in Air Are Longitudinal, Not Transverse** The standing longitudinal waves are drawn as transverse waves in Figure 18.13. Because they are in the same direction as the propagation, it is difficult to draw longitudinal displacements. Therefore, it is best to interpret the red-brown curves in Figure 18.13 as a graphical representation of the waves (our diagrams of string waves are pictorial representations), with the vertical axis representing the horizontal displacement $s(x, t)$ of the elements of the medium.

In a pipe open at both ends, the natural frequencies of oscillation form a harmonic series that includes all integral multiples of the fundamental frequency.

Because all harmonics are present and because the fundamental frequency is given by the same expression as that for a string (see Eq. 18.5), we can express the natural frequencies of oscillation as

Natural frequencies of a pipe open at both ends

$$f_n = n\frac{v}{2L} \qquad n = 1, 2, 3, \ldots \tag{18.8}$$

Despite the similarity between Equations 18.5 and 18.8, you must remember that v in Equation 18.5 is the speed of waves on the string, whereas v in Equation 18.8 is the speed of sound in air.

If a pipe is closed at one end and open at the other, the closed end is a displacement node (see Fig. 18.13b). In this case, the standing wave for the fundamental mode extends from an antinode to the adjacent node, which is one-fourth of a wavelength. Hence, the wavelength for the first normal mode is $4L$, and the fundamental frequency is $f_1 = v/4L$. As Figure 18.13b shows, the higher-frequency waves that satisfy our conditions are those that have a node at the closed end and an antinode at the open end; hence, the higher harmonics have frequencies $3f_1, 5f_1, \ldots$.

In a pipe closed at one end, the natural frequencies of oscillation form a harmonic series that includes only odd integral multiples of the fundamental frequency.

We express this result mathematically as

Natural frequencies of a pipe closed at one end and open at the other

$$f_n = n\frac{v}{4L} \quad n = 1, 3, 5, \ldots \tag{18.9}$$

It is interesting to investigate what happens to the frequencies of instruments based on air columns and strings during a concert as the temperature rises. The sound emitted by a flute, for example, becomes sharp (increases in frequency) as the flute warms up because the speed of sound increases in the increasingly warmer air inside the flute (consider Eq. 18.8). The sound produced by a violin becomes flat (decreases in frequency) as the strings thermally expand because the expansion causes their tension to decrease (see Eq. 18.6).

Musical instruments based on air columns are generally excited by resonance. The air column is presented with a sound wave that is rich in many frequencies. The air column then responds with a large-amplitude oscillation to the frequencies that match the quantized frequencies in its set of harmonics. In many woodwind instruments, the initial rich sound is provided by a vibrating reed. In brass instruments, this excitation is provided by the sound coming from the vibration of the player's lips. In a flute, the initial excitation comes from blowing over an edge at the mouthpiece of the instrument in a manner similar to blowing across the opening of a bottle with a narrow neck. The sound of the air rushing across the bottle opening has many frequencies, including one that sets the air cavity in the bottle into resonance.

Quick Quiz 18.4 A pipe open at both ends resonates at a fundamental frequency f_{open}. When one end is covered and the pipe is again made to resonate, the fundamental frequency is f_{closed}. Which of the following expressions describes how these two resonant frequencies compare? **(a)** $f_{\text{closed}} = f_{\text{open}}$ **(b)** $f_{\text{closed}} = \frac{1}{2}f_{\text{open}}$ **(c)** $f_{\text{closed}} = 2f_{\text{open}}$ (d) $f_{\text{closed}} = \frac{3}{2}f_{\text{open}}$

Quick Quiz 18.5 Balboa Park in San Diego has an outdoor organ. When the air temperature increases, the fundamental frequency of one of the organ pipes **(a)** stays the same, **(b)** goes down, **(c)** goes up, or **(d)** is impossible to determine.

Example 18.5 Wind in a Culvert

A section of drainage culvert 1.23 m in length makes a howling noise when the wind blows across its open ends.

(A) Determine the frequencies of the first three harmonics of the culvert if it is cylindrical in shape and open at both ends. Take $v = 343$ m/s as the speed of sound in air.

SOLUTION

Conceptualize The sound of the wind blowing across the end of the pipe contains many frequencies, and the culvert responds to the sound by vibrating at the natural frequencies of the air column.

Categorize This example is a relatively simple substitution problem.

Find the frequency of the first harmonic of the culvert, modeling it as an air column open at both ends:

$$f_1 = \frac{v}{2L} = \frac{343 \text{ m/s}}{2(1.23 \text{ m})} = \boxed{139 \text{ Hz}}$$

Find the next harmonics by multiplying by integers:

$$f_2 = 2f_1 = \boxed{279 \text{ Hz}}$$

$$f_3 = 3f_1 = \boxed{418 \text{ Hz}}$$

18.5 continued

(B) What are the three lowest natural frequencies of the culvert if it is blocked at one end?

SOLUTION

Find the frequency of the first harmonic of the culvert, modeling it as an air column closed at one end:

$$f_1 = \frac{v}{4L} = \frac{343 \text{ m/s}}{4(1.23 \text{ m})} = 69.7 \text{ Hz}$$

Find the next two harmonics by multiplying by odd integers:

$$f_3 = 3f_1 = 209 \text{ Hz}$$

$$f_5 = 5f_1 = 349 \text{ Hz}$$

Example 18.6 Measuring the Frequency of a Tuning Fork AM

A simple apparatus for demonstrating resonance in an air column is depicted in Figure 18.14. A vertical pipe open at both ends is partially submerged in water, and a tuning fork vibrating at an unknown frequency is placed near the top of the pipe. The length L of the air column can be adjusted by moving the pipe vertically. The sound waves generated by the fork are reinforced when L corresponds to one of the resonance frequencies of the pipe. For a certain pipe, the smallest value of L for which a peak occurs in the sound intensity is 9.00 cm.

(A) What is the frequency of the tuning fork?

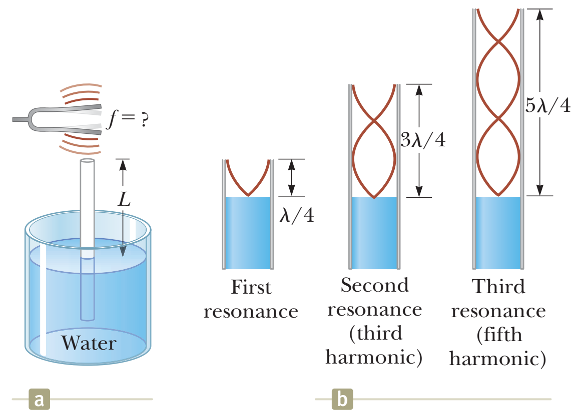


Figure 18.14 (Example 18.6) (a) Apparatus for demonstrating the resonance of sound waves in a pipe closed at one end. The length L of the air column is varied by moving the pipe vertically while it is partially submerged in water. (b) The first three normal modes of the system shown in (a).

SOLUTION

Conceptualize Sound waves from the tuning fork enter the pipe at its upper end. Although the pipe is open at its lower end to allow the water to enter, the water's surface acts like a barrier. The waves reflect from the water surface and combine with those moving downward to form a standing wave.

Categorize Because of the reflection of the sound waves from the water surface, we can model the pipe as open at the upper end and closed at the lower end. Therefore, we can apply the *waves under boundary conditions* model to this situation.

Analyze

Use Equation 18.9 to find the fundamental frequency for $L = 0.090\,0$ m:

$$f_1 = \frac{v}{4L} = \frac{343 \text{ m/s}}{4(0.090\,0 \text{ m})} = 953 \text{ Hz}$$

Because the tuning fork causes the air column to resonate at this frequency, this frequency must also be that of the tuning fork.

(B) What are the values of L for the next two resonance conditions?

SOLUTION

Use Equation 16.12 to find the wavelength of the sound wave from the tuning fork:

$$\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{953 \text{ Hz}} = 0.360 \text{ m}$$

Notice from Figure 18.14b that the length of the air column for the second resonance is $3\lambda/4$:

$$L = 3\lambda/4 = 0.270 \text{ m}$$

Notice from Figure 18.14b that the length of the air column for the third resonance is $5\lambda/4$:

$$L = 5\lambda/4 = 0.450 \text{ m}$$

Finalize Consider how this problem differs from the preceding example. In the culvert, the length was fixed and the air column was presented with a mixture of many frequencies. The pipe in this example is presented with one single frequency from the tuning fork, and the length of the pipe is varied until resonance is achieved.

18.6 Standing Waves in Rods and Membranes

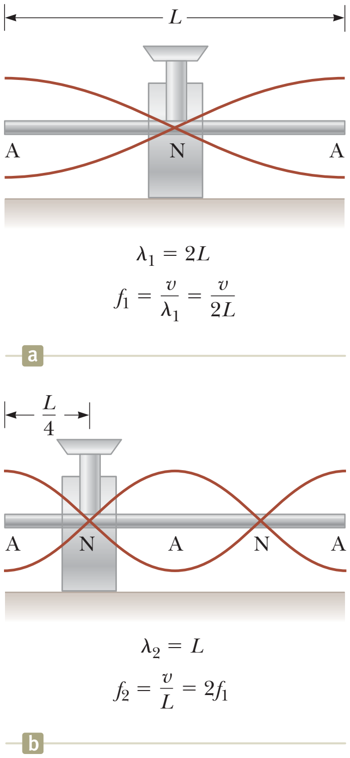


Figure 18.15 Normal-mode longitudinal vibrations of a rod of length L (a) clamped at the middle to produce the first normal mode and (b) clamped at a distance $L/4$ from one end to produce the second normal mode. Notice that the red-brown curves are graphical representations of oscillations parallel to the rod (longitudinal waves).

