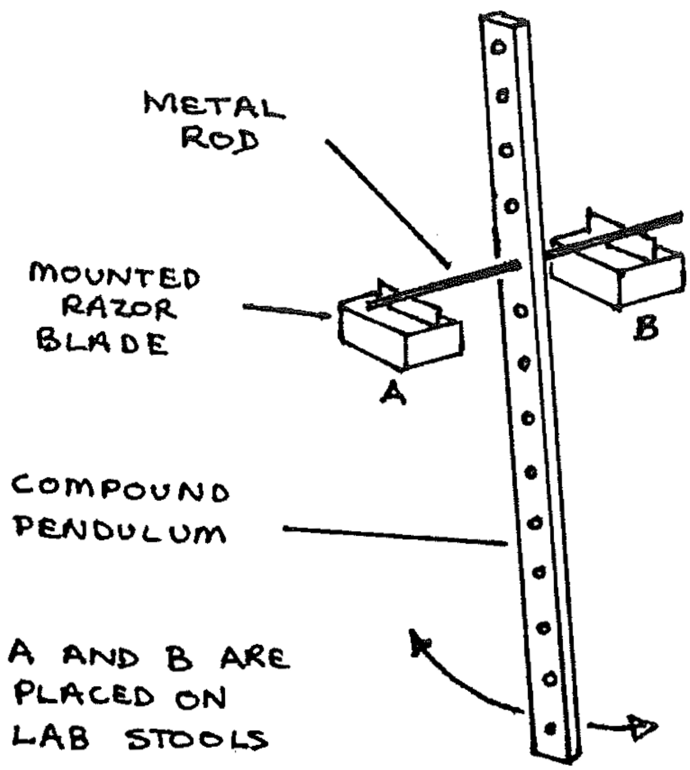
Module 23 (Experimental)

1. Compound Pendulum

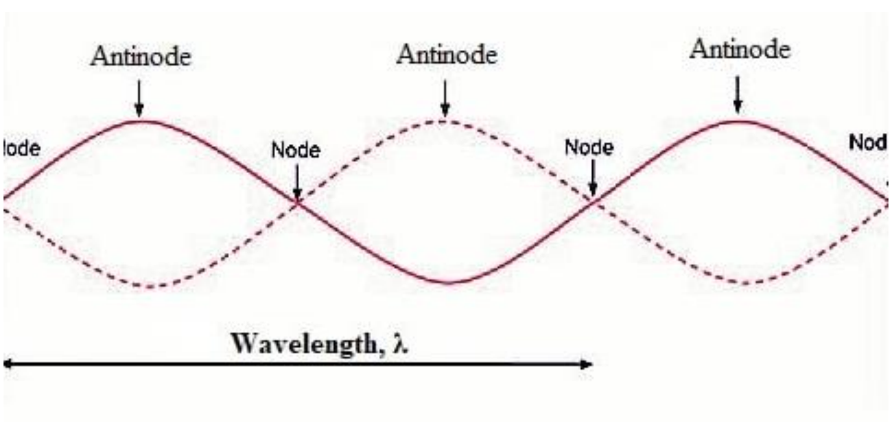
A compound pendulum consists of a long, heavy bar with holes slotted in it along the center-of-mass line. When we swing the bar from each of these holes we get a different time period because the distance from the center-of-mass is different in each case. In this experiment, we will measure the time period of oscillations for various distances from the center-of-mass and use it to determine the acceleration due to gravity 'g'. We will also learn how the moment of inertia affects the rate of oscillation.



