## Summary of Lecture 36 – THERMAL PHYSICS I

1. The ancient view was that heat is a colourless, weightless, fluid which occupies no volume has no smell, etc. This imagined substance - called phlogiston - was supposedly stored in objects and transferred between objects. It took a long time to reject this notion. Why is it wrong? Because (as we will see) heat can be created and destroyed, whereas liquids keep their volume and cannot be created or destroyed.

2. We are all familiar with an intuitive notion of temperature. We know that hotter things have higher temperature. But let us try to define temperature more rigorously. In the diagram below, the three bodies A,B,C are in contact with each other. After sufficient time passes, one thing will be common to all three - a quantity that we call temperature.
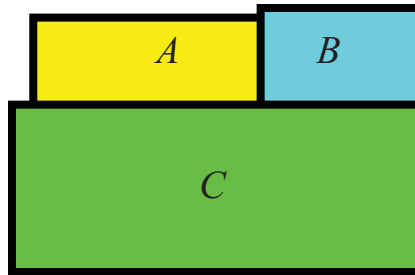

3. Now let us understand heat. Heat is energy, but it is a very special kind of energy: it is that energy which flows from a system at high temprature to a system at low temperature.


Stated in a slightly different way: heat is the flow of internal energy due to a temperature difference. (Note that we do not have to know about atoms, molecules, and the internal composition of a body to be able to define heat - all that will come later).

4. The term "thermal equilibrium" is extremely important to our understanding of heat. If some objects (say, a glass of water placed in the open atmosphere) are put in thermal contact but there is no heat exchange, then we say that the objects are in thermal equilibrium. So, if the glass of water is hot or cold initially, after sufficient time passes it will be in thermal equilibrium and will neither receive nor lose heat to the atmosphere.

5. We have an intuitive understanding of temperature, but how do we measure it? Answer: by looking at some physical property that changes when the temperature changes. So, for example, when the temperature rises most things expand, the electrical resistance changes, some things change colour, etc. These are called "thermometric properties".

6. Let's take a practical example: the constant volume gas thermometer. Here the reference level is kept fixed by raising or lowering the tube on the right side (the tube below is made of rubber or some flexible material). So the gas volume is fixed. The gas pressure is $\rho \times g \times h$, and so the pressure is known from measuring h.
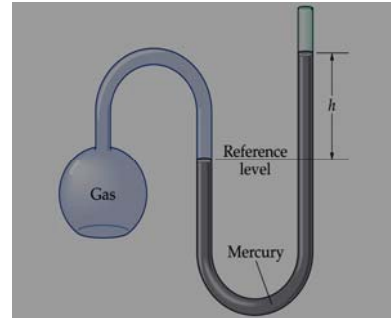
To find the temperature of a substance, the gas flask is placed in thermal contact with the substance. When the temperature is high, the pressure is large. From the graph of pressure versus temperature, you can easily read off the temperature. Note that we are using two fixed points, which we call 0^0C and 100^0C. This is called the Centigrade scale, and the two fixed points correspond to the freezing and boiling of water.
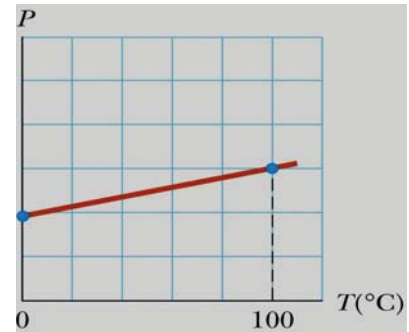
7. There is a temperature below which it is not possible to go. In other words, if you cool and cool there comes a point after which you cannot cool any more! How do we know this? Take different gases and plot how their pressure changes as you cool them down. You can see from the graph that all the lines, when drawn backwards, meet exactly at one point. This point is the absolute zero of temperature and lies at about -273.15^0C. This is also called 0^0K, or zero degrees Kelvin.
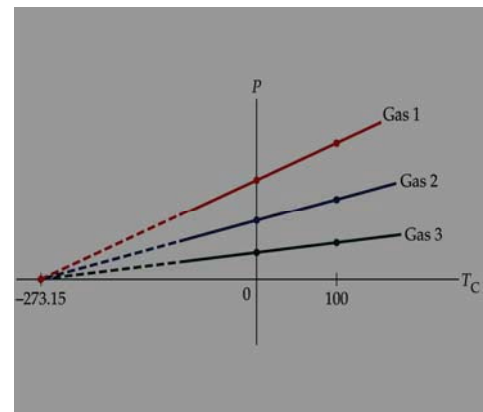
8. Temperature Scales. If you had been born 300 years ago, and if you had discovered a reliable way to tell different levels of "hotness", then maybe today there would be a temperature scale named after you! The relation between Centigrade, Fahrenheit, and Kelvin scales is illustrated to your right. You can see that absolute zero corresponds to $-460^0 K$, $-273.15^0 C$, and $0^0 K$. The Kelvin scale is the most suited for scientific purposes, and you should be careful to use this in all heat related calculations.
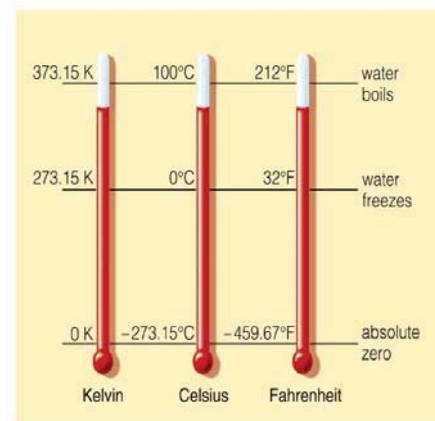
Conversion between degrees Celsius and degrees Fahrenheit: $T_F = (\frac{9}{5}F^\circ/C^\circ)T_C + 32^\circ F$