Standing waves can also be set up in rods and membranes. A rod clamped in the middle and stroked parallel to the rod at one end oscillates as depicted in Figure 18.15a. The oscillations of the elements of the rod are longitudinal, and so the red-brown curves in Figure 18.15 represent *longitudinal* displacements of various parts of the rod. For clarity, the displacements have been drawn in the transverse direction as they were for air columns. The midpoint is a displacement node because it is fixed by the clamp, whereas the ends are displacement antinodes because they are free to oscillate. The oscillations in this setup are analogous to those in a pipe open at both ends. The red-brown lines in Figure 18.15a represent the first normal mode, for which the wavelength is $2L$ and the frequency is $f = v/2L$, where v is the speed of longitudinal waves in the rod. Other normal modes may be excited by clamping the rod at different points. For example, the second normal mode (Fig. 18.15b) is excited by clamping the rod a distance $L/4$ away from one end.

It is also possible to set up transverse standing waves in rods. Musical instruments that depend on transverse standing waves in rods or bars include triangles, marimbas, xylophones, glockenspiels, chimes, and vibraphones. Other devices that make sounds from vibrating bars include music boxes and wind chimes.

Two-dimensional oscillations can be set up in a flexible membrane stretched over a circular hoop such as that in a drumhead. As the membrane is struck at some point, waves that arrive at the fixed boundary are reflected many times. The resulting sound is not harmonic because the standing waves have frequencies that are *not* related by integer multiples. Without this relationship, the sound may be more correctly described as *noise* rather than as music. The production of noise is in contrast to the situation in wind and stringed instruments, which produce sounds that we describe as musical.

Some possible normal modes of oscillation for a two-dimensional circular membrane are shown in Figure 18.16. Whereas nodes are *points* in one-dimensional standing waves on strings and in air columns, a two-dimensional oscillator has *curves* along which there is no displacement of the elements of the medium. The lowest normal mode, which has a frequency f_1, contains only one nodal curve; this curve runs around the outer edge of the membrane. The other possible normal modes show additional nodal curves that are circles and straight lines across the diameter of the membrane.

18.7 Beats: Interference in Time

The interference phenomena we have studied so far involve the superposition of two or more waves having the same frequency. Because the amplitude of the oscil-

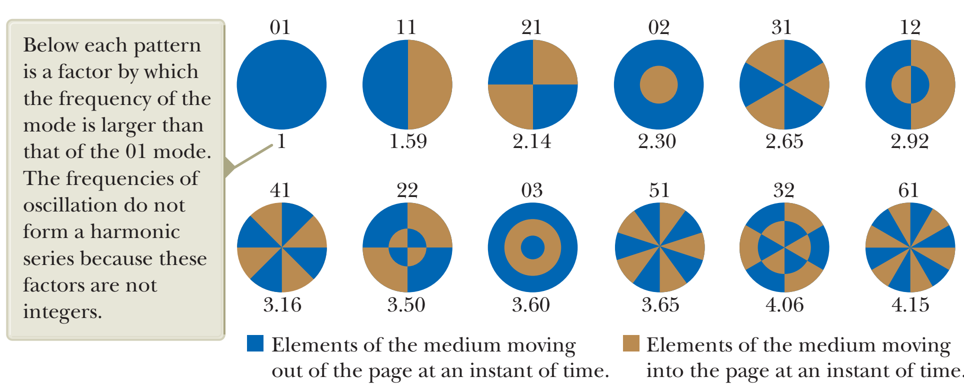


Figure 18.16 Representation of some of the normal modes possible in a circular membrane fixed at its perimeter. The pair of numbers above each pattern corresponds to the number of radial nodes and the number of circular nodes, respectively. In each diagram, elements of the membrane on either side of a nodal line move in opposite directions, as indicated by the colors. *(Adapted from T. D. Rossing,* The Science of Sound, 3rd ed., *Reading, Massachusetts, Addison-Wesley Publishing Co., 2001)*

lation of elements of the medium varies with the position in space of the element in such a wave, we refer to the phenomenon as *spatial interference.* Standing waves in strings and pipes are common examples of spatial interference.

Now let's consider another type of interference, one that results from the superposition of two waves having slightly *different* frequencies. In this case, when the two waves are observed at a point in space, they are periodically in and out of phase. That is, there is a *temporal* (time) alternation between constructive and destructive interference. As a consequence, we refer to this phenomenon as *interference in time* or *temporal interference.* For example, if two tuning forks of slightly different frequencies are struck, one hears a sound of periodically varying amplitude. This phenomenon is called **beating.**

Beating is the periodic variation in amplitude at a given point due to the superposition of two waves having slightly different frequencies.

◀ **Definition of beating**

The number of amplitude maxima one hears per second, or the *beat frequency,* equals the difference in frequency between the two sources as we shall show below. The maximum beat frequency that the human ear can detect is about 20 beats/s. When the beat frequency exceeds this value, the beats blend indistinguishably with the sounds producing them.

Consider two sound waves of equal amplitude and slightly different frequencies f_1 and f_2 traveling through a medium. We use equations similar to Equation 16.13 to represent the wave functions for these two waves at a point that we identify as $x = 0$. We also choose the phase angle in Equation 16.13 as $\phi = \pi/2$:

$$y_1 = A \sin\left(\frac{\pi}{2} - \omega_1 t\right) = A \cos\,(2\pi f_1 t)$$

$$y_2 = A \sin\left(\frac{\pi}{2} - \omega_2 t\right) = A \cos\,(2\pi f_2 t)$$

Using the superposition principle, we find that the resultant wave function at this point is

$$y = y_1 + y_2 = A\,(\cos 2\pi f_1 t + \cos 2\pi f_2 t)$$

The trigonometric identity

$$\cos a + \cos b = 2\cos\left(\frac{a-b}{2}\right)\cos\left(\frac{a+b}{2}\right)$$

allows us to write the expression for y as

$$y = \left[2A\cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t\right]\cos 2\pi\left(\frac{f_1 + f_2}{2}\right)t \qquad \textbf{(18.10)}$$

◀ **Resultant of two waves of different frequencies but equal amplitude**

Graphs of the individual waves and the resultant wave are shown in Figure 18.17. From the factors in Equation 18.10, we see that the resultant wave has an effective

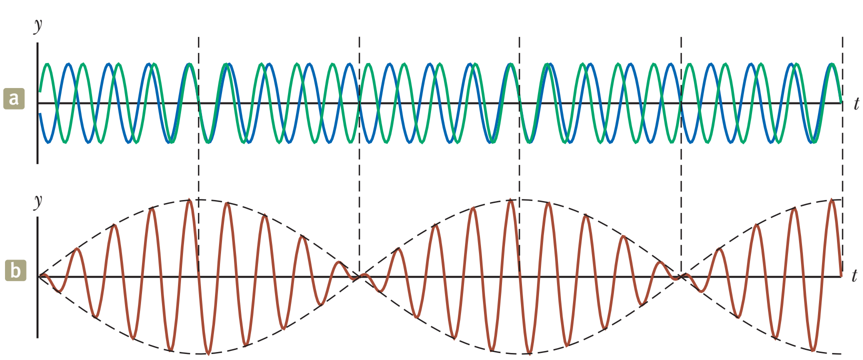


Figure 18.17 Beats are formed by the combination of two waves of slightly different frequencies. (a) The individual waves. (b) The combined wave. The envelope wave (dashed line) represents the beating of the combined sounds.

frequency equal to the average frequency $(f_1 + f_2)/2$. This wave is multiplied by an envelope wave given by the expression in the square brackets:

$$y_{\text{envelope}} = 2A\cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t \qquad \textbf{(18.11)}$$

That is, the amplitude and therefore the intensity of the resultant sound vary in time. The dashed black line in Figure 18.17b is a graphical representation of the envelope wave in Equation 18.11 and is a sine wave varying with frequency $(f_1 - f_2)/2$.

A maximum in the amplitude of the resultant sound wave is detected whenever

$$\cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t = \pm 1$$

Hence, there are *two* maxima in each period of the envelope wave. Because the amplitude varies with frequency as $(f_1 - f_2)/2$, the number of beats per second, or the **beat frequency** f_{beat}, is twice this value. That is,

Beat frequency ▶

$$f_{\text{beat}} = |f_1 - f_2| \qquad \textbf{(18.12)}$$

For instance, if one tuning fork vibrates at 438 Hz and a second one vibrates at 442 Hz, the resultant sound wave of the combination has a frequency of 440 Hz (the musical note A) and a beat frequency of 4 Hz. A listener would hear a 440-Hz sound wave go through an intensity maximum four times every second.

Example 18.7 The Mistuned Piano Strings AM

Two identical piano strings of length 0.750 m are each tuned exactly to 440 Hz. The tension in one of the strings is then increased by 1.0%. If they are now struck, what is the beat frequency between the fundamentals of the two strings?

SOLUTION

Conceptualize As the tension in one of the strings is changed, its fundamental frequency changes. Therefore, when both strings are played, they will have different frequencies and beats will be heard.

Categorize We must combine our understanding of the *waves under boundary conditions* model for strings with our new knowledge of beats.

Analyze Set up a ratio of the fundamental frequencies of the two strings using Equation 18.5:

$$\frac{f_2}{f_1} = \frac{(v_2/2L)}{(v_1/2L)} = \frac{v_2}{v_1}$$

Use Equation 16.18 to substitute for the wave speeds on the strings:

$$\frac{f_2}{f_1} = \frac{\sqrt{T_2/\mu}}{\sqrt{T_1/\mu}} = \sqrt{\frac{T_2}{T_1}}$$

Incorporate that the tension in one string is 1.0% larger than the other; that is, $T_2 = 1.010T_1$:

$$\frac{f_2}{f_1} = \sqrt{\frac{1.010T_1}{T_1}} = 1.005$$

Solve for the frequency of the tightened string:

$$f_2 = 1.005f_1 = 1.005(440\text{ Hz}) = 442\text{ Hz}$$

Find the beat frequency using Equation 18.12:

$$f_{\text{beat}} = 442\text{ Hz} - 440\text{ Hz} = \boxed{2\text{ Hz}}$$

Finalize Notice that a 1.0% mistuning in tension leads to an easily audible beat frequency of 2 Hz. A piano tuner can use beats to tune a stringed instrument by "beating" a note against a reference tone of known frequency. The tuner can then adjust the string tension until the frequency of the sound it emits equals the frequency of the reference tone. The tuner does so by tightening or loosening the string until the beats produced by it and the reference source become too infrequent to notice.

