## Summary of Lecture 26 – ELECTRIC POTENTIAL ENERGY

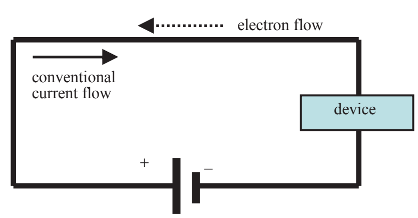
1. Electric current is the flow of electrical charge. If a small amount of charge $dq$ flows in time $dq$, then the current is $i = \frac{dq}{dt}$. If the current is constant in time, then in time $t$, the current that flows is $q = i \times t$. The unit of charge is ampere, which is define as:

$$1 \text{ ampere} = \frac{1 \text{ coulomb}}{\text{second}}$$

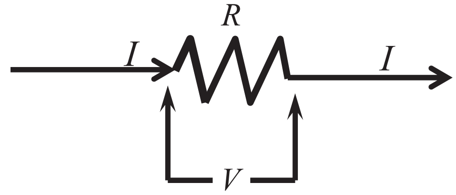
A car's battery supplies upto 50 amperes when starting the car, but often we need to deal with smaller values:

$$1 \text{ milliampere} = 1 \text{ ma} = 10^{-3} A$$
$$1 \text{ microampere} = 1 \ \mu A = 10^{-6} A$$
$$1 \text{ nanoampere} = 1 \text{ n}A = 10^{-9} A$$
$$1 \text{ picoampere} = 1 \text{ p}A = 10^{-12} A$$

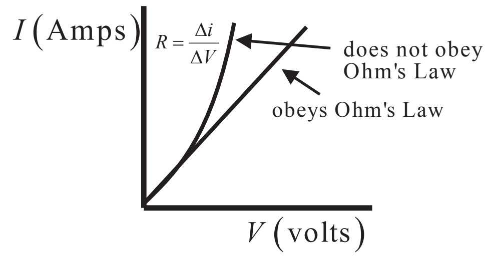
2. The direction of current flow is the direction in which positive charges move. However, in a typical wire, the positive charges are fixed to the atoms and it is really the negative charges (electrons) that move. In that case the direction of current flow is reversed.



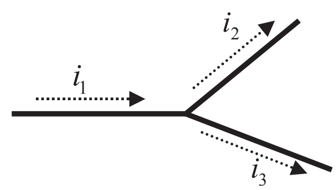

3. Current flows because something forces it around a circuit. That "something" is EMF, electromotive force. But remember that we are using bad terminology and that EMF is not a force - it is actually the difference in electric potentials between two parts of a circuit. So, in the figure below, $V = V_a - V_b$ is the EMF which causes current to flow in the resistor. How much current? Generally, the larger $V$ is , the more current will flow and we expect $I \propto V$. In general this relation will not be completely accurate but when it holds, we say Ohm's Law applies: $I = \frac{V}{R}$. Here, $R = \frac{V}{I}$ is called the resistance.



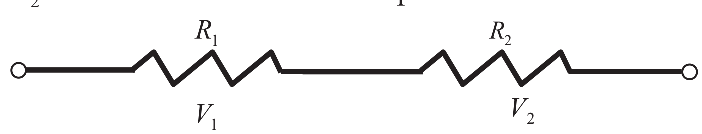
4. Be careful in understanding Ohm's Law. In general the current may depend upon the applied voltage in a complicated way. Another way of saying this is that the resistance may depend upon the current. Example: when current passes through a resistor, it gets hot and its resistance increases. Only when the graph of current versus voltage is a straight line does Ohm's Law hold. Else, we can only define the "incremental resistance".




5. Charge is always conserved, and therefore current is conserved as well. This means that when a current splits into two currents the sum remains constant, $i_1 = i_2 + i_3$.



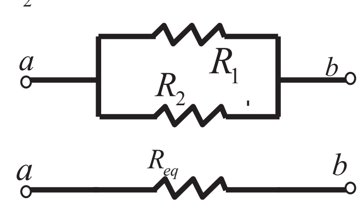

6. When resistors are put in series with each other, the same current flows through both. So, $V_1 = iR_1$ and $V_2 = iR_2$. The total potential drop across the pair is $V = V_1 + V_2 = i(R_1 + R_2)$. $\Rightarrow R_{eq} = R_1 + R_2$. So resistors in series add up.