Conversion between degrees Fahrenheit and degrees Celsius: $T_C = (\frac{5}{9}C^\circ/F^\circ)(T_F - 32^\circ F)$

Conversion between Celsius and Kelvin temperatures: $T = T_C + 273.15$

9. How hot is hot, and how cold is cold? Whenever a scientist says something is large or small, it is always relative to something. The hottest thing ever was the universe when it just came into existence (more about this in the last lecture). After this comes the hydrogen bomb (10^8K). The surface of the sun is not so hot, only 5500K. Copper melts around 1000K, water turns to steam at 373K. If you cool further, then all the gases start to solidify. The lowest temperature that has ever been achieved is $10^{-7}K$, which is one tenth of one millionth of one degree! Why not still lower? We shall later why this is not possible.
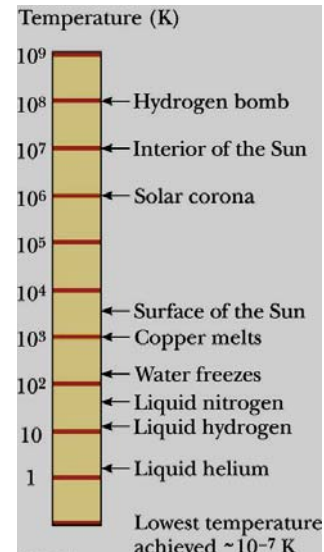
10. When you rub your hands, they get hot. Mechanical work has been converted into heat. The first person to investigate this scientifically was Joule. In the experiment below, he allowed a weight to drop. This turned a paddle that stirred up the water and caused the temperature to rise. The water got hotter if the weight was released from a greater height.
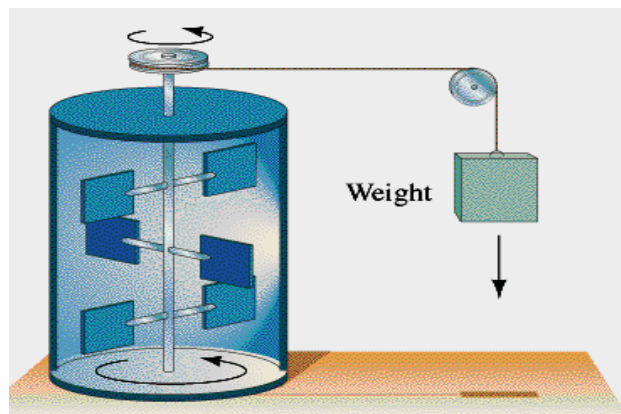

Joule established the units for the mechanical equivalent of heat. The units we use today are: 1 calorie (1 cal) raises the temperature of 1 g of water by 1 °C
            1 cal = 4.186 Joule,   1 kilocalorie (1 kcal) = 1000 cal
Remember also that joule is the unit of work: when a force of one newton acts through a distance of one metre, the work done is one joule.

