Summary of Lecture 30 – ELECTROMAGNETIC WAVES

1. Before the investigations of James Clerk Maxwell around 1865, the known laws of electromagnetism were:

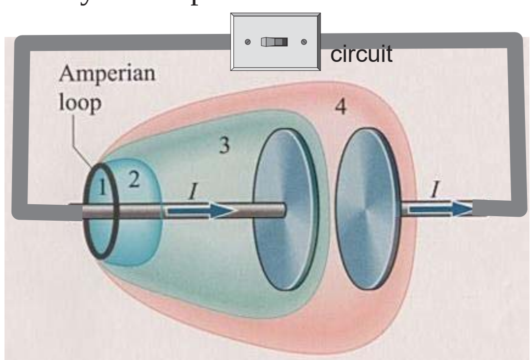
   a) Gauss' law of electricity: $\int \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0}$ (integral is over any closed surface)

   b) Gauss' law of magnetism: $\int \vec{B}\cdot d\vec{A} = 0$ (integral is over any closed surface)

   c) Faraday's law of induction: $\int \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$ (integral is over any closed loop)

   d) Ampere's law: $\int \vec{B}\cdot d\vec{s} = \mu_0 I$ (integral is over any closed loop)

2. But Maxwell realized that the above 4 laws were not consistent with the conservation of charge, which is a fundamental principle. He argued that if you take the space between two capacitors (see below) and take different surfaces 1,2,3,4 then applying Ampere's Law gives an inconsistency: $\left[\int \vec{B}\cdot d\vec{s}\right]_{(1,2,4)} \neq \left[\int \vec{B}\cdot d\vec{s}\right]_3$ because obviously charge cannot flow in the gap between plates. So Ampere's Law gives different results depending upon which surface is bounded by the loop shown!



Maxwell modified Ampere's law as follows: $\int \vec{B}\cdot d\vec{s} = \mu_0\left(I + I_d\right)$ where the "displacement current" is $I_d = \varepsilon_0 \frac{d\Phi_E}{dt}$. Let's look at the reasoning that led to Maxwell's discovery of the displacement current. The current that flows in the circuit is $I = \frac{dQ}{dt}$. But the charge on the capacitor plate is $Q = \varepsilon_0 EA$. Hence, $I = \frac{d}{dt}\left(\varepsilon_0 EA\right) = \varepsilon_0 \frac{d(EA)}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt} \equiv I_D$. In words, the changing electric field in the gap acts as source of the magnetic field in just the same way as the current in the outside wires. This is really the most important point - a magnetic field may have two separate reasons for existence - flowing charges or changing electric fields.

3. The famous Maxwell's equations are as follows:

a) $\int \vec{E} \cdot d\vec{S} = \dfrac{Q}{\varepsilon_0}$

b) $\int \vec{E} \cdot d\vec{\ell} = -\dfrac{d\Phi_B}{dt}$

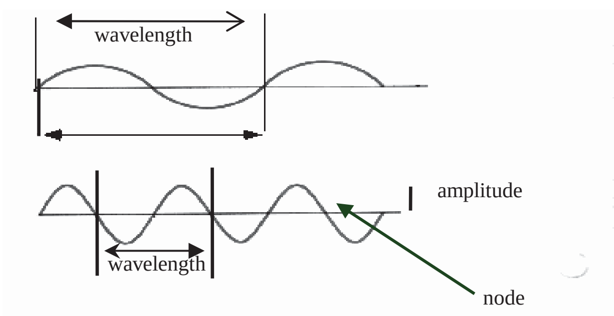
c) $\int \vec{B} \cdot d\vec{S} = 0$

d) $\int \vec{B} \cdot d\vec{\ell} = \mu_0\left(I + \varepsilon_0 \dfrac{d\Phi_E}{dt}\right)$

Together with the Lorentz Force $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$ they provide a complete description of all electromagnetic phenomena, including waves.

4. Electromagnetic waves were predicted by Maxwell and experimentally discovered many years later by Hertz. Note that for these waves:
a) Absolutely no medium is required - they travel through vacuum.
b) The speed of propagation is c for all waves in the vacuum.
c) There is no limit to the amplitude or frequency.

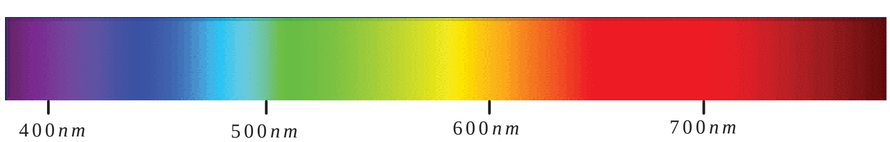
A wave is characterized by the amplitude and frequency, as illustrated below.



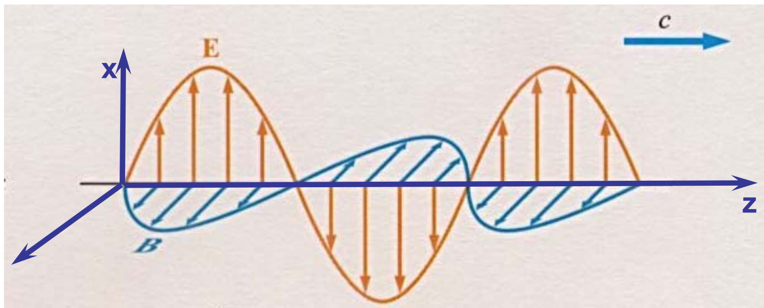
Example: Red light has λ = 700 nm. The frequency ν is calculated as follows:

$$\nu = \frac{3.0 \times 10^8 \, m/\sec}{7 \times 10^{-7} \, m} = 4.29 \times 10^{14} \, Hertz$$

By comparison, the electromagnetic waves inside a microwave oven have wavelength of 6 cm, radio waves are a few metres long. For visible light, see below. On the other hand, X-rays and gamma-rays have wavelengths of the size of atoms and even much smaller.

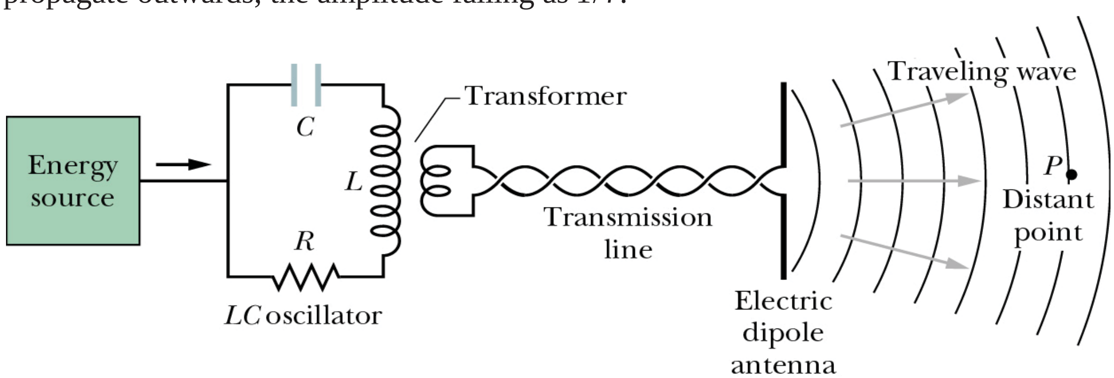


5. We now consider how electromagnetic waves can travel through empty space. Suppose an electric field (due to distant charges in an antenna) has been created. If this changes then this creates a changing electric flux $\frac{d\Phi_E}{dt}$ which, through $\int \vec{B}\cdot d\vec{s} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}$, creates a changing magnetic flux $\frac{d\Phi_B}{dt}$. This, through $\int \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$, creates a changing electric field. This chain of events in free space then allows a wave to propagate.