2. Speed of Sound using Destructive Interference

Since sound is a wave, it can undergo constructive and destructive interference when two waves superpose in the right way. In this experiment, we will use a pair of earphones to set up standing waves inside a column. We will place one earphone on top of the column and the other end at the bottom and then play a standard tone through them. As we vary the distance between the two earphones, there will come a point where the sound intensity will drop substantially, and then rise again. This is the point where destructive interference is taking place. We keep changing the distance until we find such a point again and note the separation between these two points of destructive interference. Let this separation be d. Then the wavelength λ is given by:

$$\lambda = 2d$$



Then using the given sound frequency f, we can find the speed of sound v_s by:

$$v_s = f\lambda = 2df$$

TASKS:

- When this experiment is carried out, it is noticed that the sound intensity does not go exactly to zero at the antinode, but reaches a finite minimum. Why do you think complete destructive interference is not being achieved?
- Suppose we carried out the same experiment underwater. Would we need a bigger toilet paper stack or a smaller one? What if we carried out this experiment in air but in a much hotter environment?
- What can be some sources of error in this experiment?

3. Speed of Waves on a String

If we keep a string of length l fixed at both ends and pluck it, waves will travel through it which upon interference will lead to standing waves. Since the length is fixed, only certain discrete frequencies can be generated on such a string, which is why a guitar string can only play certain fixed notes. It is common knowledge that when the different strings on a guitar have different sounds when plucked. Moreover, the sound of any given string changes when we vary tighten or loosen the string.

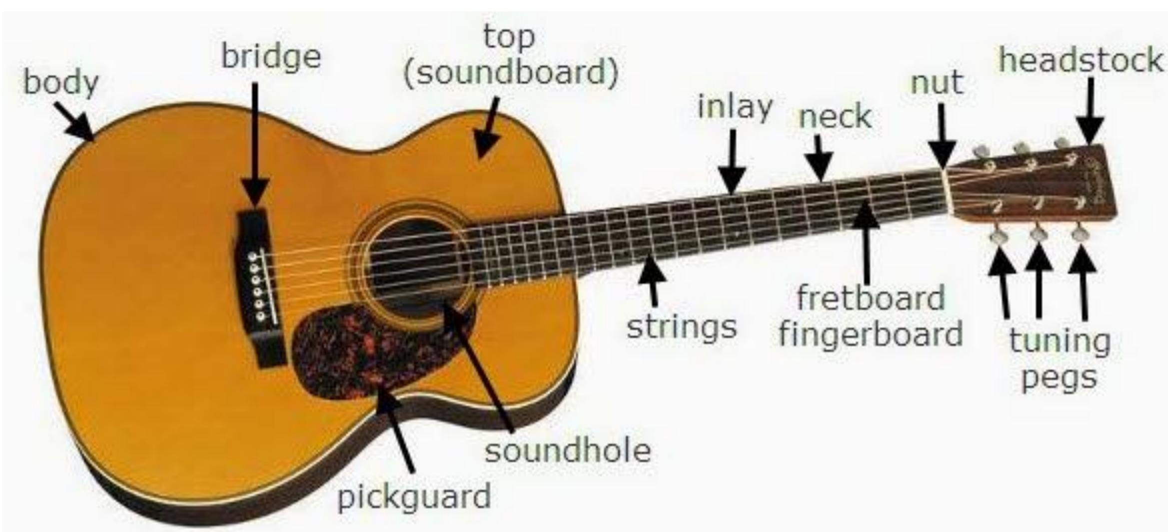
In this experiment, we will measure the speed of waves travelling on a guitar string and test its dependence of the tension and the thickness of the string. The speed of the wave can be obtained by simply measuring the frequency and wavelength. If the open string is plucked, then the relation between the length of the string and the wavelength is given by:

$$\lambda = 2l$$

We can measure the frequency of the sound generated using computer software like eTuner, which will enable us to find the speed of the wave on the string:

$$v = f\lambda$$

Now we will vary 3 parameters and see how the speed changes. These are: the length of the string, the tension of the string and the string thickness. The length can be changed by simply holding the string on different frets, the tension can be changed with the tuning pegs on the end of the guitar neck while the thickness can be changed by simply choosing a different string from among the 6 available.