11. Let's now consider one important effect of heat - most things expand when heated. Of course, our world is 3-dimensional but if there is a thin long rod then the most visible effect of heating it is that the rod increases in length. By how much?
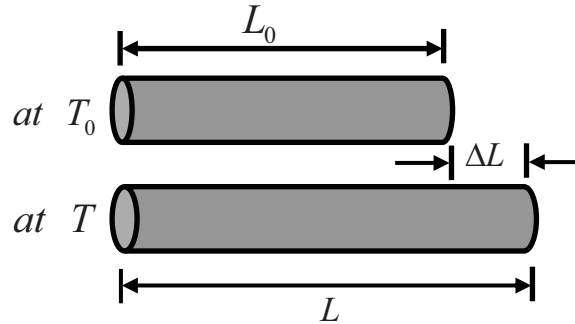

Look at the above diagram. Call the length of the rod at T_0 as L_0. When the rod is heated to T, then the length increases to $L = L_0 \left[1 + \alpha \left(T - T_0\right)\right]$. The difference of the lengths is, $L - L_0 = \alpha L_0 \left(T - T_0\right)$, or $\Delta L = \alpha L_0 \Delta T$. Here α is just a dimensionless number that tells you how a particular material expands. It is called the coefficient of linear expansion. If α was zero, then the material would not expand at all. You can also write it as $\alpha = \dfrac{\Delta L / L_0}{\Delta T}$.

Similarly, define a coefficient of volume expansion $-$ call it β $-$ as $\beta \equiv \dfrac{\Delta V / V_0}{\Delta T}$.

12. There is a relation between α and β, the linear and volume coefficients. Let's look at the change in volume due to expansion: $V' = (L + \Delta L)^3 = (L + \alpha L \Delta T)^3$

$$= L^3 + 3\alpha L^3 \Delta T + 3\alpha^2 L^3 \Delta T^2 + \alpha^3 L^3 \Delta T^3$$

$$\approx L^3 + 3\alpha L^3 \Delta T = V + 3\alpha V \Delta T$$

We are only looking for small changes, so the higher terms in ΔT can be safely dropped. Hence $\Delta V = 3\alpha V \Delta T$. From the definition, this we immediately see that $\boxed{\beta = 3\alpha}$. Just to get an idea, here is what β looks like for various different materials:

Coefficient of volume expansion β (1/°C)
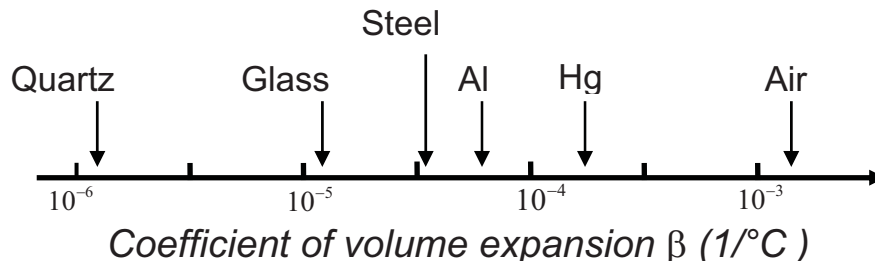
13. We can use the fact that different metals expand at different rates to make thermostats. For example, you need a thermostat to prevent an electric iron from getting too hot or a