18.8 Nonsinusoidal Wave Patterns

It is relatively easy to distinguish the sounds coming from a violin and a saxophone even when they are both playing the same note. On the other hand, a person untrained in music may have difficulty distinguishing a note played on a clarinet from the same note played on an oboe. We can use the pattern of the sound waves from various sources to explain these effects.

When frequencies that are integer multiples of a fundamental frequency are combined to make a sound, the result is a *musical* sound. A listener can assign a pitch to the sound based on the fundamental frequency. Pitch is a psychological reaction to a sound that allows the listener to place the sound on a scale from low to high (bass to treble). Combinations of frequencies that are not integer multiples of a fundamental result in a *noise* rather than a musical sound. It is much harder for a listener to assign a pitch to a noise than to a musical sound.

The wave patterns produced by a musical instrument are the result of the superposition of frequencies that are integer multiples of a fundamental. This superposition results in the corresponding richness of musical tones. The human perceptive response associated with various mixtures of harmonics is the *quality* or *timbre* of the sound. For instance, the sound of the trumpet is perceived to have a "brassy" quality (that is, we have learned to associate the adjective *brassy* with that sound); this quality enables us to distinguish the sound of the trumpet from that of the saxophone, whose quality is perceived as "reedy." The clarinet and oboe, however, both contain air columns excited by reeds; because of this similarity, they have similar mixtures of frequencies and it is more difficult for the human ear to distinguish them on the basis of their sound quality.

Pitfall Prevention 18.4

Pitch Versus Frequency Do not confuse the term *pitch* with *frequency*. Frequency is the physical measurement of the number of oscillations per second. Pitch is a psychological reaction to sound that enables a person to place the sound on a scale from high to low or from treble to bass. Therefore, frequency is the stimulus and pitch is the response. Although pitch is related mostly (but not completely) to frequency, they are not the same. A phrase such as "the pitch of the sound" is incorrect because pitch is not a physical property of the sound.

The sound wave patterns produced by the majority of musical instruments are nonsinusoidal. Characteristic patterns produced by a tuning fork, a flute, and a clarinet, each playing the same note, are shown in Figure 18.18. Each instrument has its own characteristic pattern. Notice, however, that despite the differences in the patterns, each pattern is periodic. This point is important for our analysis of these waves.

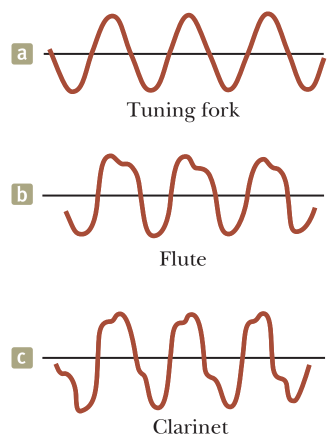


Figure 18.18 Sound wave patterns produced by (a) a tuning fork, (b) a flute, and (c) a clarinet, each at approximately the same frequency.

The problem of analyzing nonsinusoidal wave patterns appears at first sight to be a formidable task. If the wave pattern is periodic, however, it can be represented as closely as desired by the combination of a sufficiently large number of sinusoidal waves that form a harmonic series. In fact, we can represent any periodic function as a series of sine and cosine terms by using a mathematical technique based on **Fourier's theorem.**[2] The corresponding sum of terms that represents the periodic wave pattern is called a **Fourier series.** Let $y(t)$ be any function that is periodic in time with period T such that $y(t + T) = y(t)$. Fourier's theorem states that this function can be written as

$$y(t) = \sum (A_n \sin 2\pi f_n t + B_n \cos 2\pi f_n t) \tag{18.13}$$

◀ **Fourier's theorem**

where the lowest frequency is $f_1 = 1/T$. The higher frequencies are integer multiples of the fundamental, $f_n = nf_1$, and the coefficients A_n and B_n represent the amplitudes of the various waves. Figure 18.19 on page 554 represents a harmonic analysis of the wave patterns shown in Figure 18.18. Each bar in the graph represents one of the terms in the series in Equation 18.13 up to $n = 9$. Notice that a struck tuning fork produces only one harmonic (the first), whereas the flute and clarinet produce the first harmonic and many higher ones.

Notice the variation in relative intensity of the various harmonics for the flute and the clarinet. In general, any musical sound consists of a fundamental frequency f plus other frequencies that are integer multiples of f, all having different intensities.

[2] Developed by Jean Baptiste Joseph Fourier (1786–1830).

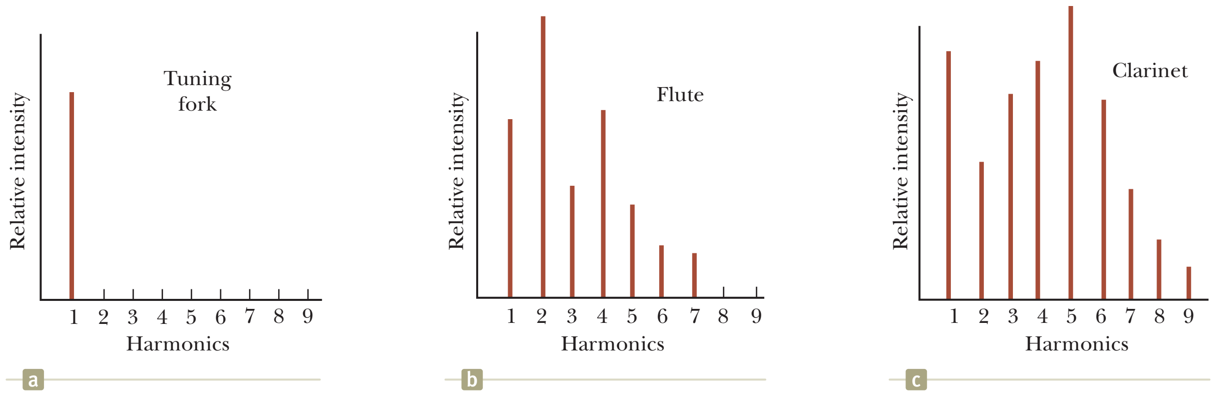


Figure 18.19 Harmonics of the wave patterns shown in Figure 18.18. Notice the variations in intensity of the various harmonics. Parts (a), (b), and (c) correspond to those in Figure 18.18.

We have discussed the *analysis* of a wave pattern using Fourier's theorem. The analysis involves determining the coefficients of the harmonics in Equation 18.13 from a knowledge of the wave pattern. The reverse process, called *Fourier synthesis,* can also be performed. In this process, the various harmonics are added together to form a resultant wave pattern. As an example of Fourier synthesis, consider the building of a square wave as shown in Figure 18.20. The symmetry of the square wave results in only odd multiples of the fundamental frequency combining in its synthesis. In Figure 18.20a, the blue curve shows the combination of f and $3f$. In Figure 18.20b, we have added $5f$ to the combination and obtained the green curve. Notice how the general shape of the square wave is approximated, even though the upper and lower portions are not flat as they should be.

Figure 18.20c shows the result of adding odd frequencies up to $9f$. This approximation (red-brown curve) to the square wave is better than the approximations in Figures 18.20a and 18.20b. To approximate the square wave as closely as possible, we must add all odd multiples of the fundamental frequency, up to infinite frequency.

Using modern technology, musical sounds can be generated electronically by mixing different amplitudes of any number of harmonics. These widely used electronic music synthesizers are capable of producing an infinite variety of musical tones.

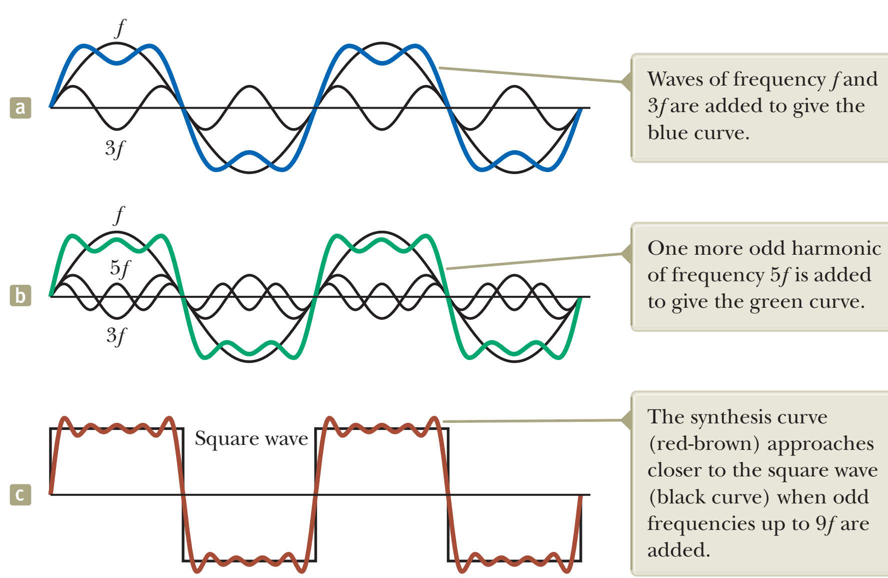


Figure 18.20 Fourier synthesis of a square wave, represented by the sum of odd multiples of the first harmonic, which has frequency f.

Summary

Concepts and Principles

The **superposition principle** specifies that when two or more waves move through a medium, the value of the resultant wave function equals the algebraic sum of the values of the individual wave functions.