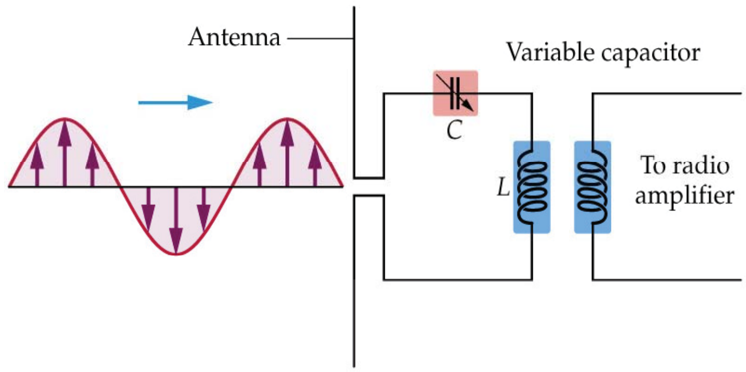
In the diagram above, an electromagnetic wave is moving in the z direction. The electric field is in the x direction, $E_x = E_0 \sin(kz - \omega t)$, and the magnetic field is perpendicular to it, $B_y = B_0 \sin(k\,z - \omega t)$. Here $\omega = kc$. From Maxwell's equations the amplitudes of the two fields are related by $E_0 = cB_0$. Note that the two fields are in phase with each other.

6. The production of electromagnetic waves is done by forcing current to vary rapidly in a small piece of wire. Consider your mobile phone, for example. Using the power from the battery, the circuits inside produce a high frequency current that goes into a "dipole antenna" made of two small pieces of conductor. The electric field between the two oppositely charged pieces is rapidly changing and so creates a magnetic field. Both fields propagate outwards, the amplitude falling as $1/r$.



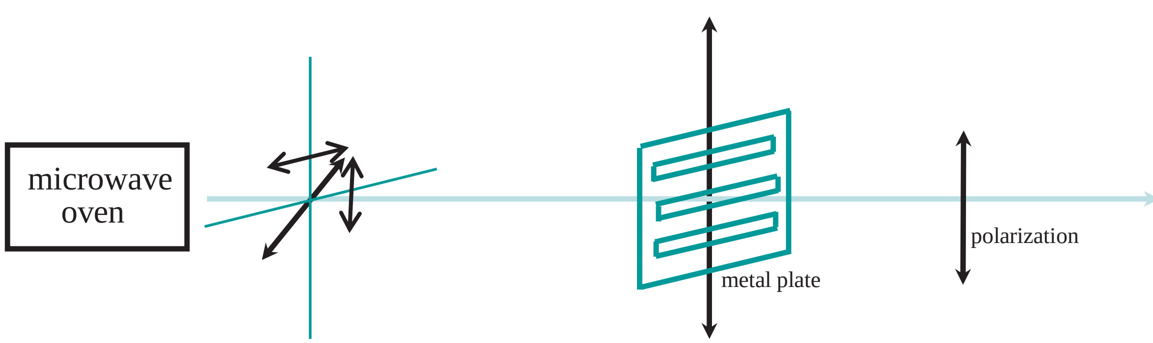
The power, which is the square of the amplitude, falls off as $I(\theta) \propto \frac{\sin^2\theta}{r^2}$. The $\sin^2\theta$ dependence shows that the power is radiated unequally as a function of direction. The maximum power is at $\theta = \pi/2$ and the least at $\theta = 0$.

7. The reception of electromagnetic waves requires an antenna. The incoming wave has an electric field that forces the electrons to run up and down the antenna wire, i.e. it produces a tiny electric current. This current is then amplified (increased in amplitude) electronically. This is schematically indicated below. Here the variable capacitor is used to tune to different frequencies.



8. As we have seen, the electric field of a wave is perpendicular to the direction of its motion. If this is a fixed direction (say, $\hat{x}$), then we say that wave is polarized in the x direction. Most sources - a candle, the sun, any light bulb - produce light that is unpolarized. In this case, there is no definite direction of the electric field, no definite phase between the orthogonal components, and the atomic or molecular dipoles that emit the light are randomly oriented in the source. But for a typical linearly polarized plane electromagnetic wave polarized along $\hat{x}$, $E_x = E_0 \sin(kz - \omega t)$, $B_y = \dfrac{E_0}{c}\sin(kz - \omega t)$ with all other components zero. Of course, it may be that the wave is polarized at an angle θ relative to $\hat{x}$, in which case $E_x = E_0 \cos\theta \cdot \sin(kz - \omega t)$, $E_y = E_0 \sin\theta \cdot \sin(k\,z - \omega t)$, $E_z = 0$.

9. Electromagnetic waves from an unpolarized source (e.g. a burning candle or microwave oven) can be polarized by passing them through a simple polarizer of the kind below. A metal plate with slits cut into will allow only the electric field component perpendicular to the slits. Thus, it will produce linearly polarized waves from unpolarized ones.



QUESTIONS AND EXERCISES – 30

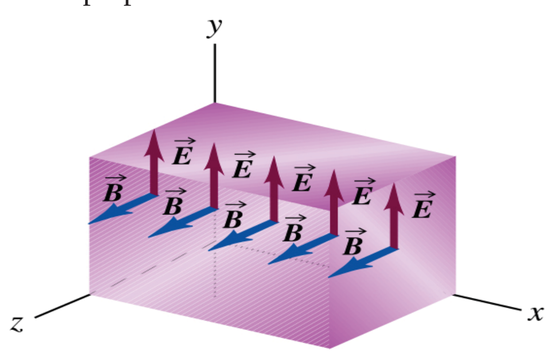
Q.1 A star that suddenly explodes is called a supernova. Looking at supernova events in different parts of the universe, do you think we can learn about what the laws of physics were at different times in the past?

Q.2 Near a wire carrying current $I(t)$, use Ampere's Law to show that the magnetic field is $B(t) = \frac{\mu_0 I(t)}{2\pi r}$. What is the direction of the field? Now suppose that the current varies rapidly with time. Do you expect your answer to be still correct? Why?

Q.3 A plane wave is moving to the right in the figure below.

a) Find the flux $\int \vec{B} \cdot d\vec{s}$ over the surface drawn here.

b) Using Maxwell's equations, can you show that $\vec{E}$, $\vec{B}$, and the direction of propagation must all be perpendicular to each other?



Q.4 Starting from Maxwell's equations, it is possible to show that the power radiated by a single charge q with acceleration a is $P = \frac{2}{3c^3} q^2 a^2$. Accepting this, answer the following:

a) Check if the expression for P is dimensionally correct.

b) Why would an answer that is proportional to q (rather than q^2) be physically wrong?

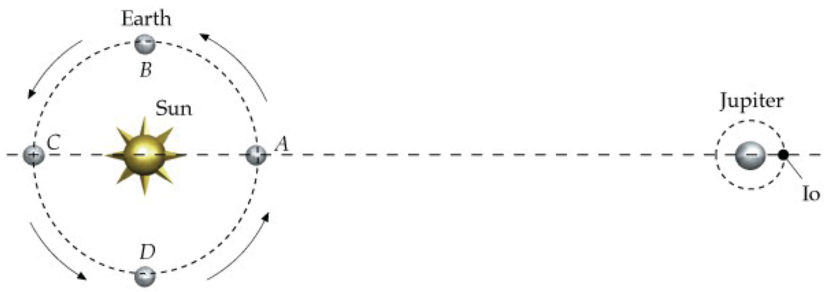
c) Why would an answer that is proportional to a (rather than a^2) be physically wrong?

d) Suppose that electrons and protons with equal velocities are injected into a constant magnetic field. Both bend around, and so both experience acceleration. Given that a proton is 2000 heavier than an electron, find the ratio of the energy radiated by a proton to that of an electron.