refrigerator from getting too cold. In the diagram below, one metal is bonded to another. When they expand together, one expands less than the other and the shape is distorted. This breaks off a circuit, as shown here.
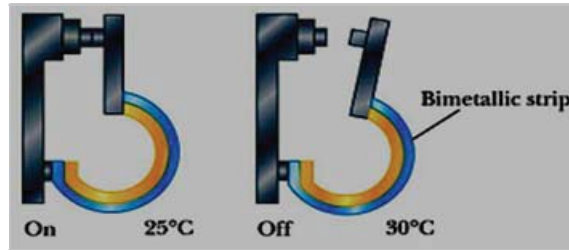
14. Water behaves strangely - for certain temperatures, it contracts when heated (instead of expanding)! Look at the graph: between 0^0C and 4^0C, it exhibits strange or *anomalous* behaviour. The reason is complicated and has to do with the molecular structure of water. If water behaved normally, it would be very bad for fish in the winter because they rely upon the bottom of the lake or sea to remain liquid even though there the surface is frozen.
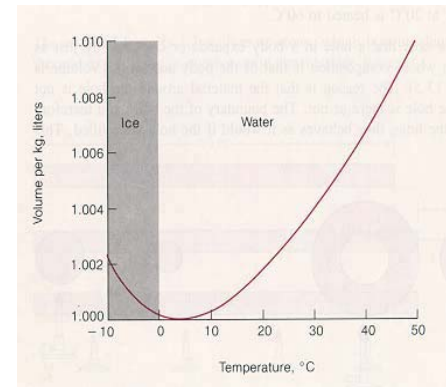
15. We define the "heat capacity" of a body as $C = \dfrac{Q}{\Delta T}$, where ΔT is the increase in temperature when an amount of heat Q is added to the body. Heat capacity is always positive; Q and ΔT have the same sign. The larger the heat capacity, the smaller is the change in the body's temperature when a fixed amount of heat is added. In general, $Q = mc\Delta T$, where $Q =$ heat added , $m =$ mass , $c =$ specific heat , and $\Delta T =$ change in temperature. Water has a very large specific heat c, $c = 1.0$ *cal* $/(^{\circ}C\ g)$; this means it takes one calorie to raise the temperature of 1 gm of water by 1 degree Celsius. In joules per kilogram this is the same as 4186. In the table are the specific heats of various common materials. You can see that metals have small c, which means that it is relatively easy to raise or lower their temperatures. The opposite is true of water. Note also that steam and ice have smaller $c's$ than water. This shows that knowing the chemical composition is not enough.

Specific Heats at Atmospheric Pressure

Substance	Specific heat, c [J / (kg·K)]
Water	4186
Ice	2090
Steam	2010
Beryllium	1820
Air	1004
Aluminum	900
Glass	837
Silicon	703
Iron (steel)	448
Copper	387
Silver	234
Gold	129
Lead	128

16. A 0.5-kg block of metal with an initial temperature of $30.0\,^\circ C$ is dropped into a container holding 1.12 kg of water at $20.0\,^\circ C$. If the final temperature of the block-water system is $20.4\,^\circ C$, what is the specific heat of the metal?

SOLUTION: Write an expression for the heat flow out of the block $Q_{block} = m_b c_b (T_b - T)$. Do the same for water, $Q_{water} = m_w c_w (T - T_w)$. Now use the fact that all the energy that is lost by the block is gained by the water: $Q_{block} = Q_{water} \Rightarrow Tm_b c_b (T_b - T) - m_w c_w (T - T_w) = 0$

From this, $c_b = \dfrac{m_w c_w (T - T_w)}{m_b (T_b - T)} = \dfrac{(1.12 kg)[4186 J/(kg \cdot K)](20.4\,^\circ C - 20.0\,^\circ C)}{(0.500 kg)(30.0\,^\circ C - 20.4\,^\circ C)}$

$$= 391 J/(kg \cdot K)$$

17. When lifting a "daigchee" from a stove, you would be wise to use a cloth. Why? Because metals transfer, or conduct, heat easily whereas cloth does not. Scientifically we define conductivity using experiments and apparatus similar to the following:
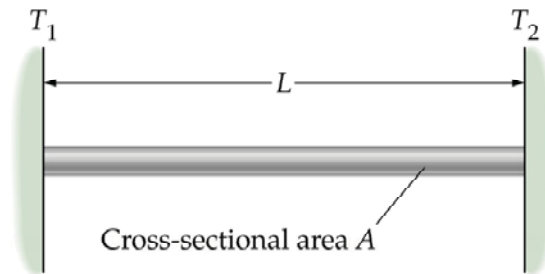
Heat flows from the hotter to the colder plate. Let us use the following symbols:

k = thermal conductivity Q = heat transferred
A = cross sectional area t = duration of heat transfer
L = length ΔT = temperature difference

Then the heat transferred in time t is, $Q = kA\left(\dfrac{\Delta T}{L}\right)t$. This formula allows us to measure k if all the other quantities in it are measured.