The phenomenon of **beating** is the periodic variation in intensity at a given point due to the superposition of two waves having slightly different frequencies. The **beat frequency** is

$$f_{\text{beat}} = |f_1 - f_2| \quad \textbf{(18.12)}$$

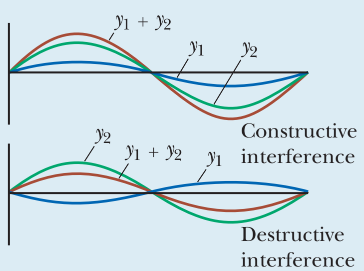
where f_1 and f_2 are the frequencies of the individual waves.

Standing waves are formed from the combination of two sinusoidal waves having the same frequency, amplitude, and wavelength but traveling in opposite directions. The resultant standing wave is described by the wave function

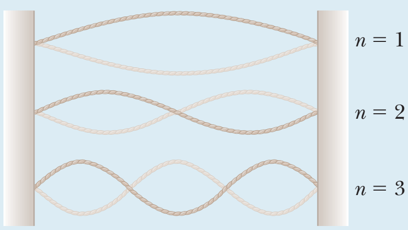
$$y = (2A \sin kx) \cos \omega t \quad \textbf{(18.1)}$$

Hence, the amplitude of the standing wave is $2A$, and the amplitude of the simple harmonic motion of any element of the medium varies according to its position as $2A \sin kx$. The points of zero amplitude (called **nodes**) occur at $x = n\lambda/2$ ($n = 0, 1, 2, 3, \ldots$). The maximum amplitude points (called **antinodes**) occur at $x = n\lambda/4$ ($n = 1, 3, 5, \ldots$). Adjacent antinodes are separated by a distance $\lambda/2$. Adjacent nodes also are separated by a distance $\lambda/2$.

Analysis Models for Problem Solving



Waves in Interference. When two traveling waves having equal frequencies superimpose, the resultant wave is described by the **principle of superposition** and has an amplitude that depends on the phase angle ϕ between the two waves. **Constructive interference** occurs when the two waves are in phase, corresponding to $\phi = 0, 2\pi, 4\pi, \ldots$ rad. **Destructive interference** occurs when the two waves are 180° out of phase, corresponding to $\phi = \pi, 3\pi, 5\pi, \ldots$ rad.



Waves Under Boundary Conditions. When a wave is subject to boundary conditions, only certain natural frequencies are allowed; we say that the frequencies are quantized.

For waves on a string fixed at both ends, the natural frequencies are

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}} \quad n = 1, 2, 3, \ldots \quad \textbf{(18.6)}$$

where T is the tension in the string and μ is its linear mass density.

For sound waves with speed v in an air column of length L open at both ends, the natural frequencies are

$$f_n = n\frac{v}{2L} \quad n = 1, 2, 3, \ldots \quad \textbf{(18.8)}$$

If an air column is open at one end and closed at the other, only odd harmonics are present and the natural frequencies are

$$f_n = n\frac{v}{4L} \quad n = 1, 3, 5, \ldots \quad \textbf{(18.9)}$$

Objective Questions

1. denotes answer available in *Student Solutions Manual/Study Guide*

1. In Figure OQ18.1 (page 556), a sound wave of wavelength 0.8 m divides into two equal parts that recombine to interfere constructively, with the original difference between their path lengths being $|r_2 - r_1| = 0.8$ m. Rank the following situations according to the intensity of sound at the receiver from the highest to the lowest. Assume the tube walls absorb no sound energy. Give equal ranks to situations in which the intensity is equal.

(a) From its original position, the sliding section is moved out by 0.1 m. (b) Next it slides out an additional 0.1 m. (c) It slides out still another 0.1 m. (d) It slides out 0.1 m more.

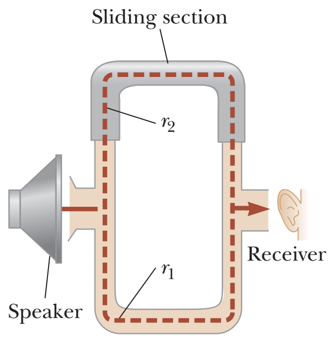


Figure OQ18.1 Objective Question 1 and Problem 6.

2. A string of length L, mass per unit length μ, and tension T is vibrating at its fundamental frequency. **(i)** If the length of the string is doubled, with all other factors held constant, what is the effect on the fundamental frequency? (a) It becomes two times larger. (b) It becomes $\sqrt{2}$ times larger. (c) It is unchanged. (d) It becomes $1/\sqrt{2}$ times as large. (e) It becomes one-half as large. **(ii)** If the mass per unit length is doubled, with all other factors held constant, what is the effect on the fundamental frequency? Choose from the same possibilities as in part (i). **(iii)** If the tension is doubled, with all other factors held constant, what is the effect on the fundamental frequency? Choose from the same possibilities as in part (i).

3. In Example 18.1, we investigated an oscillator at 1.3 kHz driving two identical side-by-side speakers. We found that a listener at point O hears sound with maximum intensity, whereas a listener at point P hears a minimum. What is the intensity at P? (a) less than but close to the intensity at O (b) half the intensity at O (c) very low but not zero (d) zero (e) indeterminate

4. A series of pulses, each of amplitude 0.1 m, is sent down a string that is attached to a post at one end. The pulses are reflected at the post and travel back along the string without loss of amplitude. **(i)** What is the net displacement at a point on the string where two pulses are crossing? Assume the string is rigidly attached to the post. (a) 0.4 m (b) 0.3 m (c) 0.2 m (d) 0.1 m (e) 0 **(ii)** Next assume the end at which reflection occurs is free to slide up and down. Now what is the net displacement at a point on the string where two pulses are crossing? Choose your answer from the same possibilities as in part (i).

5. A flute has a length of 58.0 cm. If the speed of sound in air is 343 m/s, what is the fundamental frequency of the flute, assuming it is a tube closed at one end and open at the other? (a) 148 Hz (b) 296 Hz (c) 444 Hz (d) 591 Hz (e) none of those answers

6. When two tuning forks are sounded at the same time, a beat frequency of 5 Hz occurs. If one of the tuning forks has a frequency of 245 Hz, what is the frequency of the other tuning fork? (a) 240 Hz (b) 242.5 Hz (c) 247.5 Hz (d) 250 Hz (e) More than one answer could be correct.

7. A tuning fork is known to vibrate with frequency 262 Hz. When it is sounded along with a mandolin string, four beats are heard every second. Next, a bit of tape is put onto each tine of the tuning fork, and the tuning fork now produces five beats per second with the same mandolin string. What is the frequency of the string? (a) 257 Hz (b) 258 Hz (c) 262 Hz (d) 266 Hz (e) 267 Hz

8. An archer shoots an arrow horizontally from the center of the string of a bow held vertically. After the arrow leaves it, the string of the bow will vibrate as a superposition of what standing-wave harmonics? (a) It vibrates only in harmonic number 1, the fundamental. (b) It vibrates only in the second harmonic. (c) It vibrates only in the odd-numbered harmonics 1, 3, 5, 7, ... (d) It vibrates only in the even-numbered harmonics 2, 4, 6, 8, ... (e) It vibrates in all harmonics.

9. As oppositely moving pulses of the same shape (one upward, one downward) on a string pass through each other, at one particular instant the string shows no displacement from the equilibrium position at any point. What has happened to the energy carried by the pulses at this instant of time? (a) It was used up in producing the previous motion. (b) It is all potential energy. (c) It is all internal energy. (d) It is all kinetic energy. (e) The positive energy of one pulse adds to zero with the negative energy of the other pulse.

10. A standing wave having three nodes is set up in a string fixed at both ends. If the frequency of the wave is doubled, how many antinodes will there be? (a) 2 (b) 3 (c) 4 (d) 5 (e) 6

11. Suppose all six equal-length strings of an acoustic guitar are played without fingering, that is, without being pressed down at any frets. What quantities are the same for all six strings? Choose all correct answers. (a) the fundamental frequency (b) the fundamental wavelength of the string wave (c) the fundamental wavelength of the sound emitted (d) the speed of the string wave (e) the speed of the sound emitted

12. Assume two identical sinusoidal waves are moving through the same medium in the same direction. Under what condition will the amplitude of the resultant wave be greater than either of the two original waves? (a) in all cases (b) only if the waves have no difference in phase (c) only if the phase difference is less than 90° (d) only if the phase difference is less than 120° (e) only if the phase difference is less than 180°

Conceptual Questions

1. denotes answer available in *Student Solutions Manual/Study Guide*

1. A crude model of the human throat is that of a pipe open at both ends with a vibrating source to introduce the sound into the pipe at one end. Assuming the vibrating source produces a range of frequencies, discuss the effect of changing the pipe's length.

2. When two waves interfere constructively or destructively, is there any gain or loss in energy in the system of the waves? Explain.

3. Explain how a musical instrument such as a piano may be tuned by using the phenomenon of beats.

4. What limits the amplitude of motion of a real vibrating system that is driven at one of its resonant frequencies?

5. A tuning fork by itself produces a faint sound. Explain how each of the following methods can be used to obtain a louder sound from it. Explain also any effect on the time interval for which the fork vibrates audibly. (a) holding the edge of a sheet of paper against one vibrating tine (b) pressing the handle of the tuning fork against a chalkboard or a tabletop (c) holding the tuning fork above a column of air of properly chosen length as in Example 18.6 (d) holding the tuning fork close to an open slot cut in a sheet of foam plastic or cardboard (with the slot similar in size and shape to one tine of the fork and the motion of the tines perpendicular to the sheet)

6. An airplane mechanic notices that the sound from a twin-engine aircraft rapidly varies in loudness when both engines are running. What could be causing this variation from loud to soft?

7. Despite a reasonably steady hand, a person often spills his coffee when carrying it to his seat. Discuss resonance as a possible cause of this difficulty and devise a means for preventing the spills.

8. A soft-drink bottle resonates as air is blown across its top. What happens to the resonance frequency as the level of fluid in the bottle decreases?