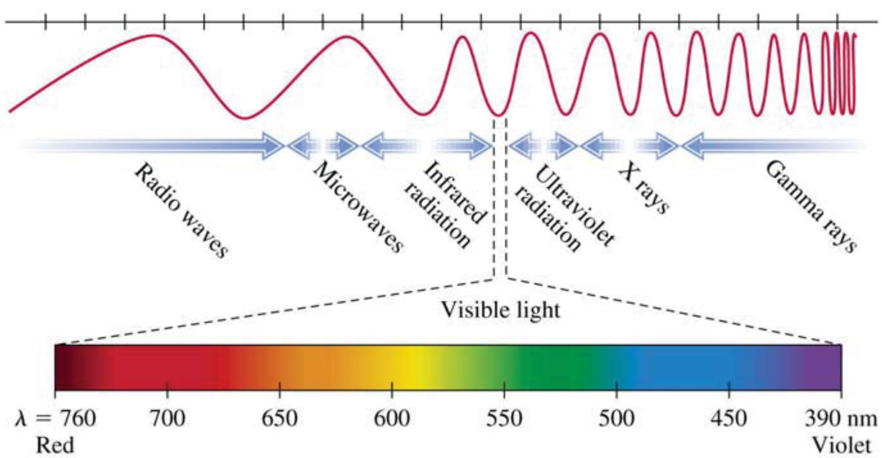
e) Repeat the above if they are injected with the same energy instead of same velocity.

Summary of Lecture 31 – LIGHT

1. Light travels very fast but its speed is not infinite. Early attempts to measure the speed using earth based experiments failed. Then in 1675 the astronomer Roemer studied timing of the eclipse of one of Jupiter's moon called Io. In the diagram below Io is observed with Earth at A and then at C. The eclipse is 16.6 minutes late, which is the time taken for light to travel AC. Roemer estimated that $c = 3\times10^8$ metres/sec, a value that is remarkably close to the best modern measurement, c = 299792458.6 metres/sec.

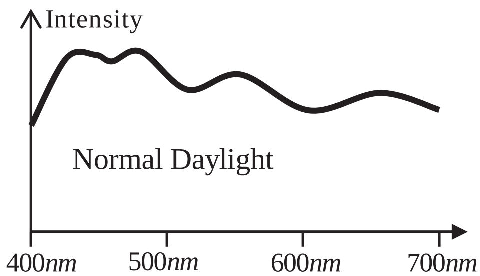


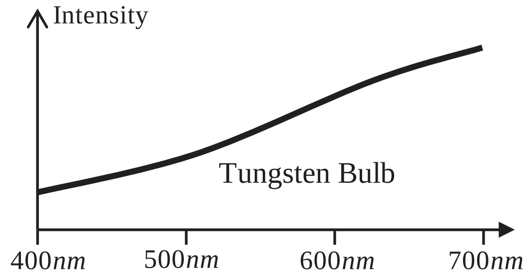
2. Light is electromagnetic waves. Different frequencies ν correspond to different colours. Equivalently, different wavelength λ correspond to different colours. Recall that the product $\nu\times\lambda = c$. In the diagram below you see that visible light is only one small part of the total electromagnetic spectrum. Here nm means nanometres or 10^{-9}metres.



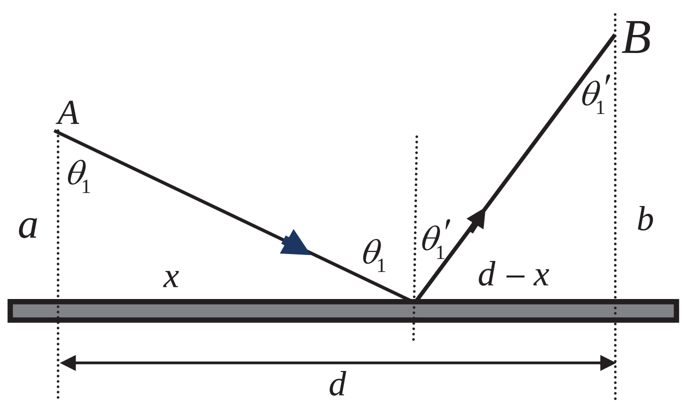
3. If light contained all frequencies with equal strength, it would appear as white to us. course, most things around us appear coloured. That is because they radiate more strongly in one range of frequency than in others. If there is more intensity in the yellow range than the green range, we will see mostly yellow. The sky appears blue to us on a clear day because tiny dust particles high above in the atmosphere reflect a lot of the blue light coming from the sun. In the figure below you can see the hump at smaller wavelengths.

Compare this with the spectrum of light emitted from a tungsten bulb. You can see that this is smoother and yellow dominates.





4. What path does light travel upon? If there is no obstruction, it obviously likes to travel on straight line which is the shortest path between any two points, say A and B. Fermat's Principle states that in all situations, light will always take that path for which it takes the least time. As an example, let us apply Fermat's Principle to the case of light reflected from a mirror, as below.



The total distance travelled by the ray is $L = \sqrt{a^2 + x^2} + \sqrt{b^2 + (d-x)^2}$. The time taken is $t = L/c$. To find the smallest time, we must differentiate and then set the derivative to zero,

$$0 = \frac{dt}{dx} = \frac{1}{c}\frac{dL}{dx}$$
$$= \frac{1}{2c}\left(a^2 + x^2\right)^{-1/2}(2x) + \frac{1}{2c}\left[b^2 + (d-x)^2\right]^{1/2}(2)(d-x)(-1)$$

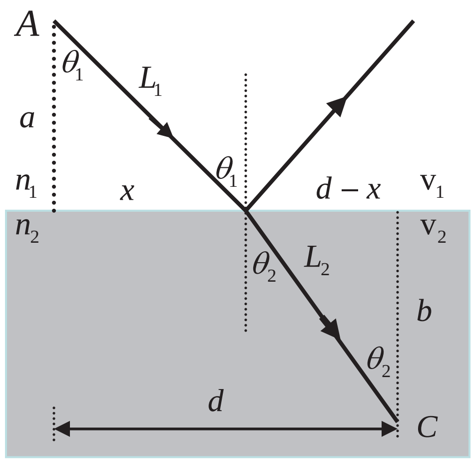
From here we immediately see that $\dfrac{x}{\sqrt{a^2 + x^2}} = \dfrac{d-x}{\sqrt{b^2 + (d-x)^2}}$ From the above diagram, $\sin\theta_1 = \sin\theta_1'$. Of course, it is no surprise that the angle of reflection equals the angle of incidence. You knew this from before, and this seems like a very complicated derivation of a simple fact. But it is still nice to see that there is a deeper principle behind it.

5. The speed of light in vacuum is a fixed constant of nature which we usually call c, but in a medium light can travel slower or faster than c. We define the "refractive index" of that medium as: $\text{Refractive index} = \dfrac{\text{Speed of light in vacuum}}{\text{Speed of light in material}}$ (or $n = \dfrac{c}{\mathrm{v}}$). Usually the values of n are bigger than one (e.g. for glass it is around 1.5) but in some special media, its value can be less than one. The value of n also depends on the wavelength (or frequency) of light. This is called dispersion, and it means that different colours travel at different speeds inside a medium. This is why, as in the diagram below, white light gets separated into different colours. The fact that in a glass prism blue light travels faster than red light is responsible for the many colours we see here.