18. Conduction is one possible way by which heat is transferred from one portion of a system to another. It does not involve physical transport of particles. However, there is another way by which heat can be transferred - convection. In convection, heat is carried by a moving fluid. So when you heat a pot of water, molecules at the bottom move up, and the ones at the top come down - the water has currents inside it that transfer heat. Another mechanism for transferring heat is through radiation. We have already talked about this while discussing blackbody radiation and the Stefan-Boltzman Law.

QUESTIONS AND EXERCISES – 36

Q.1 The brakes of a car get hot as the car descends a mountain. Explain where the energy is coming from.

Q.2 A thermometer calibrated in degrees Fahrenheit has the same reading as a thermometer calibrated in degrees Celsius at one particular temperature. What is that temperature?

Q.3 A lathe machine without a load draws one kilowatt, but when it is turning its tool against a piece of iron of 3 kilograms, it requires 2.4 kw. Assume that the heat created in the iron has no way out. Using the specific heat given in the table shown earlier, find the rate at which the temperature of the iron increases.

Q.4 A thin sheet of metal has length L_0 and width W_0. It is heated through an amount ΔT. If the coefficient of linear expansion is α, and ΔT is small, find the increase in area of the plate. Now, following the way in which we defined the coefficient of volume expansion, define the coefficient of area exapnsion.

Q.5 a) A 0.5-kg block of nearly melted ice with an initial temperature of $0°C$ is dropped into a container holding 1.5 kg of water at $20.0°C$. Find the final temperature of the ice-water system.
   b) A 0.5-kg block of very cold ice with an initial temperature of $-30°C$ is dropped into a container holding 1.5 kg of water at $20.0°C$. Find the final temperature of the ice-water system now.

Q.6 Suppose that the rate at which a hot body cools is proportional to its temperature. In other words, suppose that in time dt the body cools by amount dT where $dT \propto -T$ (the minus sign shows that the temperature is decreasing rather than increasing). Use this to show that the temperature of a body decreases exponentially with time.

Q.7 When a lake freezes, the freezing begins at the top. Why? Are conduction and radiation involved, or just convection? Discuss.

Summary of Lecture 37 – THERMAL PHYSICS II

1. Let us agree to call whatever we are studying the "system" (a mixture of ice and water, a hot gas, etc). The state of this system is specified by giving its pressure, volume, temperature, etc. These are called "thermodynamic variables". The relation between these variables is called the "equation of state". This equation relates P,V,T. So, for example:

$$P = f(V,T) \quad \text{(knowing } V \text{ and } T \text{ gives you } P\text{)}$$
$$V = g(P,T) \quad \text{(knowing } P \text{ and } T \text{ gives you } V\text{)}$$
$$T = h(P,V) \quad \text{(knowing } P \text{ and } V \text{ gives you } T\text{)}$$

For an ideal gas, the equation of state is $PV = Nk_BT$. Here N is the number of molecules in the gas, T is the temperature in degrees Kelvin, and k_B is called the Boltzman cosntant. This EOS is easy to derive, although I shall not do it here. In principle it is possible to mathematically derive the EOS from the underlying properties of the atoms and molecules in the system. In practice, however, this is a very difficult task (except for an ideal gas). However the EOS can be discovered experimentally.

2. Thermodynamics is the study of heat and how it flows. The First Law of Thermodynamics is actually just an acknowledgment that heat is a form of energy (and, of course, energy is always conserved!). Mathematically, the First Law states that:

$$\Delta E = \Delta q + \Delta w$$

where: E = internal energy of the system

$\quad\quad\quad q$ = heat transferred to the system from the surroundings

$\quad\quad\quad w$ = work done on the system by the surroundings.

In words, what the above formula says is this: if you do an amount of work Δw and also transfer an amount of heat Δq, then the sum of these two quantities will be the additional amount of energy ΔE that is stored in the system. There could be nothing simpler! Note: Δq and Δw are positive if heat is added to or work is done on a system, and negative if heat is removed from the system, or if the system does work on the environment.