7. Resistors can also be put in parallel. This means that the same voltage $V$ is across both. So the currents are $i_1 = \dfrac{V}{R_1},\ i_2 = \dfrac{V}{R_2}$.

Since $i = i_1 + i_2$ it follows that $i = \dfrac{V}{R_{eq}} = \dfrac{V}{R_1} + \dfrac{V}{R_2}$

$$\Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \text{ or } R_{eq} = \frac{R_1 R_2}{R_1 + R_2}.$$



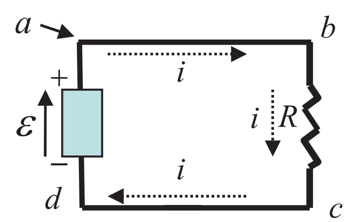

This makes sense: with two possible paths the current will find less resistance than if only one was present.

8. When current flows in a circuit work is done. Suppose a small amount of charge $dq$ is moved through a potential difference V. Then the work done is $dW = Vdq = V\,idt$. Hence

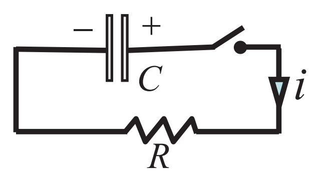
$Vidt = i^2 Rdt$ ( because $i = \frac{V}{R}$ ). The rate of doing work, i.e. power, is $P = \frac{dW}{dt} = i^2 R$.

This is an important formula. It can also be written as $P = \frac{V^2}{R}$, or as $P = iV$. The unit of power is: 1 volt-ampere $= 1\frac{\text{joule}}{\text{coulomb}} \cdot \frac{\text{coulomb}}{\text{second}} = 1\frac{\text{joule}}{\text{second}} = 1$ watt.

9. Kirchoff's Law: The sum of the potential differences encountered in moving around a closed circuit is zero. This law is easy to prove: since the electric field is conservative, therefore no work is done in taking a charge all around a circuit and putting it back where it was. However, it is very useful in solving problems. As a trivial example, consider the circuit below. The statement that, starting from any point a we get back to the same potential after going around is: $V_a - iR + \mathcal{E} = V_a$. This says $-iR + \mathcal{E} = 0$.




10. We can apply Kirchoff's Law to a circuit that consists of a resistor and capacitor in order to see how current flows through it. Since $q = VC$, we can see that $\frac{q}{C} + iR = 0$. Now differentiate with respect to time to find $\frac{1}{C}\frac{dq}{dt} + \frac{di}{dt}R = 0$, or $\frac{di}{dt} = -\frac{1}{RC}i$. This equation has solution: $i = i_0 e^{-\frac{t}{RC}}$. The product $RC$ is called the time constant $\tau$, and it gives the time by which the current has fallen to $1/e \approx 1/2.7$ of the initial value.

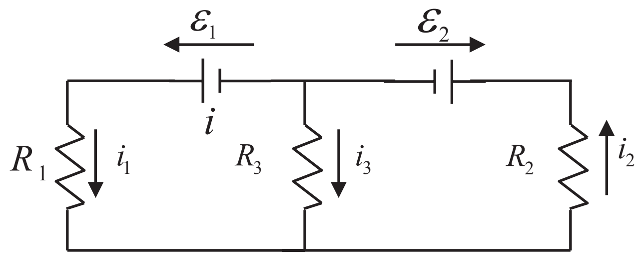



A reminder about the exponential function, $e^x \equiv 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$

From this, $\frac{\mathrm{d}}{\mathrm{dx}} e^x = 1 + \frac{2x}{2!} + \frac{3x^2}{3!} + \frac{4x^3}{4!} + \cdots = e^x$ Similarly, $\frac{\mathrm{d}}{\mathrm{dx}} e^{-x} = -e^{-x}$

11. Circuits often have two or more loops. To find the voltages and currents in such situations, it is best to apply Kirchoff's Law. In the figure below, you see that there are 3 loops and you can see that:

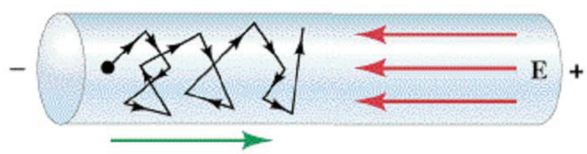
$$i_1 + i_3 = i_2$$
$$\mathcal{E}_1 - i_1 R_1 + i_3 R_3 = 0$$
$$-i_3 R_3 - i_2 R_2 + \mathcal{E}_2 = 0$$




Make sure that you understand each of these, and then check that the solution is:

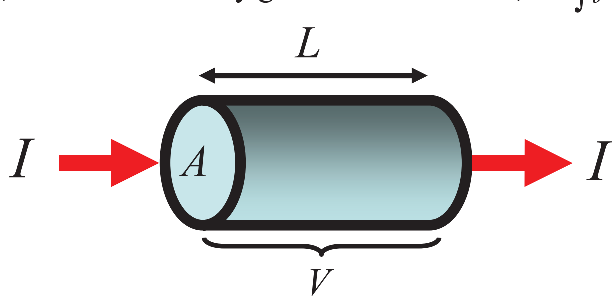
$$i_1 = \frac{\mathcal{E}_1 (R_2 + R_3) - \mathcal{E}_2 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3} \qquad i_2 = \frac{\mathcal{E}_1 R_3 - \mathcal{E}_2 (R_1 + R_3)}{R_1 R_2 + R_2 R_3 + R_1 R_3} \qquad i_3 = \frac{-\mathcal{E}_1 R_2 - \mathcal{E}_2 R_1}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

12. A charge inside a wire moves under the influence of the applied electric field and suffers many collisions that cause it to move on a highly irregular, jagged path as shown below.




Nevertheless, it moves on the average to the right at the "drift velocity" (or speed).

13. Consider a wire through which charge is flowing. Suppose that the number of charges per unit volume is $n$. If we multiply $n$ by the crossectional area of the wire $A$ and the length $L$, then the charge in this section of the wire is $q = (nAL)e$. If the drift velocity of the charges is $\mathrm{v_d}$, then the time taken for the charge to move through the wire is $t = \dfrac{L}{\mathrm{v_d}}$. hence the current is $i = \dfrac{q}{t} = \dfrac{nALe}{L/\mathrm{v_d}} = nAe\mathrm{v_d}$. From this we can calculate the drift velocity of the charges in terms of the measured current, $\mathrm{v_d} = \dfrac{i}{nAe}$. The current density, which is the current per unit crossectional area is defined as $j = \dfrac{i}{A} = ne\mathrm{v_d}$. If $j$ varies inside a volume, then we can easily generalize and write, $i = \int \vec{j} \cdot d\vec{A}$.



## QUESTIONS AND EXERCISES – 26

Q.1 a) Sometimes people replace thin fuse wire with thick fuse wire. Why is this a wrong and dangerous practice?

b) Give two examples where currents flow outside of wires.

Q.2 In the lecture, you learned about the internal resistance of a battery. Suppose that the terminals of a battery of 1.5 V and internal resistance 3Ω are connected by an ammeter. The ammeter shows that 300 ma flows. What is the internal resistance of the ammeter?

Q.3 A certain electronic device has a current voltage behaviour which is very non-linear,

$$I = 3(e^{V/V_0} - 1),\ \text{where } V_0 = 2.$$

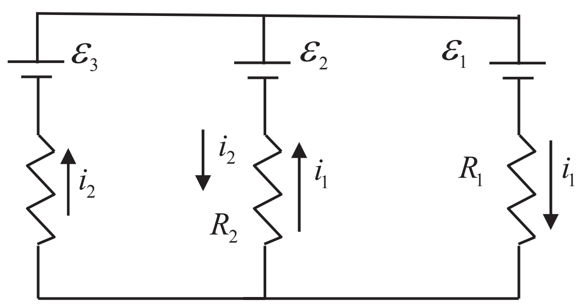
a) Plot the current versus voltage upto a maximum voltage of 3.

b) What is the incremental resistance at V=2 ?

c) Expand the exponential for very small values of V, and hence find the incremental resistance as $V \to 0$.