6. We can apply Fermat's Principle to find the path followed by a ray of light when it goes from one medium to another. Part of the light is reflected, and part is "refracted", i.e. it bends away or towards the normal. The total time is,

$$t = \frac{L_1}{\mathrm{v}_1} + \frac{L_2}{\mathrm{v}_2},\ n = \frac{c}{\mathrm{v}} \Rightarrow t = \frac{n_1 L_1 + n_2 L_2}{c} = \frac{L}{c}$$

$$L = n_1 L_1 + n_2 L_2 \ = n_1\sqrt{a^2 + x^2} + n_2\sqrt{b^2 + (d-x)^2}$$

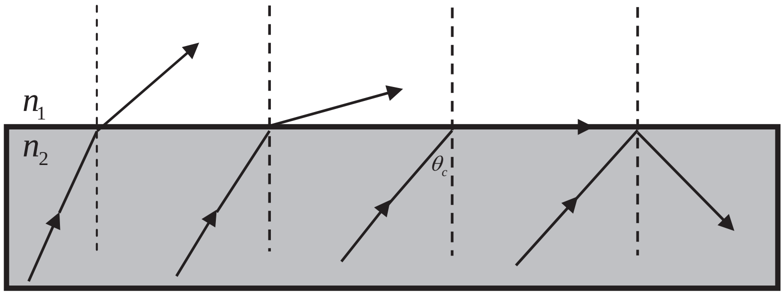


Fermat's Principle says that the time t must be minimized: $0 = \dfrac{dt}{dx} = \dfrac{1}{c}\dfrac{dL}{dx} = \dfrac{n_1}{2c}\left(a^2 + x^2\right)^{-1/2}(2x) + \dfrac{n_2}{2c}\left[b^2 + (d-x)^2\right]^{1/2}(2)(d-x)(-1)$

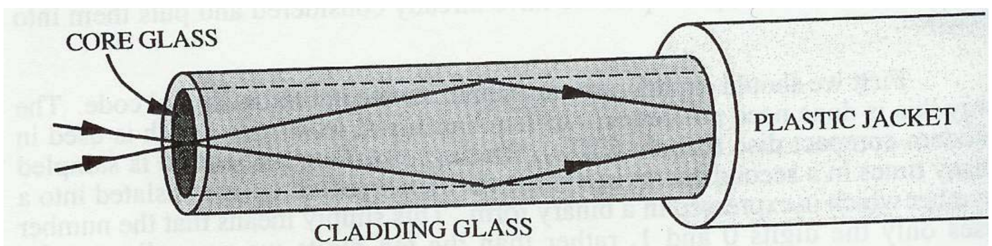
And so we get "Snell's Law", $n_1 \dfrac{x}{\sqrt{a^2 + x^2}} = n_2 \dfrac{d-x}{\sqrt{b^2 + (d-x)^2}}$ or $n_1 \sin\theta_1 = n_2 \sin\theta_2$. This required a little bit of mathematics, but you can see how powerful Fermat's Principle is!

7. Light coming from air into water bends toward the normal. Conversely, light from a source in the water will bend away from the normal. What if you keep increasing the angle with respect to the normal so that the light bends and begins to just follow the surface? This phenomenon is called total internal reflection and θ_c is called the critical

angle. It obeys: $n_1 \sin\theta_c = n_2 \sin 90^0$ from which $\theta_c = \sin^{-1}\frac{n_2}{n_1}$.



8. Fibre optic cables, which are now common everywhere, make use of the total internal reflection principle to carry light. Here is what a fibre optic cable looks like from inside:



Even if the cable is bent, the light will continue to travel along it. The glass inside the cable must have exceedingly good consistency - if it thicker or thinner in any part, the refractive index will become non-uniform and a lot of light will get lost. Optical fibres now carry thousands of telephone calls in a cable whose diameter is only a little bigger than a human hair!

QUESTIONS AND EXERCISES – 31

Q.1 If you look at the reflection of street light from across a body of water, it appears long in one direction but not the other. Explain.

Q.2 You are given a glass cube and a laser. How will you find the speed of light inside the glass?

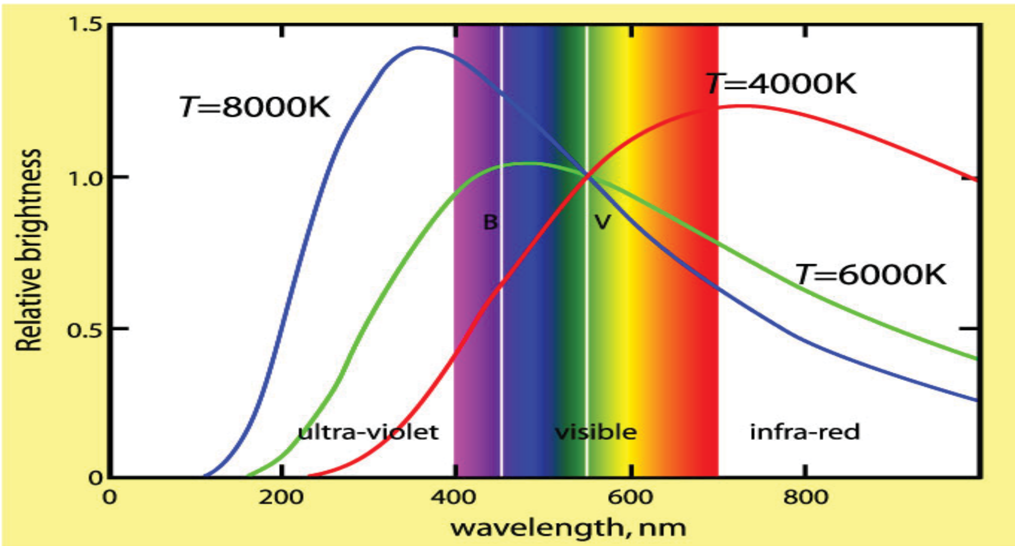
Q.3 With reference to the fibre optic cable above, which refractive index must be bigger - that of the core glass or the cladding glass? If the core glass has $n = 1.5$, what must be n for the cladding if $\theta_c = 10^o$.

Q.4 If light passes through a transparent medium, can we say that there is interaction between that medium and the light?

Summary of Lecture 32 – INTERACTION OF LIGHT WITH MATTER

1. In this lecture I shall deal with the 4 basic ways in which light interacts with matter:
   a) Emission - matter releases energy as light.
   b) Absorption - matter takes energy from light.
   c) Transmission - matter allows light to pass through it.
   d) Reflection - matter repels light in another direction.

2. When an object (for example, an iron rod or the filament of a tungsten bulb) is heated, it emits light. When the temperature is around 800°C, it is red hot. Around 2500°C it is yellowish-white. At temperatures lower than 800°C, infrared (IR) light is emitted but our eyes cannot see this. This kind of emission is called blackbody radiation. Blackbody radiation is continuous - all wavelengths are emitted. However most of the energy is radiated close to the peak. As you can see in the graph, the position of the peak goes to smaller wavelengths (or higher frequencies) as the object becomes hotter. The scale of temperature is shown in degrees Kelvin (^{o}K). To convert from ^{o}C to ^{o}K, simply add 273. We shall have more to say about the Kelvin scale later.



Where exactly does the peak occur? Wien's Law states that $\lambda_{\max} T = 2.90 \times 10^{-3}$ m K. We can derive this in an advanced physics course, but for now you must take this as given.

3. In the lecture on electromagnetic waves you had learnt that these waves are emitted when charges accelerate. Blackbody radiation occurs for exactly this reason as well. When a body is heated up, the electrons, atoms, and molecules which it contains undergo violent random motion. Light may emitted by electrons in one atom and absorbed in another. Even an empty box will be filled with blackbody radiation because the sides of the box are

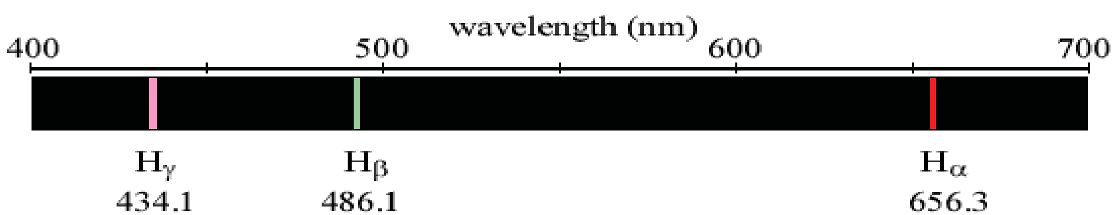
made up of material that has charged constituents that radiate energy when they undergo acceleration during their random motion.