3. Work and heat are called *path variables* - their values depend on the steps leading from one state to another. To give an obvious example: suppose you drag a heavy box on the floor in a straight line from point A to point B, and then in a very roundabout way from A to B again. Obviously the work done by you will be very different. On the other hand, the internal energy does not depend upon the path. Example: a gas has internal energy proportional to its temperature. It makes no difference whether the gas had been slowly heated or rapidly; it will have the same internal energy. Since the internal energy does not depend on the path we can write $\int_{State1}^{State2} dE = E_2 - E_1 = \Delta E.$

4. Let us calculate the work done by an expanding gas. In the diagram below, suppose the piston moves out by distance dx, then the work done is $dw = Fdx$ where F is the force exerted on the gas. But $F = P \times A = -P_{ext} \times A$. (The minus sign is important to understand; remember that the force exerted by the gas on the piston is the negative of the force that the piston exerts on the gas). So $dw = Fdx = -P_{ext} \times A \times dx = -P_{ext}dV$. The work done by the expanding gas is $w = -\int_{V_1}^{V_2} P_{ext}dV$ where V_1 and V_2 are the initial and final volumes.

Closed system
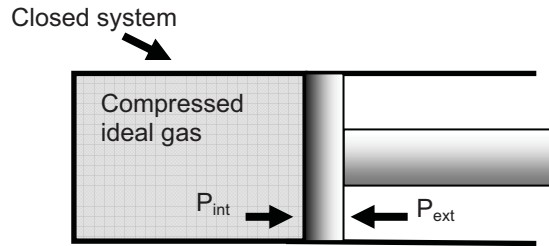
Note that w depends on the path. Let's consider 3 different paths.

a) Suppose that the gas expands into the vacuum, $P_{ext} = 0$. Then $w = 0$. So, if in the figure below, the valve is opened, then gas will flow into the vacuum without doing work.


b) Suppose the pressure outside has some constant value. Then, $w = P_{ext} \times (V_2 - V_1)$ or, $w = P_{ext}\Delta V$. You can see that this is just the area of the rectangle below.
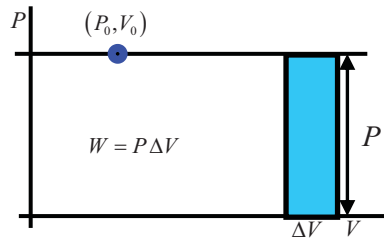
c) We can also let the gas expand so that the internal and external pressures are almost the same (this is called reversible expansion). Then the work done *on* the system is: $dw = -P_{ext}dV$. Using $PV = Nk_BT$ gives,

$$w = -\int_{V_1}^{V_2} \frac{Nk_BT}{V}dV = -Nk_BT(\ln V_2 - \ln V_1) = -Nk_BT\ln\frac{V_2}{V_1}$$

Of course, the work done *by* the gas as it expands is positive since V_2 is larger than V_1. Remember that the log function is positive if its argument is bigger than 1.
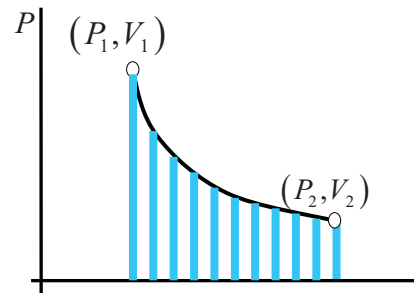
5. The internal energy of a gas depends only on the number of molecules it contains and on the temperature, $E = \frac{3}{2} Nk_B T$. In a free expansion of gas (it doesn't do any work in this case), the initial and final internal energies are equal, $E_f = E_i$.

6. Imagine that a substance is heated while keeping its volume constant. Obviously, you have to supply more heat as you raise the temperature higher, $\Delta Q_V = C_V \Delta T$, where C_V is given the name "specific heat at constant volume". No work is done since there is no expansion, $\Delta W = 0$. Hence, using the First Law, $\Delta Q_V = \Delta E + \Delta W = \Delta E$, and so $\Delta E = C_V \Delta T$.