TASKS:

- Use dimensional analysis to figure out the relationship between speed v, tension T, length l and mass density (thickness) μ.
- Using the data you obtain, plot v vs T, v vs l and v vs μ. Verify whether this agrees with your dimensional analysis.
- Would our measurements of v change if we conducted the same experiment in outer space (assuming we could measure the frequency without hearing it)?
- Assume the wave speed on a guitar string is $425\ m/s$ and we are dealing with the 1st harmonic (n = 1). What is the range of lengths the guitar string needs to be in for the first harmonic to be audible to the average human ear?
- Identify some possible sources of error in this experiment.

4. Coupled Pendulums

Coupled Pendulums on a Clothesline 36

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The importance of coupled oscillations for natural and engineering sciences is undisputed. But due to the complex processes behind the phenomena, its educational arrangement remains challenging.

Based on traditional setups of coupled spring pendulums [1–3] and torsion pendulums [3, 4], we investigate a low-cost experiment consisting of two clothespins that are mounted on a fine string [5]. We use a simple approach via spring pendulums to describe the dependencies of the coupling between the clothespins, particularly with regard to the distance between their mounting points. The theoretical approach coincides with experimental data that is extracted via mobile video analysis using tablet PCs [6] and the apps Viana [7] and Graphical Analysis [8].

36.1 Theoretical Background

Compared to a traditional setup of two spring-coupled spring pendulums with identical masses, we use two identical clothespins mounted on a fine string with identical distances between the pins and the mounting of the string (Figs. 36.1 and 36.2), resulting in two string-coupled torsion pendulums.

Writing Newton's second law for x_1 and x_2 of the system in Fig. 36.1a yields

$$-kx_1 - k_{12}(x_1 - x_2) = m\ddot{x}_1 \text{ and } -kx_2 - k_{12}(x_2 - x_1) = m\ddot{x}_2,$$

which, when written in terms of the center of mass $X = \frac{1}{2}(x_1 + y_2)$ and difference coordinate, $Y = \frac{1}{2}(x_1 - y_2)$ become simple harmonic oscillator equations [3]:

$$\ddot{X} = -\frac{k}{m} \cdot X \text{ and } \ddot{Y} = \frac{k + 2k_{12}}{m} \cdot Y.$$

Thus, the system has two "normal" modes of oscillation: one with frequency squared given by $\omega^2 = k/m$, corresponding to the symmetric motion of both masses together, and one given by

$$\Omega^2 = \frac{k + 2k_{12}}{m},$$

corresponding to the anti-symmetric motion. The actual motion of each clothespin can be described as a superposition of those modes. The energy is transferred back

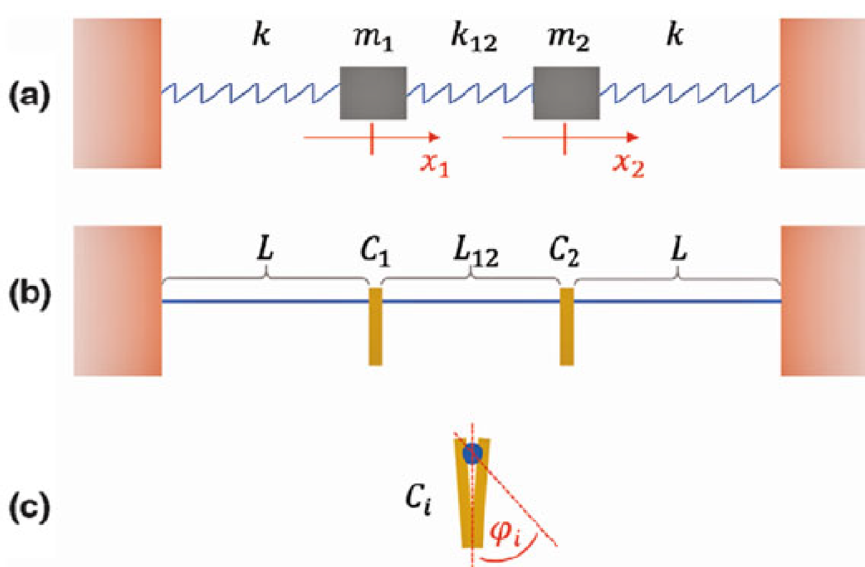

Fig. 36.1 Schematic comparison between coupled spring pendulums (**a**) and coupled torsion pendulums ([**b**] frontal view, [**c**] side view); x_i and φ_i describe the corresponding deviations from the rest positions