4. When can you use Wien's Law? More generally, when can you expect a body to emit blackbody radiation? Answer: only for objects that emit light, not for those that merely reflect light (e.g. flowers). The Sun and other stars obey Wien's Law since the gases they are composed of emit radiation that is in equilibrium with the other materials. Wien's law allows astronomers to determine the temperature of a star because the wavelength at which a star is brightest is related to its temperature.

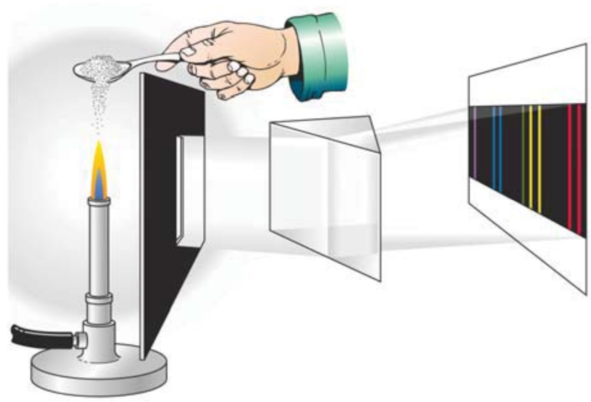
5. All heated matter radiates energy, and hotter objects radiate more energy. The famous Stefan-Boltzman Law, which we unfortunately cannot derive in this introductory course, states that the power radiated per unit area of a hot body is $P = \sigma T^4$, where the Stefan-Boltzman constant is $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}.

6. Let us apply $P = \sigma T^4$ for finding the temperature of a planet that is at distance R from the sun. The sun has temperature T_{sun} and radius R_{sun}. In equilibrium, the energy received from the sun is exactly equal to the energy radiated by the planet. Now, the total energy radiated by the sun is $\sigma T_0^4 \times 4\pi R_{sun}^2$. But on a unit area of the planet, only $\dfrac{1}{4\pi R^2}$ of this is received. So the energy received per unit area on the planet is $\sigma T_0^4 \times 4\pi R_{sun}^2 \times \dfrac{1}{4\pi R^2}$. This must be equal to $\sigma T^4 \Rightarrow \quad T = T_0\sqrt{\dfrac{R_{sun}}{R}}$.

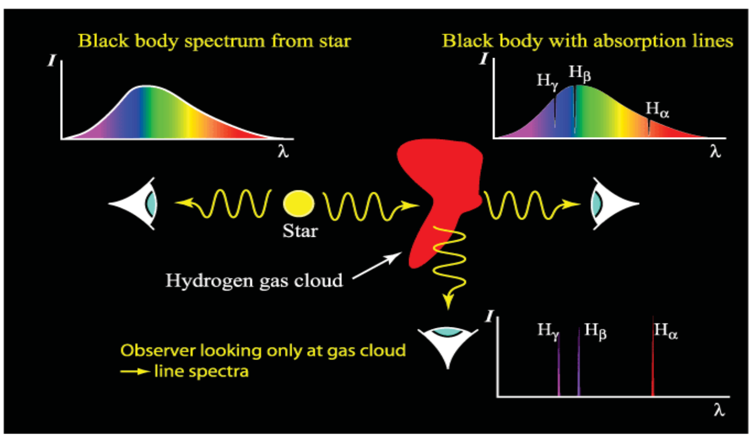
7. The above was for blackbody radiation where the emitted light has a continuous spectrum. But if a gas of identical atoms is excited by some mechanism, then only a few discrete wavelengths are emitted. Each chemical element produces a very distinct pattern of colors called an emission spectrum. So, for example, laboratory hydrogen gas lamps emit 3 lines in the visible region, as you can see below. Whenever we see 3 lines spaced apart in this way, we immediately know that hydrogen gas is present. It is as good as the thumbprint of a man!



8. But how do we get atoms excited so that they can start revealing their identity? One way is to simply heat material containing those atoms. You saw in the lecture how different colours come from sprinkling different materials on a flame.



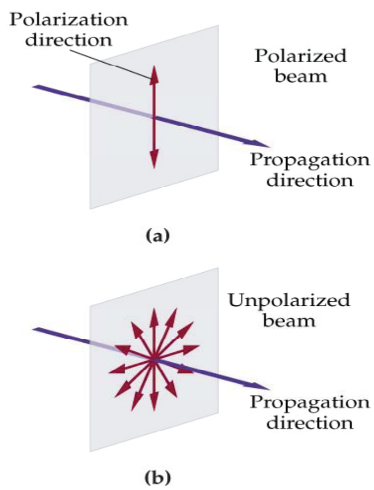
9. Everything that I have said about the emission of light applies exactly to the *absorption* of light as well. So, for example, when white light (which has all different frequencies within it) passes through hydrogen gas, you will see that all wavelengths survive except the three on the previous page. So the absorption spectrum looks exactly the same as the emission spectrum - the same lines are emitted and absorbed. This how we know that there are huge clouds of hydrogen floating in outer space. See the diagram below.



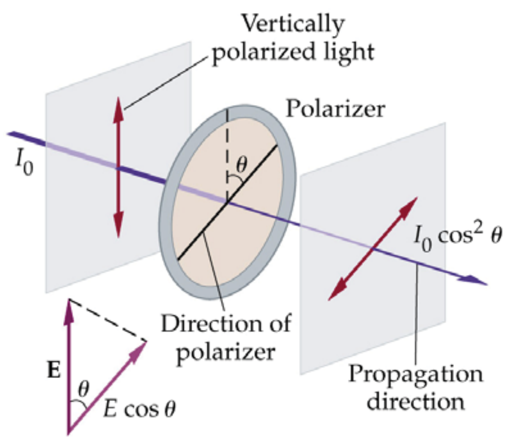
10. The atmosphere contains various gases which absorb light at many different wavelengths. Molecules of oxygen, nitrogen, ozone, and water have their own absorption spectra, just as atoms have their own.

11. All the beauty of colours we see is due to the selective absorption by molecules of certain frequencies. So, for example, *carotene* is a long, complicated molecule that makes carrots orange, tomatoes red, sarson yellow, and which absorbs blue light. Similarly *chlorophyll* makes leaves green and which absorbs red and blue light.

11. As you learned earlier, light is an electromagnetic wave that has an electric field vector. This vector is always perpendicular to the direction of travel, but it can be pointing anywhere in the plane. If it is pointing in a definite direction, we say that the wave is polarized, else it is unpolarized. In general, the light emitted from a source, such as a flame, will be unpolarized and it is equally possible to find any direction of the electric field in the wave. Of course, the magnetic field is perpendicular to both the direction of travel and the electric field.



12. From unpolarized light we can make polarized light by passing it through a polarizer as shown below.



Each wave is reduced in amplitude by $\cos\theta$, and in intensity by $\cos^2\theta$. The wave that emerges is now polarized in the θ direction.

13. We can design materials (crystals or stressed plastics) so that they have different optical properties in the two transverse directions. These are called birefringent materials. They are used to make commonly used liquid crystal displays (LCD) in watches and mobile phones. Birefringence can occur in any material that possesses some asymmetry in its structure where the material is more springy in one direction than another.

QUESTIONS AND EXERCISES – 32

Q.1 a) Taking the earth's average temperature to be 300^{o}K, find the frequency and wavelength at which maximum emission occurs.

b) Repeat the above for the sun, for T ≈ 4700^{o}K.

Q.2 Estimate the total amount of radiation emitted in 24 hours by a human being. Make a reasonable estimate of the surface area of the body. For the purposes of this calculation, does it make any difference that he/she is wearing clothes?