7. Now take the same system as above, but allow it to expand at constant pressure as it is heated. Then you will have to supply an amount of heat, $\Delta Q_P = C_P \Delta T$. Here, C_P is called the specific heat at constant pressure, $\Delta Q_P = \Delta E + \Delta w = \Delta E + P \Delta V$. Hence, $C_P \Delta T = \Delta U + P \Delta V$. If we supply only a small amount of heat then, $C_P dT = dU + P dV$. This gives, $C_P dT = C_V dT + P dV$. If we consider the special case of an ideal gas, then $PV = Nk_B T$. Hence, $P dV = Nk_B dT$ and $C_P dT = C_V dT + Nk_B dT$. Hence, $C_P = C_V + Nk_B$. Since the internal energy is $E = \frac{3}{2} Nk_B T$, it follows that $C_V = \frac{dU}{dT} = \frac{3}{2} Nk_B$. From this, the specific heat at constant pressure is: $C_P = \frac{3}{2} Nk_B + Nk_B = \frac{5}{2} Nk_B$. From the equation $C_P = C_V + Nk_B$, it follows that $\frac{Nk_B}{C_V} = \frac{C_P - C_V}{C_V}$. Let us define a new symbol, $\frac{C_P}{C_V} \equiv \gamma$.

8. We can find the EOS of an ideal gas when it expands without losing any heat (this is called adiabiatic expansion). In this case, $dQ = dE + dW = 0$. Hence, $C_V dT + P dV = 0$, or $C_V dT + Nk_B T \frac{dV}{V} = 0$. Dividing by dT gives, $\frac{dT}{T} + \frac{Nk_B}{C_V} \frac{dV}{V} = 0$ or $\frac{dT}{T} + (\gamma - 1) \frac{dV}{V} = 0$. Integrating this gives, $\ln T + (\gamma - 1) \ln V = $ Constant. Equivalently, $\ln\left(TV^{\gamma-1}\right) = $ Constant. A more convenient form is: $TV^{\gamma-1} = $ Constant. This holds for the entire adiabatic expansion.

9. Almost nothing beats the importance of the Second Law Of Thermodynamics. There are many equivalent ways of stating it. The one I like is: "There can be no process whose only final result is to transfer thermal energy from a cooler object to a hotter object". This seems extremely simple (and almost useless), but in fact it tells you, among other things, that no perpetual motion machine (such as that which generates electricity without any fuel input) can ever be built. If someone could build such a machine, then it could also be used to transfer heat from a cold object to a hot object - a contradiction with the Second Law.

10. Before we consider some implications of the Second Law, I want to introduce the concept of a *thermal reservoir*. Suppose you put a thermometer in your mouth. You will transfer a small amount of heat to the thermometer, but because your body is so big this will make no measureable difference to your body temperature. Your body therefore is a thermal reservoir as far as the thermometer is concerned. More generally, any large mass of material will act as a thermal reservoir if it is at constant temperature. A *heat engine* (e.g. a car engine, refrigerator, etc) operates between reservoirs at two different temperatures.

11. A heat engine works between a high temperature T_H and a low temperature T_C. It aborbs heat $Q_{in,h}$ from the hot reservoir and rejects heat $Q_{out,c}$ into the cold reservoir. Since the internal energy of the engine does not change, $\Delta U = 0$ and the First Law gives simply, $Q = \Delta U + W = W$. The work done by the engine is $W = Q_{in,h} - Q_{out,c}$. Now define the efficiency of the machine as $\varepsilon = \dfrac{\text{work done by engine}}{\text{heat put in}} = \dfrac{W}{Q_{in,h}}$. Then