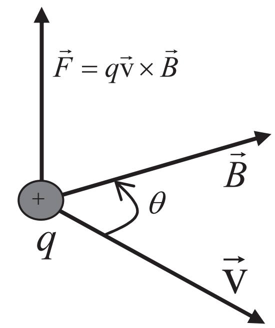
d) Explain in your own words why Ohm's law does not apply for this device.

Q.4 Apply Kirchoff's Law to the circuit below and find the currents. Take the following parameters: $R_1 = 5\Omega,\ R_2 = 5\Omega,\ R_3 = 15\Omega,\ \mathcal{E}_1 = 4V,\ \mathcal{E}_2 = 6V,\ \mathcal{E}_3 = 25V.$




Q.5 A metal wire of diameter 0.5mm has resistivity 6 ohms per metre. A different kind of metal wire with diameter 0.25mm has resistivity 12 ohms per metre. Calculate the ratio of the drift velocities in the two metals.

**Summary of Lecture 27 – THE MAGNETIC FIELD**




1. The magnetic field exerts a force upon any charge that moves in the field. The greater the size of the charge, and the faster it moves, the larger the force. The direction of the force is perpendicular to both the direction of motion and the magnetic field. If $\theta$ is the angle between $\vec{v}$ and $\vec{B}$, then $F = qvB\sin\theta$ is the magnitude of the force. This vanishes when $\vec{v}$ and $\vec{B}$ are parallel ($\theta = 0$), and is maximum when they are perpendicular.

2. The unit of magnetic field that is used most commonly is the *tesla*. A charge of one coulomb moving at 1 metre per second perpendicularly to a field of one tesla experiences a force of 1 newton. Equivalently,

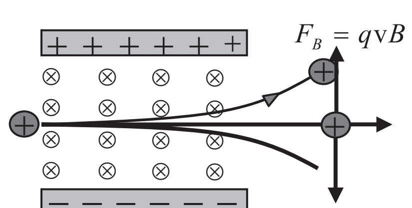
$$1 \text{ tesla} = 1 \frac{\text{newton}}{\text{coulomb}\cdot\text{meter/second}} = 1 \frac{\text{newton}}{\text{ampere}\cdot\text{meter}} = 10^4 \text{ gauss (CGS unit)}$$

In order to have an appreciation for how much a tesla is, here are some typical values of the magnetic field in these units:

| | |
|---|---|
| Earth's surface | $10^{-4}$ T |
| Bar magnet | $10^{-2}$ T |
| Powerful electromagnet | 1 T |
| Superconducting magnet | 5 T |

3. When both magnetic and electric fields are present at a point, the total force acting upon a charge is the vector sum of the electric and magnetic forces, $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$. This is known as the *Lorentz Force.* Note that the electric force and magnetic force are very different. The electric force is non-zero even if the charge is stationary, and it is in the same direction as $\vec{E}$.

4. The Lorentz Force can be used to select charged particles of whichever velocity we want. In the diagram below, particles enter from the left with velocity v. They experience a force due to the perpendicular magnetic field, as well as force downwards because of an electric field. Only particles with speed $v = E/B$ are undeflected and keep going straight.



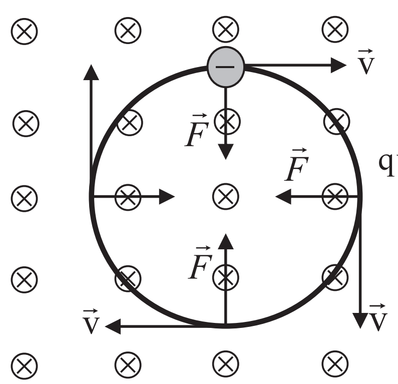

$qE = qvB \Rightarrow v = \dfrac{E}{B}$ velocity selector !!

5. A magnetic field can be strong enough to lift an elephant, but it can never increase or decrease the energy of a particle. Proof: suppose the magnetic force $\vec{F}$ moves a particle through a displacement $d\vec{r}$. Then the small amount of work done is,

$$\begin{aligned} dW = \vec{F}\cdot d\vec{r} &= q(\vec{\mathrm{v}}\times\vec{\mathrm{B}})\cdot d\vec{r} \\ &= q(\vec{\mathrm{v}}\times\vec{\mathrm{B}})\cdot\frac{d\vec{r}}{dt}dt \\ &= q(\vec{\mathrm{v}}\times\vec{\mathrm{B}})\cdot\vec{\mathrm{v}}\,dt = 0. \end{aligned}$$

Basically the force and direction of force are orthogonal, and hence there can be no work done on the particle or an increase in its energy.