Q.3 In calculating the temperature of a planet in the solar system, I had made the assumption that all the energy absorbed was re-radiated into space.

a) Why is this assumption correct and under what circumstances?

b) Suppose that only 60% of the sun's radiation is absorbed, and that the remainder is reflected back into space. Redo the calculation under this assumption.

c) Carbon dioxide gas in the atmosphere absorbs certain frequencies in the infrared region. What effect will this have on the temperature of the earth?

Q.4 A wave that is linearly polarized in the x direction has the following electric and magnetic fields: $E_x = E_0 \sin(kz - \omega t),\quad E_y = 0,\ E_z = 0,\ B_x = 0,\quad B_y = \frac{E_0}{c}\sin(kz - \omega t),\ B_z = 0$

a)Write the fields for a wave that is polarized in the y direction.

b)Write the fields for a wave that is equally polarized in the x and y directions.

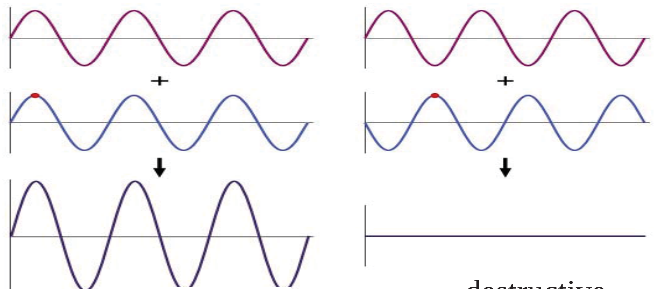
Q.5 A birefringent material has refractive indices 1.51 and 1.53 in the x and y directions respectively. A ray of red light enters the material, which has thickness 2cm.

a) Suppose the light is polarized in the x direction. How long will the light take to cross the crystal? What if it is polarized in the y direction?

b) Suppose the light is equally polarized in the x and y directions? Compute the phase difference between the two waves.

Summary of Lecture 33 – INTERFERENCE AND DIFFRACTION

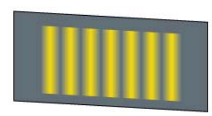
1. Two waves (of any kind) add up together, with the net result being the simple sum of the two waves. Consider two waves, both of the same frequency, shown below. If they start together (i.e. are *in phase* with each other) then the net amplitude is increased. This is called *constructive interference.* But if they start at different times (i.e. are *out of phase* with each other) then the net amplitude is decreased. This is called *destructive interference.*

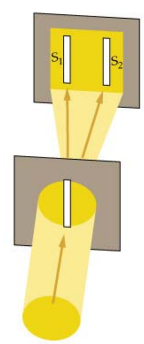


constructive   destructive

In the example above, both waves have the same frequency and amplitude, and so the resulting amplitude is doubled (constructive) or zero (destructive). But interference occurs for any two waves even when their amplitudes and frequencies are different.

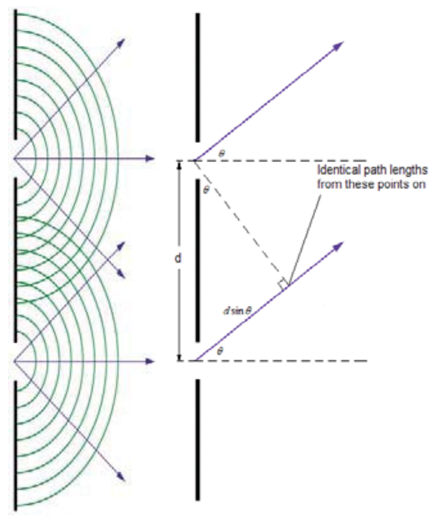
2. Although any waves from different sources interfere, if one wants to observe the interference of light then it is necessary to have a *coherent* source of light. Coherent means that both waves should have a fixed phase relative to each other. Even with lasers, it is very difficult to produce coherent light from two separate sources. Observing interference usually requires taking two waves from a single source, with each going along a different path. In the figure, an incoherent light source illuminates the first slit. This creates a uniform and coherent illumination of the second screen. Then waves from the slits S_1 and S_2 meet on the third screen and create a pattern of alternating light and dark fringes.





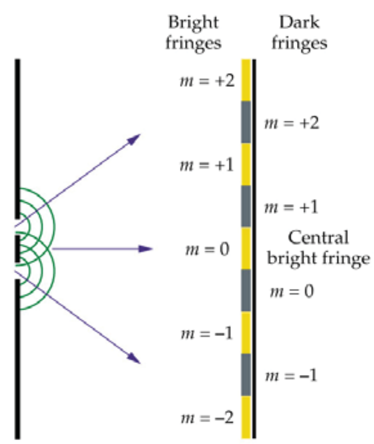
3. Wherever there is a bright fringe, constructive interference has occured, and wherever there is a dark fringe, destructive interference has occured. We shall now calculate where on the third screen the interference is constructive. Take any point on the third screen. Light reaches this point from both S_1 and S_2, but it will take different amounts of time to get there. Hence there will be a phase difference that we can calculate. Look at the diagram below. You can see that light from one of the slits has to travel an extra

distance equal to $d\sin\theta$, and so the extra amount of time it takes is $(d\sin\theta)/c$. There will be constructive interference if this is equal to $T, 2T, 3T, \cdots$ (remember that the time period is inversely related to the frequency, $T = 1/\nu$, and that $c = \lambda\nu$). We find that $\frac{d\sin\theta}{c} = nT = \frac{n}{\nu} \quad \Rightarrow \quad d\sin\theta = n\lambda$, where $n = 1, 2, 3, \cdots$ What about for destructive interference? Here the waves will cancel each other if the extra amount of time is $\frac{T}{2}, \frac{3T}{2}, \frac{5T}{2}, \cdots$ The condition then becomes $d\sin\theta = (n + \frac{1}{2})\lambda$.



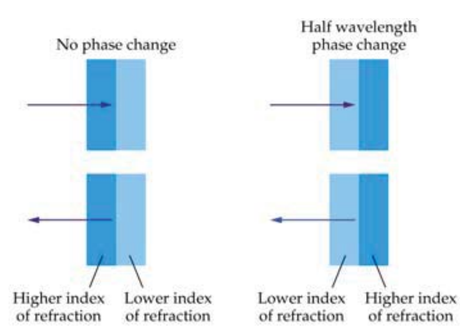
4. Example: two slits with a separation of 8.5× 10^{-5}m create an interference pattern on a screen 2.3m away. If the $n = 10$ bright fringe above the central is a linear distance of 12 cm from it, what is the wavelength of light used in the experiment?

Answer: First calculate the angle to the tenth bright fringe using $y = L\tan\theta$. Solving for θ gives, $\theta = \tan^{-1}\left(\frac{y}{L}\right) = \tan^{-1}\left(\frac{0.12m}{2.3m}\right) = 3.0^\circ$. From this,

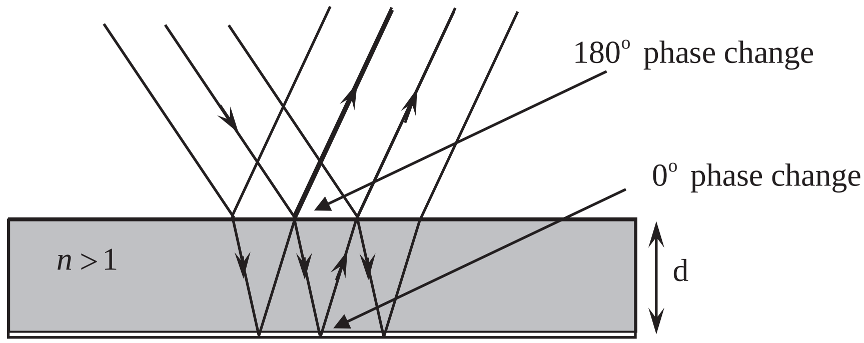


$$\lambda = \frac{d}{n}\sin\theta = \left(\frac{8.5\times10^{-5}}{10}\right)\sin(3.0^\circ) = 4.4\times10^{-7}m = 440nm \text{ (nanometres).}$$

5. When a wave is reflected at the interface of two media, the phase will not change if it goes from larger refractive index to a smaller one. But for smaller to larger, there will be phase change of a half-wavelength. One can show this using Maxwell's equations and applying the boundary conditions, but this will require some more advanced studies. Instead let's just use this fact below.