9. Does the phenomenon of wave interference apply only to sinusoidal waves?

Problems

ENHANCED WebAssign The problems found in this chapter may be assigned online in Enhanced WebAssign

1. straightforward; **2.** intermediate; **3.** challenging

1. full solution available in the *Student Solutions Manual/Study Guide*

AMT Analysis Model tutorial available in Enhanced WebAssign

GP Guided Problem

M Master It tutorial available in Enhanced WebAssign

W Watch It video solution available in Enhanced WebAssign

Note: Unless otherwise specified, assume the speed of sound in air is 343 m/s, its value at an air temperature of 20.0°C. At any other Celsius temperature T_C, the speed of sound in air is described by

$$v = 331\sqrt{1 + \frac{T_C}{273}}$$

where v is in m/s and T is in °C.

Section 18.1 Analysis Model: Waves in Interference

1. W Two waves are traveling in the same direction along a stretched string. The waves are 90.0° out of phase. Each wave has an amplitude of 4.00 cm. Find the amplitude of the resultant wave.

2. Two wave pulses A and B are moving in opposite directions, each with a speed v = 2.00 cm/s. The amplitude of A is twice the amplitude of B. The pulses are shown in Figure P18.2 at t = 0. Sketch the resultant wave at t = 1.00 s, 1.50 s, 2.00 s, 2.50 s, and 3.00 s.

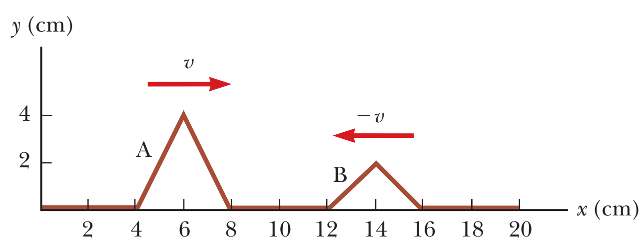


Figure P18.2

3. W Two waves on one string are described by the wave functions

$$y_1 = 3.0 \cos (4.0x - 1.6t) \qquad y_2 = 4.0 \sin (5.0x - 2.0t)$$

where x and y are in centimeters and t is in seconds. Find the superposition of the waves $y_1 + y_2$ at the points (a) x = 1.00, t = 1.00; (b) x = 1.00, t = 0.500; and (c) x = 0.500, t = 0. *Note:* Remember that the arguments of the trigonometric functions are in radians.

4. Two pulses of different amplitudes approach each other, each having a speed of v = 1.00 m/s. Figure P18.4 shows the positions of the pulses at time t = 0. (a) Sketch the resultant wave at t = 2.00 s, 4.00 s, 5.00 s, and 6.00 s. (b) **What If?** If the pulse on the right is inverted so that it is upright, how would your sketches of the resultant wave change?

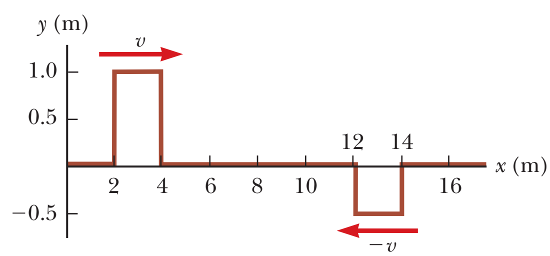


Figure P18.4

5. A tuning fork generates sound waves with a frequency of 246 Hz. The waves travel in opposite directions along a hallway, are reflected by end walls, and return. The hallway is 47.0 m long and the tuning fork is located 14.0 m from one end. What is the phase difference between the reflected waves when they meet at the tuning fork? The speed of sound in air is 343 m/s.

6. The acoustical system shown in Figure OQ18.1 is driven by a speaker emitting sound of frequency 756 Hz. (a) If constructive interference occurs at a particular location of the sliding section, by what minimum amount should the sliding section be moved upward so that destructive interference occurs instead? (b) What minimum distance from the original position of the sliding section will again result in constructive interference?

7. Two pulses traveling on the same string are described by

$$y_1 = \frac{5}{(3x - 4t)^2 + 2} \qquad y_2 = \frac{-5}{(3x + 4t - 6)^2 + 2}$$

(a) In which direction does each pulse travel? (b) At what instant do the two cancel everywhere? (c) At what point do the two pulses always cancel?

8. AMT Two identical loudspeakers are placed on a wall 2.00 m apart. A listener stands 3.00 m from the wall directly in front of one of the speakers. A single oscillator is driving the speakers at a frequency of 300 Hz. (a) What is the phase difference in radians between the waves from the speakers when they reach the observer? (b) **What If?** What is the frequency closest to 300 Hz to which the oscillator may be adjusted such that the observer hears minimal sound?

9. M Two traveling sinusoidal waves are described by the wave functions

$$y_1 = 5.00 \sin [\pi(4.00x - 1\,200t)]$$

$$y_2 = 5.00 \sin [\pi(4.00x - 1\,200t - 0.250)]$$

where x, y_1, and y_2 are in meters and t is in seconds. (a) What is the amplitude of the resultant wave function $y_1 + y_2$? (b) What is the frequency of the resultant wave function?

10. *Why is the following situation impossible?* Two identical loudspeakers are driven by the same oscillator at frequency 200 Hz. They are located on the ground a distance $d = 4.00$ m from each other. Starting far from the speakers, a man walks straight toward the right-hand speaker as shown in Figure P18.10. After passing through three minima in sound intensity, he walks to the next maximum and stops. Ignore any sound reflection from the ground.

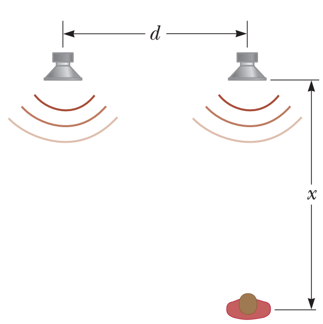


Figure P18.10

11. M Two sinusoidal waves in a string are defined by the wave functions

$$y_1 = 2.00 \sin (20.0x - 32.0t) \qquad y_2 = 2.00 \sin (25.0x - 40.0t)$$

where x, y_1, and y_2 are in centimeters and t is in seconds. (a) What is the phase difference between these two waves at the point $x = 5.00$ cm at $t = 2.00$ s? (b) What is the positive x value closest to the origin for which the two phases differ by $\pm\pi$ at $t = 2.00$ s? (At that location, the two waves add to zero.)

12. Two identical sinusoidal waves with wavelengths of 3.00 m travel in the same direction at a speed of 2.00 m/s. The second wave originates from the same point as the first, but at a later time. The amplitude of the resultant wave is the same as that of each of the two initial waves. Determine the minimum possible time interval between the starting moments of the two waves.

13. Two identical loudspeakers 10.0 m apart are driven by the same oscillator with a frequency of $f = 21.5$ Hz (Fig. P18.13) in an area where the speed of sound is 344 m/s. (a) Show that a receiver at point A records a minimum in sound intensity from the two speakers. (b) If the receiver is moved in the plane of the speakers, show that the path it should take so that the intensity remains at a minimum is along the hyperbola $9x^2 - 16y^2 = 144$ (shown in red-brown in Fig. P18.13). (c) Can the receiver remain at a minimum and move very far away from the two sources? If so, determine the limiting form of the path it must take. If not, explain how far it can go.

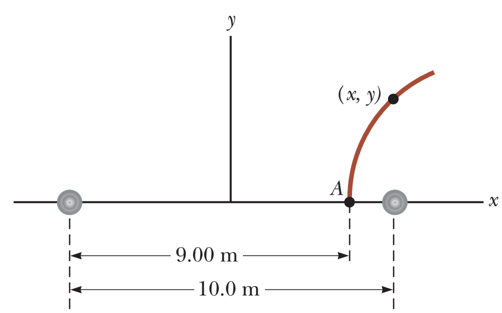


Figure P18.13

Section 18.2 Standing Waves

14. Two waves simultaneously present on a long string have a phase difference ϕ between them so that a standing wave formed from their combination is described by

$$y(x, t) = 2A \sin\left(kx + \frac{\phi}{2}\right) \cos\left(\omega t - \frac{\phi}{2}\right)$$

(a) Despite the presence of the phase angle ϕ, is it still true that the nodes are one-half wavelength apart? Explain. (b) Are the nodes different in any way from the way they would be if ϕ were zero? Explain.

15. W Two sinusoidal waves traveling in opposite directions interfere to produce a standing wave with the wave function

$$y = 1.50 \sin (0.400x) \cos (200t)$$

where x and y are in meters and t is in seconds. Determine (a) the wavelength, (b) the frequency, and (c) the speed of the interfering waves.

16. Verify by direct substitution that the wave function for a standing wave given in Equation 18.1,

$$y = (2A \sin kx) \cos \omega t$$

is a solution of the general linear wave equation, Equation 16.27:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

17. M Two transverse sinusoidal waves combining in a medium are described by the wave functions

$$y_1 = 3.00 \sin \pi(x + 0.600t) \qquad y_2 = 3.00 \sin \pi(x - 0.600t)$$

where x, y_1, and y_2 are in centimeters and t is in seconds. Determine the maximum transverse position of an element of the medium at (a) $x = 0.250$ cm, (b) $x = 0.500$ cm, and (c) $x = 1.50$ cm. (d) Find the three smallest values of x corresponding to antinodes.

18. A standing wave is described by the wave function

$$y = 6 \sin\left(\frac{\pi}{2}x\right) \cos(100\pi t)$$

where x and y are in meters and t is in seconds. (a) Prepare graphs showing y as a function of x for five instants: t = 0, 5 ms, 10 ms, 15 ms, and 20 ms. (b) From the graph, identify the wavelength of the wave and explain how to do so. (c) From the graph, identify the frequency of the wave and explain how to do so. (d) From the equation, directly identify the wavelength of the wave and explain how to do so. (e) From the equation, directly identify the frequency and explain how to do so.

19. M Two identical loudspeakers are driven in phase by a common oscillator at 800 Hz and face each other at a distance of 1.25 m. Locate the points along the line joining the two speakers where relative minima of sound pressure amplitude would be expected.