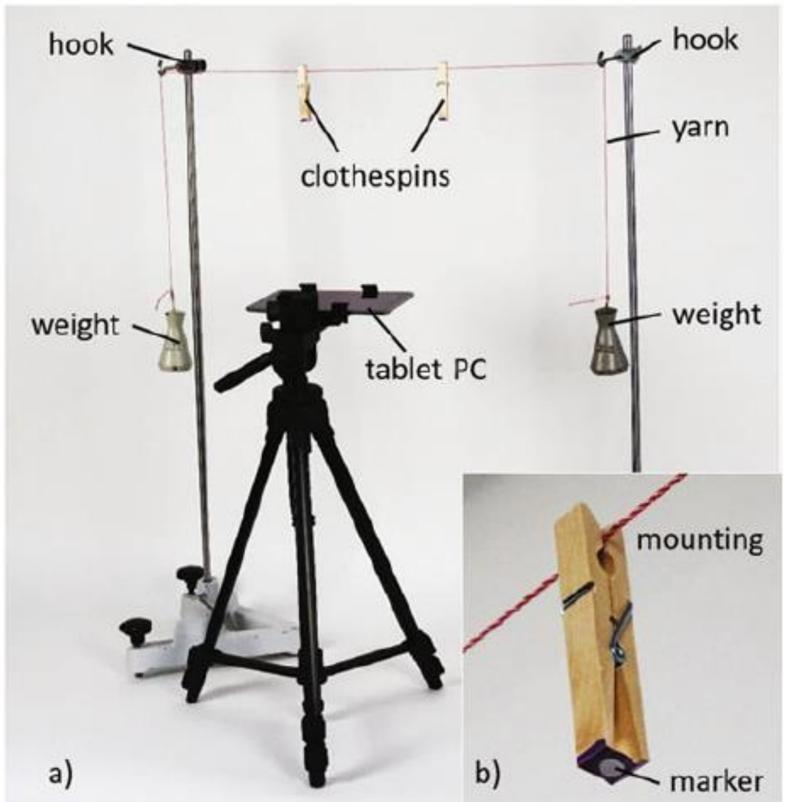

Fig. 36.2 (**a**) Photo of the experimental setup. (**b**) Detailed view of the tight mounting and one of the markers beneath the clothespins

and forth between the pins with a frequency given by half the beat frequency of the superposition, so the time it takes the energy to transfer from one pin to the other is

$$\tau_{\text{transfer}} = \frac{T_{\text{beat}}}{4} = \frac{\pi}{\Omega - \omega} = \pi \cdot \frac{\Omega + \omega}{\Omega^2 - \omega^2} = \pi \cdot \frac{\Omega + \omega}{2k_{12}/m}. \tag{36.1}$$

Likewise, we can apply the torque equation [3] to each of the clothespins, using c and c_{12} for the Hooke's law proportionality constants of the small angular displacements of the pins from their equilibrium position, arriving at

$$-M \cdot g \cdot r \cdot \sin \varphi_1 - c\varphi_1 - c_{12}(\varphi_1 - \varphi_2) = I\ddot{\varphi}_1$$

and

$$-M \cdot g \cdot r \cdot \sin \varphi_2 - c\varphi_1 - c_{12}(\varphi_2 - \varphi_1) = I\ddot{\varphi}_2,$$

where M, r, and I are the mass, momentum arm to the center of mass, and moment of inertia of one single pin, respectively. Thus, for small angles, and with the identifications $k \rightarrow (Mgr + c)$, $k_{12} \rightarrow c_{12}$, and $m \rightarrow I$, we expect that the energy will be transferred between the clothespins in a time

$$\tau_{\text{transfer}} = \pi \cdot \frac{\Omega + \omega}{2c_{12}/I}. \tag{36.2}$$