5. When light falls upon a thin film of soapy water, oil, etc. it is reflected from two surfaces. On the top surface, the reflection is with change of phase by π whereas at the lower surface there is no change of phase. This means that when waves from the two surfaces combine at the detector (your eyes), they will interfere.



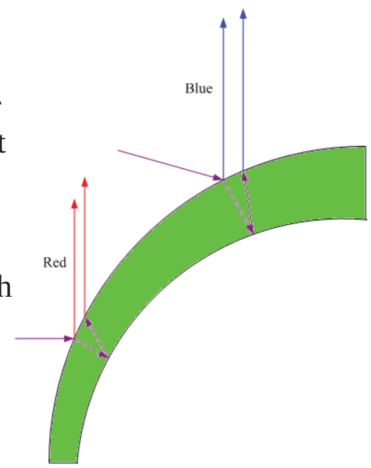
To simplify matters, suppose that you are looking at the thin film almost directly from above. Here n is the index of refraction for the medium. Then,

The condition for destructive interference is: $2nd = m\lambda \quad (m = 0,1,2,...)$

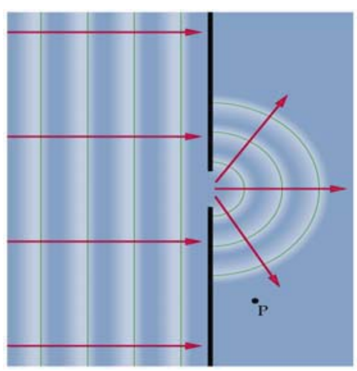
The condition for constructive interference is : $2nd = (m + \frac{1}{2})\lambda \quad (m = 0,1,2,...)$.

Prove it!

6. Interference is why thin films give rise to colours. A drop of oil floating on water spreads out until it is just a few microns thick. It will have thick and thin portions. Thick portions of any non-uniform thin film appear blue because the long-wavelength red light experiences destructive interference. Thinner portions appear red because the short-wavelength blue light interferes destructively.

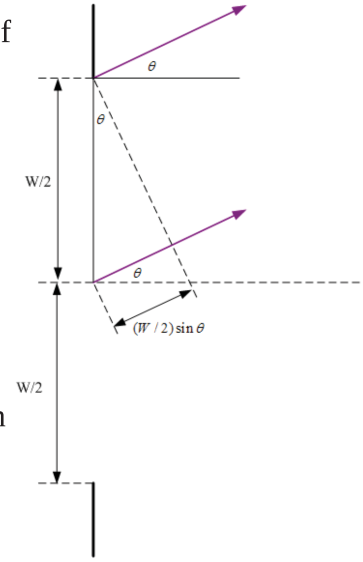


7. **Diffraction :** The bending of light around objects (into what would otherwise be a shadowed region) is known as diffraction. Diffraction occurs when light passes through very small apertures or near sharp edges. Diffraction is actually just interference, with the difference being only in the source of the interfering waves. Interference from a single slit, as in the figure here, is called diffraction.



We can get an interference pattern with a single slit provided its size is approximately equal to the wavelength of the light (neither too small nor too large).

8. Let's work out the condition necessary for diffraction of light from a single slit. With reference to the figure, imagine that a wave is incident from the left. It will cause secondary waves to be radiated from the edges of the slit. If one looks at angle θ, the extra distance that the wave emitted from the lower slit must travel is $W\sin\theta$. If this is a multiple of the wavelength λ, then constructive interference will occur. So the condition becomes $W\sin\theta = m\lambda$ with $m = \pm1, \pm2, \pm3...$ So, even from a single slit one will see a pattern of light and dark fringes when observed from the other side.



9. Light with wavelength of 511 nm forms a diffraction pattern after passing through a single slit of width 2.2×10^{-6}m. Find the angle associated with (a) the first and (b) the second bright fringe above the central bright fringe.

SOLUTION: For $m=1$, $\theta = \sin^{-1}\left(\frac{m\lambda}{W}\right) = \sin^{-1}\left(\frac{(1)(511\times10^{-9}m}{2.20\times10^{-6}m}\right) = 13.4^\circ$

For $m=2$, $\theta = \sin^{-1}\left(\frac{m\lambda}{W}\right) = \sin^{-1}\left(\frac{(2)(511\times10^{-9}m}{2.20\times10^{-6}m}\right) = 27.7^\circ$

10. Diffraction puts fundamental limits on the capacity of telescopes and microscopes to separate the objects being observed because light from the sides of a circular aperture interferes. One can calculate that the first dark fringe is at $\theta_{\min} = 1.22\frac{\lambda}{D}$, where D is diameter of the aperture. Two objects can be barely resolved if the diffraction maximum of one object lies in the diffraction minimum of the second object. Clearly, the larger D is, the smaller the angular diameter separation. We say that larger apertures lead to better resolution.

QUESTIONS AND EXERCISES – 33

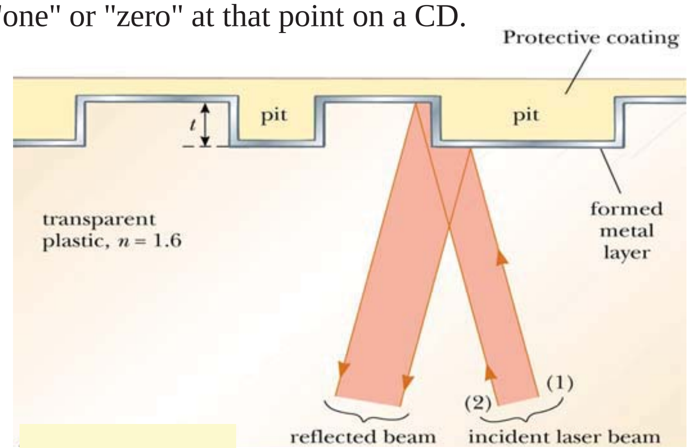
Q.1 In the lecture on sound, I demonstrated interference of waves coming from two separate loudspeakers. Why is not possible to demonstrate interference of waves coming from two separate light sources?

Q.2 In the two-slit experiment (see the diagram next to point no. 2 above) what would happen to the pattern on the third screen if one of the slits is covered up?

Q.3 Simplify $f(t) = \sin\omega t + \cos\omega t$ using a trigonometric identity that you certainly know. Then make a plot of $f^2(t)$ from $t = 0$ to $2\pi/T$.

Q.4 In a double slit experiment, green light of wavelength 550 nm illuminates slits that are 1.5mm apart. The screen is 2 m away. What will be the separation between the dark fringes?

Q.5 A compact disc player uses a laser to reflect light from the metal layer deposited on a protective coating, as shown below. When the light reflected from both segments is combined at the receiver, interference from the two waves results. We can therefore whether there is "one" or "zero" at that point on a CD.



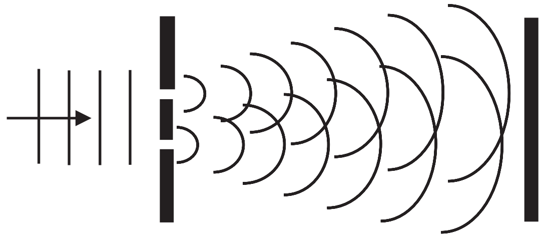
Answer the following:

a) A CD has about 700 MB (megabytes) recorded upon it. Give a rough estimate of how much area is needed for one byte.