Section 18.3 Analysis Model: Waves Under Boundary Conditions

20. A standing wave is established in a 120-cm-long string fixed at both ends. The string vibrates in four segments when driven at 120 Hz. (a) Determine the wavelength. (b) What is the fundamental frequency of the string?

21. A string with a mass $m = 8.00$ g and a length $L = 5.00$ m has one end attached to a wall; the other end is draped over a small, fixed pulley a distance $d = 4.00$ m from the wall and attached to a hanging object with a mass $M = 4.00$ kg as in Figure P18.21. If the horizontal part of the string is plucked, what is the fundamental frequency of its vibration?

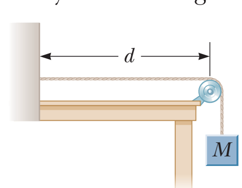


Figure P18.21

22. The 64.0-cm-long string of a guitar has a fundamental frequency of 330 Hz when it vibrates freely along its entire length. A fret is provided for limiting vibration to just the lower two-thirds of the string. (a) If the string is pressed down at this fret and plucked, what is the new fundamental frequency? (b) **What If?** The guitarist can play a "natural harmonic" by gently touching the string at the location of this fret and plucking the string at about one-sixth of the way along its length from the other end. What frequency will be heard then?

23. W The A string on a cello vibrates in its first normal mode with a frequency of 220 Hz. The vibrating segment is 70.0 cm long and has a mass of 1.20 g. (a) Find the tension in the string. (b) Determine the frequency of vibration when the string vibrates in three segments.

24. A taut string has a length of 2.60 m and is fixed at both ends. (a) Find the wavelength of the fundamental mode of vibration of the string. (b) Can you find the frequency of this mode? Explain why or why not.

25. A certain vibrating string on a piano has a length of 74.0 cm and forms a standing wave having two antinodes. (a) Which harmonic does this wave represent? (b) Determine the wavelength of this wave. (c) How many nodes are there in the wave pattern?

26. A string that is 30.0 cm long and has a mass per unit length of 9.00×10^{-3} kg/m is stretched to a tension of 20.0 N. Find (a) the fundamental frequency and (b) the next three frequencies that could cause standing-wave patterns on the string.

27. AMT W In the arrangement shown in Figure P18.27, an object can be hung from a string (with linear mass density μ = 0.002 00 kg/m) that passes over a light pulley. The string is connected to a vibrator (of constant frequency f), and the length of the string between point P and the pulley is $L = 2.00$ m. When the mass m of the object is either 16.0 kg or 25.0 kg, standing waves are observed; no standing waves are observed with any mass between these values, however. (a) What is the frequency of the vibrator? *Note:* The greater the tension in the string, the smaller the number of nodes in the standing wave. (b) What is the largest object mass for which standing waves could be observed?

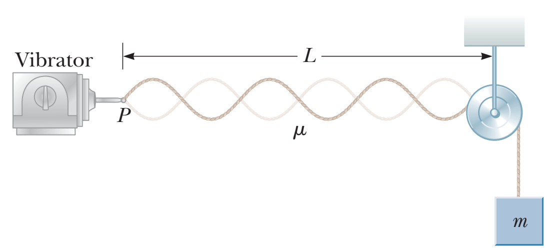


Figure P18.27 Problems 27 and 28.

28. M In the arrangement shown in Figure P18.27, an object of mass $m = 5.00$ kg hangs from a cord around a light pulley. The length of the cord between point P and the pulley is $L = 2.00$ m. (a) When the vibrator is set to a frequency of 150 Hz, a standing wave with six loops is formed. What must be the linear mass density of the cord? (b) How many loops (if any) will result if m is changed to 45.0 kg? (c) How many loops (if any) will result if m is changed to 10.0 kg?

29. Review. A sphere of mass $M = 1.00$ kg is supported by a string that passes over a pulley at the end of a horizontal rod of length $L = 0.300$ m (Fig. P18.29). The string makes an angle $\theta = 35.0°$ with the rod. The fundamental frequency of standing waves in the portion of the string above the rod is $f = 60.0$ Hz. Find the mass of the portion of the string above the rod.

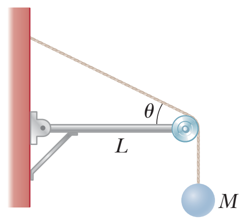


Figure P18.29 Problems 29 and 30.

30. Review. A sphere of mass M is supported by a string that passes over a pulley at the end of a horizontal rod of length L (Fig. P18.29). The string makes an angle θ with the rod. The fundamental frequency of standing waves in the portion of the string above the rod is f. Find the mass of the portion of the string above the rod.

31. A violin string has a length of 0.350 m and is tuned to concert G, with $f_G = 392$ Hz. (a) How far from the end of the string must the violinist place her finger to play concert A, with $f_A = 440$ Hz? (b) If this position is to remain correct to one-half the width of a finger (that is, to within 0.600 cm), what is the maximum allowable percentage change in the string tension?

32. Review. A solid copper object hangs at the bottom of a steel wire of negligible mass. The top end of the wire is fixed. When the wire is struck, it emits sound with a fundamental frequency of 300 Hz. The copper object is then submerged in water so that half its volume is below the water line. Determine the new fundamental frequency.

33. A standing-wave pattern is observed in a thin wire with a length of 3.00 m. The wave function is

$$y = 0.002\,00 \sin (\pi x) \cos (100\pi t)$$

where x and y are in meters and t is in seconds. (a) How many loops does this pattern exhibit? (b) What is the fundamental frequency of vibration of the wire? (c) **What If?** If the original frequency is held constant and the tension in the wire is increased by a factor of 9, how many loops are present in the new pattern?

Section 18.4 Resonance

34. The Bay of Fundy, Nova Scotia, has the highest tides in the world. Assume in midocean and at the mouth of the bay the Moon's gravity gradient and the Earth's rotation make the water surface oscillate with an amplitude of a few centimeters and a period of 12 h 24 min. At the head of the bay, the amplitude is several meters. Assume the bay has a length of 210 km and a uniform depth of 36.1 m. The speed of long-wavelength water waves is given by $v = \sqrt{gd}$, where d is the water's depth. Argue for or against the proposition that the tide is magnified by standing-wave resonance.

35. An earthquake can produce a *seiche* in a lake in which the water sloshes back and forth from end to end with remarkably large amplitude and long period. Consider a seiche produced in a farm pond. Suppose the pond is 9.15 m long and assume it has a uniform width and depth. You measure that a pulse produced at one end reaches the other end in 2.50 s. (a) What is the wave speed? (b) What should be the frequency of the ground motion during the earthquake to produce a seiche that is a standing wave with antinodes at each end of the pond and one node at the center?

36. High-frequency sound can be used to produce standing-wave vibrations in a wine glass. A standing-wave vibration in a wine glass is observed to have four nodes and four antinodes equally spaced around the 20.0-cm circumference of the rim of the glass. If transverse waves move around the glass at 900 m/s, an opera singer would have to produce a high harmonic with what frequency to shatter the glass with a resonant vibration as shown in Figure P18.36?

Steve Bronstein/Stone/Getty Images

Figure P18.36

Section 18.5 Standing Waves in Air Columns

37. The windpipe of one typical whooping crane is 5.00 feet long. What is the fundamental resonant frequency of the bird's trachea, modeled as a narrow pipe closed at one end? Assume a temperature of 37°C.

38. If a human ear canal can be thought of as resembling an organ pipe, closed at one end, that resonates at a fundamental frequency of 3 000 Hz, what is the length of the canal? Use a normal body temperature of 37°C for your determination of the speed of sound in the canal.

39. Calculate the length of a pipe that has a fundamental frequency of 240 Hz assuming the pipe is (a) closed at one end and (b) open at both ends.

40. W The overall length of a piccolo is 32.0 cm. The resonating air column is open at both ends. (a) Find the frequency of the lowest note a piccolo can sound. (b) Opening holes in the side of a piccolo effectively shortens the length of the resonant column. Assume the highest note a piccolo can sound is 4 000 Hz. Find the distance between adjacent antinodes for this mode of vibration.

41. The fundamental frequency of an open organ pipe corresponds to middle C (261.6 Hz on the chromatic musical scale). The third resonance of a closed organ pipe has the same frequency. What is the length of (a) the open pipe and (b) the closed pipe?

42. The longest pipe on a certain organ is 4.88 m. What is the fundamental frequency (at 0.00°C) if the pipe is (a) closed at one end and (b) open at each end? (c) What will be the frequencies at 20.0°C?

43. An air column in a glass tube is open at one end and closed at the other by a movable piston. The air in the tube is warmed above room temperature, and a 384-Hz tuning fork is held at the open end. Resonance is heard when the piston is at a distance d_1 = 22.8 cm from the open end and again when it is at a distance d_2 = 68.3 cm from the open end. (a) What speed of sound is implied by these data? (b) How far from the open end will the piston be when the next resonance is heard?

44. A tuning fork with a frequency of f = 512 Hz is placed near the top of the tube shown in Figure P18.44. The water level is lowered so that the length L slowly increases from an initial value of 20.0 cm. Determine the next two values of L that correspond to resonant modes.

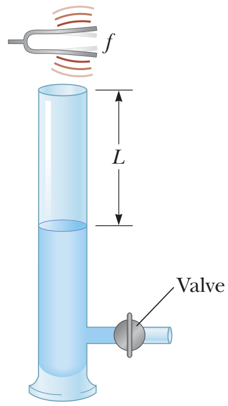


Figure P18.44

45. With a particular fingering, a flute produces a note with frequency 880 Hz at 20.0°C. The flute is open at both ends. (a) Find the air column length. (b) At the beginning of the halftime performance at a late-season football game, the ambient temperature is −5.00°C and the flutist has not had a chance to warm up her instrument. Find the frequency the flute produces under these conditions.

46. A shower stall has dimensions 86.0 cm × 86.0 cm × 210 cm. Assume the stall acts as a pipe closed at both ends, with nodes at opposite sides. Assume singing voices range from 130 Hz to 2 000 Hz and let the speed of sound in the hot air be 355 m/s. For someone singing in this shower, which frequencies would sound the richest (because of resonance)?