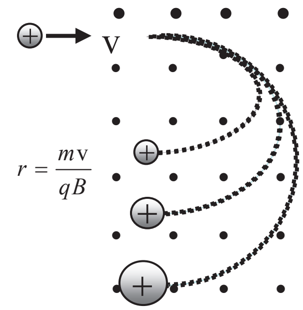
6. A magnetic field bends a charged particle into a circular orbit because the particle feels a force that is directed perpendicular to the magnetic field. As we saw above, the particle cannot change its speed, but it certainly does change direction! So it keeps bending and bending until it makes a full circle. The radius of orbit can be easily calculated: the magnetic and centrifugal forces must balance each other for equilibrium. So, $\mathrm{qv}B = m\frac{\mathrm{v}^2}{r}$ and we find that $r = \frac{m\mathrm{v}}{qB}$. A strong B forces the particle into a tighter orbit, as you can see. We can also calculate the angular frequency, $\omega = \frac{\mathrm{v}}{r} = \frac{qB}{m}$. This shows that a strong B makes the particle go around many times in unit time. There are a very large number of applications of these facts.



$$\mathrm{qv}B = m\frac{\mathrm{v}^2}{r} \Rightarrow r = \frac{m\mathrm{v}}{qB}.$$

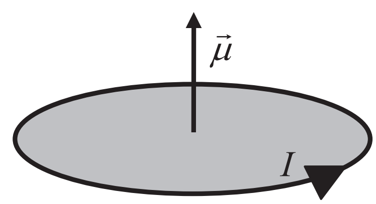
7. The fact that a magnetic field bends charged particles is responsible for shielding the earth from harmful effects of the "solar wind". A large number of charged particles are released from the sun and reach the earth. These can destroy life. Fortunately the earth's magnetic field deflects these particles, which are then trapped in the "Van Allen" belt around the earth.

8. The mass spectrometer is an extremely important equipment that works on the above principle. Ions are made from atoms by stripping away one electron. Then they pass through a velocity selector so that they all have the same speed. In a beam of many different ions, the heavier ones bend less, and lighter ones more, when they are passed through a $B$ field.

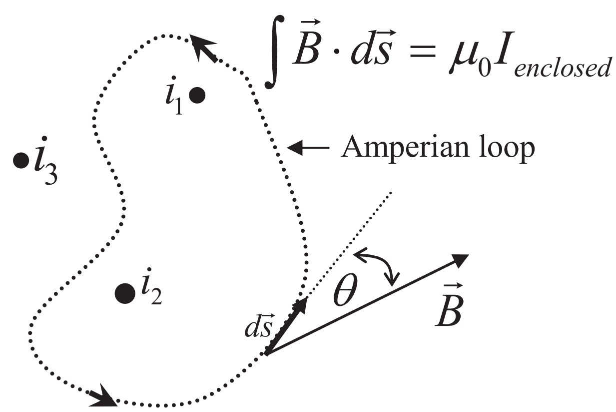



9. A wire carries current, and current is flowing charges. Since each charge experiences a force when placed in a magnetic field, you might expect the same for the current. Indeed, that is exactly the case, and we can easily calculate the force on a wire from the force on individual charges. Suppose N is the total number of charges and they are moving at the average (or drift) velocity $\vec{\text{v}}_{\text{d}}$. Then the total force is $\vec{F} = Ne\vec{v}_d \times \vec{B}$. Now suppose that the wire has length $L$, crossectional area $A$, and it has $n$ charges per unit volume. Then clearly $N = nAL$, and so $\vec{F} = nALe\vec{\text{v}}_{\text{d}} \times \vec{B}$. Remember that the current is the charge that flows through the wire per unit time, and so $nAe\vec{\text{v}}_{\text{d}} = \vec{I}$. We get the important result that the force per unit length on the wire is $\vec{F} = \vec{I} \times \vec{B}$.