$\varepsilon = \dfrac{Q_{in,h} - Q_{out,c}}{Q_{in,h}} = 1 - \dfrac{Q_{out,c}}{Q_{in,h}}$. Now, the heat that is transferred out of or into a reservoir is proportional to its absolute temperature. Hence we arrive at the important result that $\varepsilon = 1 - \dfrac{T_C}{T_H}$. This number is always less than one, showing that no machine can convert all the input heat into useful work. As an example, a nuclear reactor has temperature $300^0 C$ at the core and rejects heat into a river at $30^0 C$. The maximum efficiency it can have is $\varepsilon = 1 - \dfrac{30 + 273}{300 + 273} = 0.471$
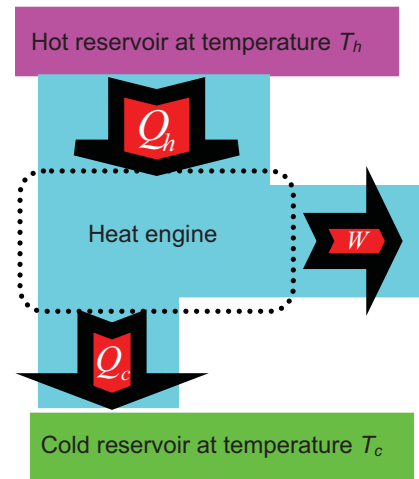
12. A refrigerator is a heat engine working in reverse. Work is done (by an electric motor) to pump heat from a cold reservoir (inside of refrigerator) to the hot exterior. The Second Law can be shown to imply the following: it is impossible for a refrigerator to produce no other effect than the transfer of thermal energy from a cold object to a hot object. Again, the efficiency of a refrigerator is always less than one.
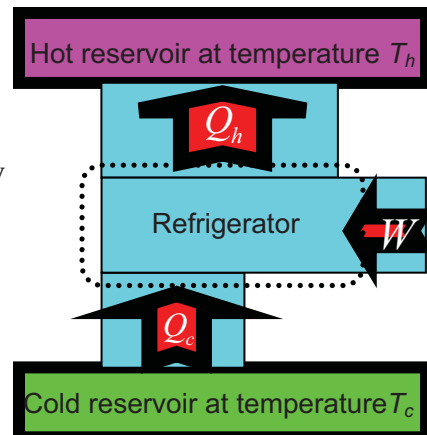
QUESTIONS AND EXERCISES – 37

Q.1 a) Give an example where no heat is transferred to or from a system but the temperature of the system changes.

b) Why is it wrong to think of heat as a form of stored energy?

Q.2 An ideal gas expands from a given initial state to a given final state along the two paths shown below, A and B.

a) Calculate the work done along path A and path B.

b) Is the internal energy of the gas changed during the expansions, and if so by how much?
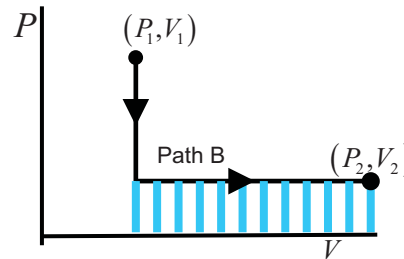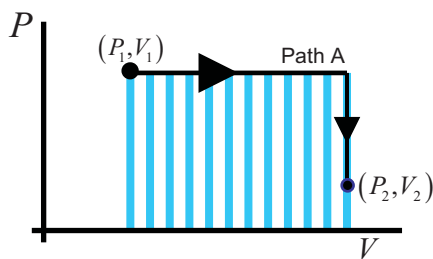
Q.3 Calculate the pdV work done in taking an ideal gas around the cycle shown here along each of the segments. Then, calculate the total work done. Will the internal energy of the gas change as you go around? Will the temperature remain constant on each of the segments?
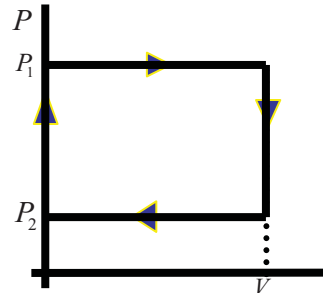
Q.4 A steam engine is shown schematically in the figure. Discuss the function of each of its parts. What does efficiency of this engine mean? For increasing efficiency, would it be better to use a liquid with a higher boiling point than water? If so, what would be the disavantage?
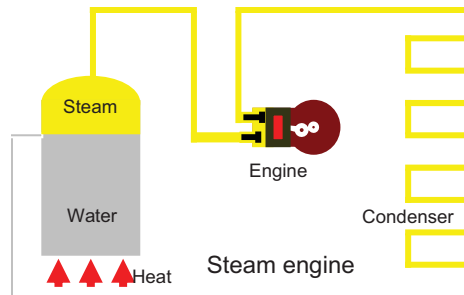
Q.5. An airconditioner takes air from a room at $20^0 C$ and transfers it to outdoors, where the temperature is $40^0 C$. For each joule of electricity that is delivered to the refrigerator, how many joules of energy are transferred to the outside?