47. A glass tube (open at both ends) of length L is positioned near an audio speaker of frequency f = 680 Hz. For what values of L will the tube resonate with the speaker?

48. A tunnel under a river is 2.00 km long. (a) At what frequencies can the air in the tunnel resonate? (b) Explain whether it would be good to make a rule against blowing your car horn when you are in the tunnel.

49. As shown in Figure P18.49, water is pumped into a tall, vertical cylinder at a volume flow rate R = 1.00 L/min. The radius of the cylinder is r = 5.00 cm, and at the open top of the cylinder a tuning fork is vibrating with a frequency f = 512 Hz. As the water rises, what time interval elapses between successive resonances?

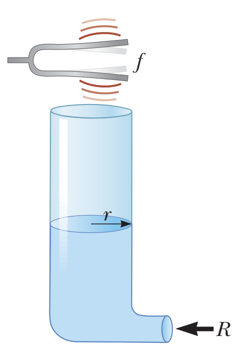


Figure P18.49
Problems 49 and 50.

50. As shown in Figure P18.49, water is pumped into a tall, vertical cylinder at a volume flow rate R. The radius of the cylinder is r, and at the open top of the cylinder a tuning fork is vibrating with a frequency f. As the water rises, what time interval elapses between successive resonances?

51. AMT M Two adjacent natural frequencies of an organ pipe are determined to be 550 Hz and 650 Hz. Calculate (a) the fundamental frequency and (b) the length of this pipe.

52. *Why is the following situation impossible?* A student is listening to the sounds from an air column that is 0.730 m long. He doesn't know if the column is open at both ends or open at only one end. He hears resonance from the air column at frequencies 235 Hz and 587 Hz.

53. A student uses an audio oscillator of adjustable frequency to measure the depth of a water well. The student reports hearing two successive resonances at 51.87 Hz and 59.85 Hz. (a) How deep is the well? (b) How many antinodes are in the standing wave at 51.87 Hz?

Section 18.6 Standing Waves in Rods and Membranes

54. An aluminum rod is clamped one-fourth of the way along its length and set into longitudinal vibration by a variable-frequency driving source. The lowest frequency that produces resonance is 4 400 Hz. The speed of sound in an aluminum rod is 5 100 m/s. Determine the length of the rod.

55. An aluminum rod 1.60 m long is held at its center. It is stroked with a rosin-coated cloth to set up a longitudinal vibration. The speed of sound in a thin rod of aluminum is 5 100 m/s. (a) What is the fundamental frequency of the waves established in the rod? (b) What harmonics are set up in the rod held in this manner? (c) **What If?** What would be the fundamental frequency if the rod were copper, in which the speed of sound is 3 560 m/s?

Section 18.7 Beats: Interference in Time

56. W While attempting to tune the note C at 523 Hz, a piano tuner hears 2.00 beats/s between a reference oscillator and the string. (a) What are the possible frequencies of the string? (b) When she tightens the string slightly, she hears 3.00 beats/s. What is the frequency of the string now? (c) By what percentage should the piano tuner now change the tension in the string to bring it into tune?

57. M In certain ranges of a piano keyboard, more than one string is tuned to the same note to provide extra loudness. For example, the note at 110 Hz has two strings at this frequency. If one string slips from its normal tension of 600 N to 540 N, what beat frequency is heard when the hammer strikes the two strings simultaneously?

58. **Review.** Jane waits on a railroad platform while two trains approach from the same direction at equal speeds of 8.00 m/s. Both trains are blowing their whistles (which have the same frequency), and one train is some distance behind the other. After the first train passes Jane but before the second train passes her, she hears beats of frequency 4.00 Hz. What is the frequency of the train whistles?

59. M **Review.** A student holds a tuning fork oscillating at 256 Hz. He walks toward a wall at a constant speed of 1.33 m/s. (a) What beat frequency does he observe between the tuning fork and its echo? (b) How fast must he walk away from the wall to observe a beat frequency of 5.00 Hz?

Section 18.8 Nonsinusoidal Wave Patterns

60. An A-major chord consists of the notes called A, C#, and E. It can be played on a piano by simultaneously striking strings with fundamental frequencies of 440.00 Hz, 554.37 Hz, and 659.26 Hz. The rich consonance of the chord is associated with near equality of the frequencies of some of the higher harmonics of the three tones. Consider the first five harmonics of each string and determine which harmonics show near equality.

61. Suppose a flutist plays a 523-Hz C note with first harmonic displacement amplitude $A_1 = 100$ nm. From Figure 18.19b read, by proportion, the displacement amplitudes of harmonics 2 through 7. Take these as the values A_2 through A_7 in the Fourier analysis of the sound and assume $B_1 = B_2 = \cdots = B_7 = 0$. Construct a graph of the waveform of the sound. Your waveform will not look exactly like the flute waveform in Figure 18.18b because you simplify by ignoring cosine terms; nevertheless, it produces the same sensation to human hearing.

Additional Problems

62. M A pipe open at both ends has a fundamental frequency of 300 Hz when the temperature is 0°C. (a) What is the length of the pipe? (b) What is the fundamental frequency at a temperature of 30.0°C?

63. A string is 0.400 m long and has a mass per unit length of 9.00×10^{-3} kg/m. What must be the tension in the string if its second harmonic has the same frequency as the second resonance mode of a 1.75-m-long pipe open at one end?

64. Two strings are vibrating at the same frequency of 150 Hz. After the tension in one of the strings is decreased, an observer hears four beats each second when the strings vibrate together. Find the new frequency in the adjusted string.

65. The ship in Figure P18.65 travels along a straight line parallel to the shore and a distance $d = 600$ m from it. The ship's radio receives simultaneous signals of the same frequency from antennas A and B, separated by a distance $L = 800$ m. The signals interfere constructively at point C, which is equidistant from A and B. The signal goes through the first minimum at point D, which is directly outward from the shore from point B. Determine the wavelength of the radio waves.

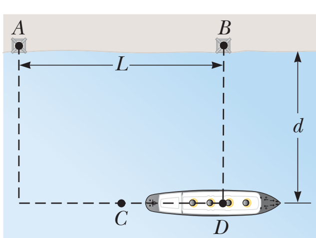


Figure P18.65

66. A 2.00-m-long wire having a mass of 0.100 kg is fixed at both ends. The tension in the wire is maintained at 20.0 N. (a) What are the frequencies of the first three allowed modes of vibration? (b) If a node is observed at a point 0.400 m from one end, in what mode and with what frequency is it vibrating?

67. The fret closest to the bridge on a guitar is 21.4 cm from the bridge as shown in Figure P18.67. When the thinnest string is pressed down at this first fret, the string produces the highest frequency that can be played on that guitar, 2 349 Hz. The next lower note that is produced on the string has frequency 2 217 Hz. How far away from the first fret should the next fret be?

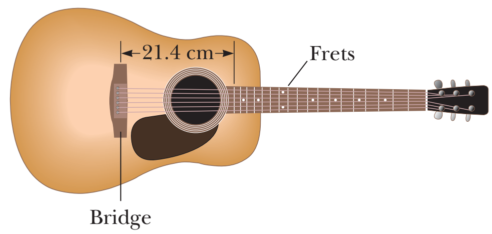


Figure P18.67

68. A string fixed at both ends and having a mass of 4.80 g, a length of 2.00 m, and a tension of 48.0 N vibrates in its second ($n = 2$) normal mode. (a) Is the wavelength in air of the sound emitted by this vibrating string larger or smaller than the wavelength of the wave on the string? (b) What is the ratio of the wavelength in air of the sound emitted by this vibrating string and the wavelength of the wave on the string?

69. A quartz watch contains a crystal oscillator in the form of a block of quartz that vibrates by contracting and expanding. An electric circuit feeds in energy to maintain the oscillation and also counts the voltage pulses to keep time. Two opposite faces of the block, 7.05 mm apart, are antinodes, moving alternately toward each other and away from each other. The plane halfway between these two faces is a node of the vibration. The speed of sound in quartz is equal to 3.70×10^3 m/s. Find the frequency of the vibration.

70. GP **Review.** For the arrangement shown in Figure P18.70, the inclined plane and the small pulley are frictionless; the string supports the object of mass M at the bottom of the plane; and the string has mass m. The system is in equilibrium, and the vertical part of the string has a length h. We wish to study standing waves set up in the vertical section of the string. (a) What analysis model describes the object of mass M? (b) What analysis model describes the waves on the vertical part of the

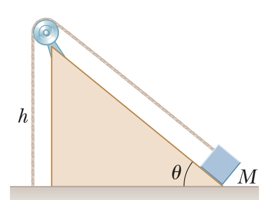


Figure P18.70

string? (c) Find the tension in the string. (d) Model the shape of the string as one leg and the hypotenuse of a right triangle. Find the whole length of the string. (e) Find the mass per unit length of the string. (f) Find the speed of waves on the string. (g) Find the lowest frequency for a standing wave on the vertical section of the string. (h) Evaluate this result for $M = 1.50$ kg, $m = 0.750$ g, $h = 0.500$ m, and $\theta = 30.0°$. (i) Find the numerical value for the lowest frequency for a standing wave on the sloped section of the string.

71. A 0.010 0-kg wire, 2.00 m long, is fixed at both ends and vibrates in its simplest mode under a tension of 200 N. When a vibrating tuning fork is placed near the wire, a beat frequency of 5.00 Hz is heard. (a) What could be the frequency of the tuning fork? (b) What should the tension in the wire be if the beats are to disappear?

72. Two speakers are driven by the same oscillator of frequency f. They are located a distance d from each other on a vertical pole. A man walks straight toward the lower speaker in a direction perpendicular to the pole as shown in Figure P18.72. (a) How many times will he hear a minimum in sound intensity? (b) How far is he from the pole at these moments? Let v represent the speed of sound and assume that the ground does not reflect sound. The man's ears are at the same level as the lower speaker.