Finally, we expect that the Hooke's law constants c and c_{12} will depend inversely on the length of the corresponding clothesline [3, 4], so we write $c = C/L$ and $c_{12} = C_{12}/L_{12}$, giving

$$\tau_{\text{transfer}} = \frac{\pi}{2} \cdot I \cdot (\Omega + \omega) \cdot L_{12}/C_{12}. \tag{36.3}$$

This result suggests that if L_{12} is about the same length as L, i.e., $L_{12} = L + x$ with $x \ll L$, then the transfer period should be approximately linear in x, since $(\Omega + \omega)$ depends so weakly on x. Altogether, we obtain

$$\tau_{\text{transfer}} = \frac{\pi}{2} \cdot \frac{I}{C_{12}} \cdot \left(\sqrt{\left(Mgr + \frac{C}{L} + 2\frac{C_{12}}{L+x}\right)/I} + \sqrt{\left(Mgr + \frac{C}{L}\right)/I} \right) \cdot (L + x).$$

Furthermore, the ratio of the y-intercept of the line to the slope of the line in this theoretical model should be given by L; the experimental value comes to

$$\frac{y - \text{intercept}}{\text{slope}} = \frac{1.45\ \text{s}}{7.37\frac{\text{s}}{\text{m}}} = (0.197 \pm 0.006)\text{m}$$

(Fig. 36.4), a reasonably satisfying result.

36.2 Experimental Setup

The two clothespins are mounted tightly on the fine string ("the clothesline"), which is under tension with the help of two weights (we used $m = 0.5$ kg). The tablet is placed beneath the pins with its display pointing upwards in order to record a video of the pins' motion with the front camera of the device (the default recording rate is about 30 fps, iPad 4 mini). To improve tracking with the video analysis software, it is helpful to attach a colored marker beneath each clothespin (Fig. 36.2b).

To conduct the experiment, one clothespin is deflected manually and released, while the oscillation is recorded by the tablet PC. Due to the coupling via the string, the energy of the oscillation is transferred between the clothespins periodically. After several transfer periods, the recording is stopped and the video is imported into the app Viana, where the position of each marker is tracked and displayed automatically. We obtain the following diagram (Fig. 36.3) of the oscillation over time.

To determine τ_{transfer}, the data are imported into Graphical Analysis [8], where we can read out the period with the help of a precise cursor. A systematical variation of the distance between the clothespins resulted in an almost linear relation between the distance $x = L_{12} - L$ and τ_{transfer} (Fig. 36.4), in accordance to our theoretical approach (Eq. 36.4).

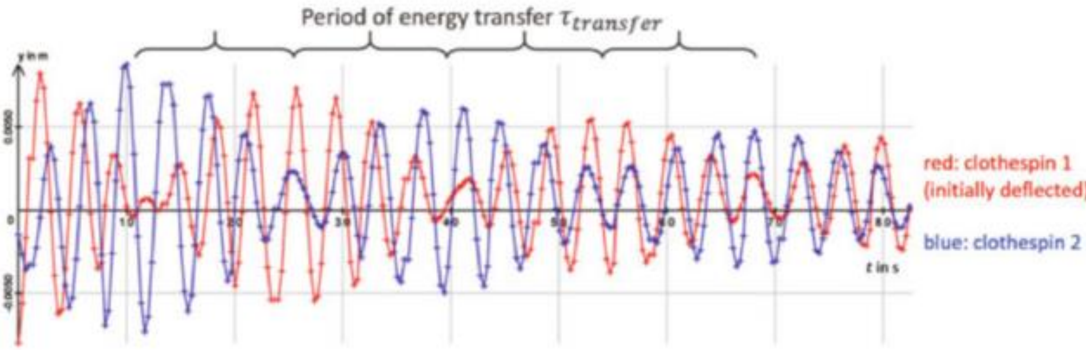

Fig. 36.3 Screenshot of the deviations over time of each clothespin, using $L_{12} = 0.2$ m and $L = 0.2$ m (the y-axis points in the direction of movement); the period of energy transfer τ_{transfer} is marked