10. A current that goes around a loop (any shape) produces a magnetic field. We define the *magnetic moment* as the product of current and area, $\vec{\mu} = IA\hat{z}$. Here A is the area of the loop and I the current flowing around it. The direction is perpendicular to the plane of the loop, as shown.

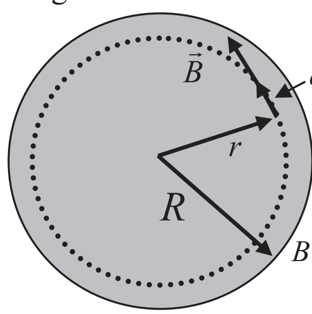



11. Magnetic fields are produced by currents. Every small bit of current produces a small amount of the $B$ field. Ampere's Law, illustrated below, says that if one goes around a loop (of any shape or size) then the integral of the B field around the loop is equal to the enclosed current. In the loop below $I = I_1 + I_2$. Here $I_3$ is excluded as it lies outside.



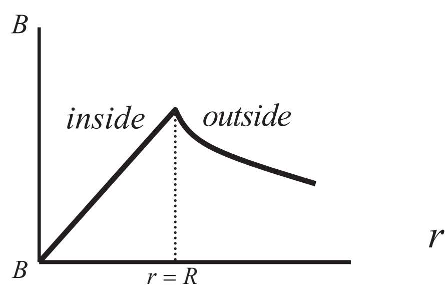
12. Let us apply Ampere's Law to a circular loop of radius $r$ outside an infinitely long wire carrying current $I$ through it. The magnetic field goes around in circles, and so $\vec{B}$ and $d\vec{s}$ are both in the same direction. Hence, $\int \vec{B} \cdot d\vec{s} = B\int ds = B(2\pi r) = \mu_0 I$. We get the important result that $B = \dfrac{\mu_0 I}{2\pi r}$.

13. Assuming that the current flows uniformly over the crossection, we can use Ampere's Law to calculate the magnetic field at distance $r$, where $r$ now lies inside the wire.




$$B(2\pi r) = \mu_0 I\left(\frac{\pi r^2}{\pi R^2}\right) \Rightarrow B = \frac{\mu_0 I r}{2\pi R^2}$$

Here is a sketch of the B field inside and outside the wire as a function of distance $r$.



## QUESTIONS AND EXERCISES – 27

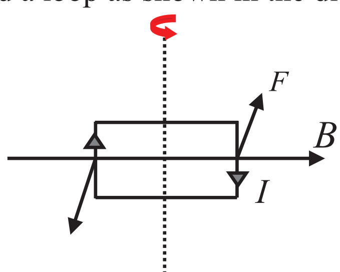
Q.1 In the lecture and notes, I have emphasized that a magnetic field can only be measured through the magnetic force acting upon a moving charged particle. Yet the commonest way to measure the field is by means of a compass. How are the statements consistent with each other?

Q.2 A pair of capacitor plates is charged upto some potential and an electric field exists in the gap between them. Suppose an electron is released from the negative plate. Discuss qualitatively its path if a magnetic field is applied perpendicular to the electric field if:

a) the B field is not too strong,

b) the B field is very strong.

Q.3 A current goes around a loop as shown in the diagram below.

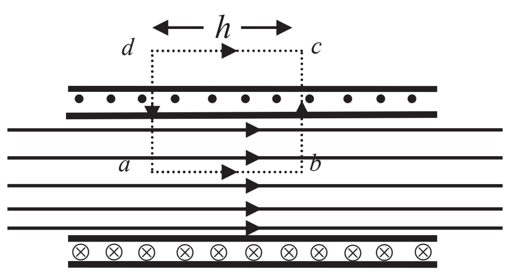



a) Find the torque about the vertical axis.

b) Express the torque in terms of the magnetic moment of the current loop.

c) If $B = 1.5\ T$, $I = 10\ A$, and the loop is $3 \times 5$ cm, find $\tau$.

Q.4 In the figure below, an infinitely long solenoid has N turns of wire per unit length and the current in the wire is $I$. The goal is to find the magnetic field along the axis. The $\otimes$ and $\bullet$ denote current going in/out of the paper.




a) Why is the magnetic field zero outside the solenoid?

b) Apply Ampere's Law around the loop abcda and hence show that the magnetic field is $B = \mu_0 In$.