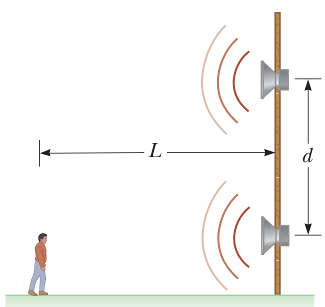


Figure P18.72

73. **Review.** Consider the apparatus shown in Figure 18.11 and described in Example 18.4. Suppose the number of antinodes in Figure 18.11b is an arbitrary value n. (a) Find an expression for the radius of the sphere in the water as a function of only n. (b) What is the minimum allowed value of n for a sphere of nonzero size? (c) What is the radius of the largest sphere that will produce a standing wave on the string? (d) What happens if a larger sphere is used?

74. **Review.** The top end of a yo-yo string is held stationary. The yo-yo itself is much more massive than the string. It starts from rest and moves down with constant acceleration 0.800 m/s^2 as it unwinds from the string. The rubbing of the string against the edge of the yo-yo excites transverse standing-wave vibrations in the string. Both ends of the string are nodes even as the length of the string increases. Consider the instant 1.20 s after the motion begins from rest. (a) Show that the rate of change with time of the wavelength of the fundamental mode of oscillation is 1.92 m/s. (b) **What if?** Is the rate of change of the wavelength of the second harmonic also 1.92 m/s at this moment? Explain your answer. (c) **What if?** The experiment is repeated after more mass has been added to the yo-yo body. The mass distribution is kept the same so that the yo-yo still moves with downward acceleration 0.800 m/s^2. At the 1.20-s point in this case, is the rate of change of the fundamental wavelength of the string vibration still equal to 1.92 m/s? Explain. (d) Is the rate of change of the second harmonic wavelength the same as in part (b)? Explain.

75. On a marimba (Fig. P18.75), the wooden bar that sounds a tone when struck vibrates in a transverse standing wave having three antinodes and two nodes. The lowest-frequency note is 87.0 Hz, produced by a bar 40.0 cm long. (a) Find the speed of transverse waves on the bar. (b) A resonant pipe suspended vertically below the center of the bar enhances the loudness of the emitted sound. If the pipe is open at the top end only, what length of the pipe is required to resonate with the bar in part (a)?

Figure P18.75

76. A nylon string has mass 5.50 g and length $L = 86.0$ cm. The lower end is tied to the floor, and the upper end is tied to a small set of wheels through a slot in a track on which the wheels move (Fig. P18.76). The wheels have a mass that is negligible compared with that of the string, and they roll without friction on the track so that the upper end of the string is essentially free. At equilibrium, the string is vertical and motionless. When it is carrying a small-amplitude wave, you may assume the string is always under uniform tension 1.30 N. (a) Find the speed of transverse waves on the string. (b) The string's vibration possibilities are a set of standing-wave states, each with a node at the fixed bottom end and an antinode at the free top end. Find the node–antinode distances for each of the three simplest states. (c) Find the frequency of each of these states.

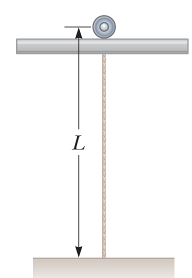


Figure P18.76

77. M Two train whistles have identical frequencies of 180 Hz. When one train is at rest in the station and the other is moving nearby, a commuter standing on the station platform hears beats with a frequency of 2.00 beats/s when the whistles operate together. What are the two possible speeds and directions the moving train can have?

78. **Review.** A loudspeaker at the front of a room and an identical loudspeaker at the rear of the room are being driven by the same oscillator at 456 Hz. A student walks at a uniform rate of 1.50 m/s along the length of the room. She hears a single tone repeatedly becoming louder and softer. (a) Model these variations as beats between the Doppler-shifted sounds the student receives. Calculate the number of beats the student hears each second. (b) Model the two speakers as producing a standing wave in the room and the student as walking between antinodes. Calculate the number of intensity maxima the student hears each second.

79. **Review.** Consider the copper object hanging from the steel wire in Problem 32. The top end of the wire is fixed. When the wire is struck, it emits sound with a fundamental frequency of 300 Hz. The copper object is then submerged in water. If the object can be positioned with any desired fraction of its volume submerged, what is the lowest possible new fundamental frequency?

80. M Two wires are welded together end to end. The wires are made of the same material, but the diameter of one is twice that of the other. They are subjected to a tension of 4.60 N. The thin wire has a length of 40.0 cm and a linear mass density of 2.00 g/m. The combination is fixed at both ends and vibrated in such a way that two antinodes are present, with the node between them being right at the weld. (a) What is the frequency of vibration? (b) What is the length of the thick wire?

81. A string of linear density 1.60 g/m is stretched between clamps 48.0 cm apart. The string does not stretch appreciably as the tension in it is steadily raised from 15.0 N at $t = 0$ to 25.0 N at $t = 3.50$ s. Therefore, the tension as a function of time is given by the expression $T = 15.0 + 10.0t/3.50$, where T is in newtons and t is in seconds. The string is vibrating in its fundamental mode throughout this process. Find the number of oscillations it completes during the 3.50-s interval.

82. A standing wave is set up in a string of variable length and tension by a vibrator of variable frequency. Both ends of the string are fixed. When the vibrator has a frequency f, in a string of length L and under tension T, n antinodes are set up in the string. (a) If the length of the string is doubled, by what factor should the frequency be changed so that the same number of antinodes is produced? (b) If the frequency and length are held constant, what tension will produce $n + 1$ antinodes? (c) If the frequency is tripled and the length of the string is halved, by what factor should the tension be changed so that twice as many antinodes are produced?

83. Two waves are described by the wave functions

$$y_1(x, t) = 5.00 \sin (2.00x - 10.0t)$$

$$y_2(x, t) = 10.0 \cos (2.00x - 10.0t)$$

where x, y_1, and y_2 are in meters and t is in seconds. (a) Show that the wave resulting from their superposition can be expressed as a single sine function. (b) Determine the amplitude and phase angle for this sinusoidal wave.

84. A flute is designed so that it produces a frequency of 261.6 Hz, middle C, when all the holes are covered and the temperature is 20.0°C. (a) Consider the flute as a pipe that is open at both ends. Find the length of the flute, assuming middle C is the fundamental. (b) A second player, nearby in a colder room, also attempts to play middle C on an identical flute. A beat frequency of 3.00 Hz is heard when both flutes are playing. What is the temperature of the second room?

85. AMT **Review.** A 12.0-kg object hangs in equilibrium from a string with a total length of $L = 5.00$ m and a linear mass density of $\mu = 0.001\,00$ kg/m. The string is wrapped around two light, frictionless pulleys that are separated by a distance of $d = 2.00$ m (Fig. P18.85a). (a) Determine the tension in the string. (b) At what frequency must the string between the pulleys vibrate to form the standing-wave pattern shown in Figure P18.85b?

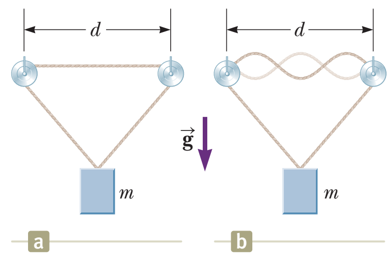


Figure P18.85 Problems 85 and 86.

86. **Review.** An object of mass m hangs in equilibrium from a string with a total length L and a linear mass density μ. The string is wrapped around two light, frictionless pulleys that are separated by a distance d (Fig. P18.85a). (a) Determine the tension in the string. (b) At what frequency must the string between the pulleys vibrate to form the standing-wave pattern shown in Figure P18.85b?

Challenge Problems

87. **Review.** Consider the apparatus shown in Figure P18.87a, where the hanging object has mass M and the string is vibrating in its second harmonic. The vibrating blade at the left maintains a constant frequency. The wind begins to blow to the right, applying a con-

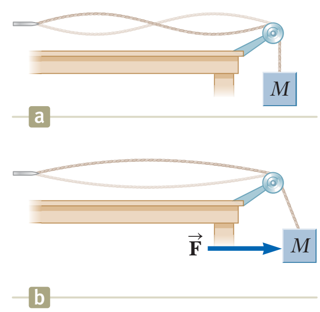


Figure P18.87

stant horizontal force $\vec{\mathbf{F}}$ on the hanging object. What is the magnitude of the force the wind must apply to the hanging object so that the string vibrates in its first harmonic as shown in Figure 18.87b?

88. In Figures 18.20a and 18.20b, notice that the amplitude of the component wave for frequency f is large, that for $3f$ is smaller, and that for $5f$ smaller still. How do we know exactly how much amplitude to assign to each frequency component to build a square wave? This problem helps us find the answer to that question. Let the square wave in Figure 18.20c have an amplitude A and let $t = 0$ be at the extreme left of the figure. So, one period T of the square wave is described by

$$y(t) = \begin{cases} A & 0 < t < \dfrac{T}{2} \\ -A & \dfrac{T}{2} < t < T \end{cases}$$

Express Equation 18.13 with angular frequencies:

$$y(t) = \sum_n (A_n \sin n\omega t + B_n \cos n\omega t)$$

Now proceed as follows. (a) Multiply both sides of Equation 18.13 by $\sin m\omega t$ and integrate both sides over one period T. Show that the left-hand side of the resulting equation is equal to 0 if m is even and is equal to $4A/m\omega$ if m is odd. (b) Using trigonometric identities, show that all terms on the right-hand side involving B_n are equal to zero. (c) Using trigonometric identities, show that all terms on the right-hand side involving A_n are equal to zero *except* for the one case of $m = n$. (d) Show that the entire right-hand side of the equation reduces to $\frac{1}{2}A_m T$. (e) Show that the Fourier series expansion for a square wave is

$$y(t) = \sum_n \frac{4A}{n\pi} \sin n\omega t$$