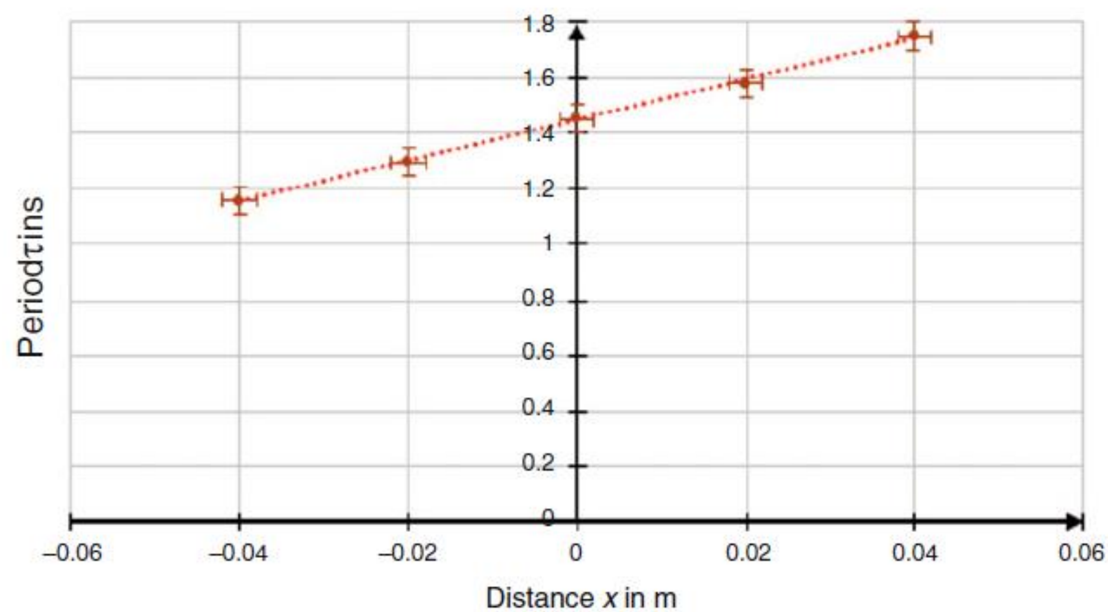

Fig. 36.4 Period-distance-diagram for $L = 0.2$ m; a simple linear regression yields $\tau_{\text{transfer}}(x) = (7.37 \pm 0.17)\,\frac{\text{s}}{\text{m}}x + (1.45 \pm 0.01)$ s and the coefficient of determination $R^2 = 0.99$

36.3 Discussion

Combining this low-cost experiment with the simple but effective method of mobile video analysis results in a low-cost setup that can easily be used for educational purposes in school laboratories. Based on concepts of traditional spring-coupled pendulums, this case is useful for qualitative and quantitative investigations, e.g., with the presented simple theoretical approach.

Alternatively, the coupling could be varied via the tension of the string using different weights. Furthermore, the focus of the investigations could also be on the phase relationship between the oscillating clothespins, e.g., their normal modes.

The validation of our approach and further investigations are limited due to additional vibrations of the string and the pins, leading to a nonlinear impact on the energy transfer. Anyway research shows that learning with mobile video analysis can also increase conceptual understanding [9, 11] while decreasing irrelevant cognitive effort and negative emotions [10, 11].

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