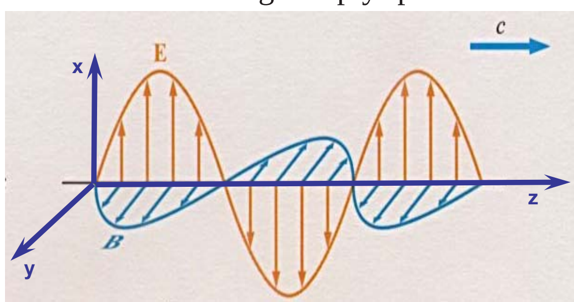
b) For a byte to be detected by interference using a green light laser, what should be t approximate depth t of the pit?

Summary of Lecture 34 – THE PARTICLE NATURE OF LIGHT

1. In the previous lecture I gave you some very strong reasons to believe that light is waves. Else, it is impossible to explain the interference and diffraction phenomena that we see in innumerable situations. Interference from two slits produces the characteristic pattern.

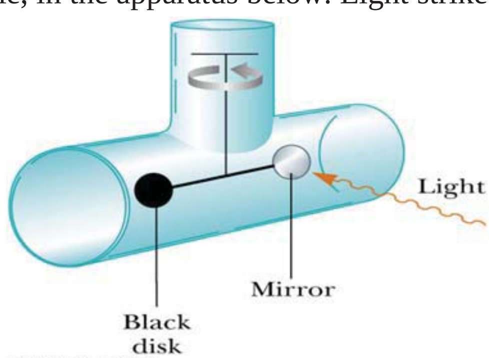


2. Light is waves, but waves in what? of what? The thought that there is some invisible medium (given the name *aether*) turned out to be wrong. Light is actually electric and magnetic waves that can travel through empty space. The electric and magnetic waves



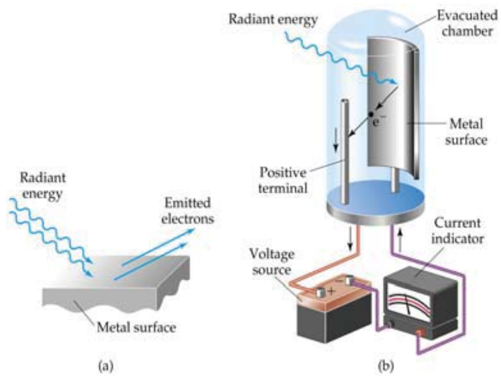
are perpendicular to each other and to the direction of travel (here the z direction).

3. Electromagnetic waves transport linear momentum and energy. If the energy per unit volume in a wave is U then it is carrying momentum p, where $p = U/c$. Waves with large amplitude carry more energy and momentum. For the sun's light on earth the momentum is rather small (although it is very large close to or inside the sun). Nevertheless, it is easily measureable as, for example, in the apparatus below. Light strikes a mirror and rebounds.



Thus the momentum of the light changes and this creates a force that rotates the mirror. The force is quite small - just 5×10^{-6} Newtons per unit area (in metre^2) of the mirror.

4. So strong was the evidence of light as waves that observation of the photoelectric effect came as a big shock to everybody. In the diagram below, light hits a metal surface and

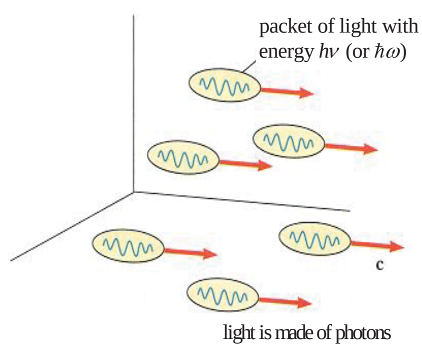


and knocks out electrons that travel to the anode. A current flows only as long as the light is shining. Above the threshold frequency, the number of electrons ejected depends on the intensity of the light. This was called the photoelectric effect. The following was observed:

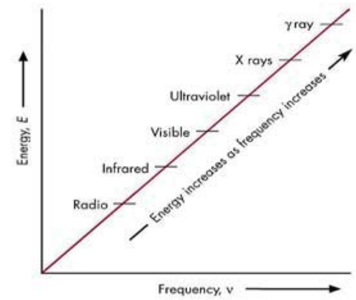
a) The photoelectric effect does not occur for all frequencies ν; it does not occur at all when ν is below a certain value. *But classically (meaning according to the Maxwell nature of light as an electromagnetic wave) electrons should be ejected at any ν. If an electron is shaken violently enough by the wave, it should surely be ejected!*

b) It is observed that the first photoelectrons are emitted instantaneously. *But classically the first photoelectrons should be emitted some time after the light first strikes the surface and excites its atoms enough to cause ionization of their electrons.*

5. Explanation of the photoelectric puzzle came from Einstein, for which he got the Nobel Prize in 1905. Einstein proposed that the light striking the surface was actually made of little packets (called quanta in plural, quantum in singular). Each quantum has an energy energy $\varepsilon = h\nu$ (or $\varepsilon = \hbar\omega$ where $\hbar = h/2\pi$ and $\omega = 2\pi\nu$) where h is the famous Planck's constant, 6.626×10^{-34} Joule-seconds. An electron is kicked out of the metal only when a quantum has energy (and frequency) big enough to do the job. It doesn't matter how many quanta of light - called photons - are fired at the metal. No photoelectrons will be released unless ν is large enough. Furthermore the photoelectrons are released immediately when the photon hits an electron.



6. Microwaves, radio and TV waves, X-rays, γ-rays, etc. are all photons but of very different frequencies. Because Planck's constant h is an extremely small number, the energy of each photon is very small.



7. How many photons do we see? Here is a table that gives us some interesting numbers:

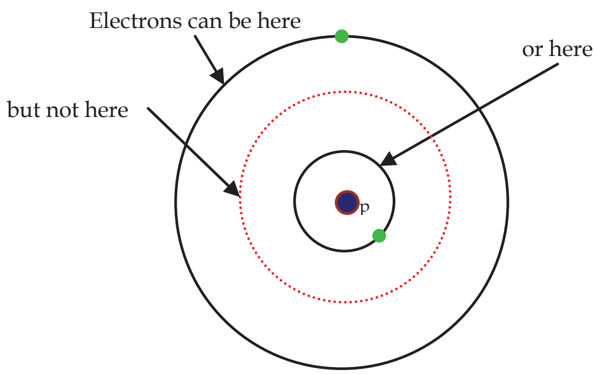
a) Sunny day (outdoors): 10^{15} photons per second enter eye (2 mm pupil).

b) Moonlit night (outdoors): 5×10^{10} photons/sec (6 mm pupil).

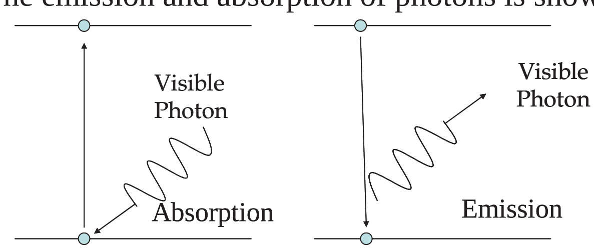
c) Moonless night (clear, starry sky): 10^{8} photons/sec (6 mm pupil).

d) Light from dimmest naked eye star (mag 6.5): 1000 photons/sec entering eye.

8. Where do photons come from? For this it is necessary to first understand that electrons inside an atom can only be in certain definite energy states. When an electron drops from a state with higher energy to one with lower energy, a photon is released whose energy is exactly equal to the difference of energies. Similarly a photon is absorbed when a photon of just the right energy hits an electron in the lower state and knocks it into a higher state.

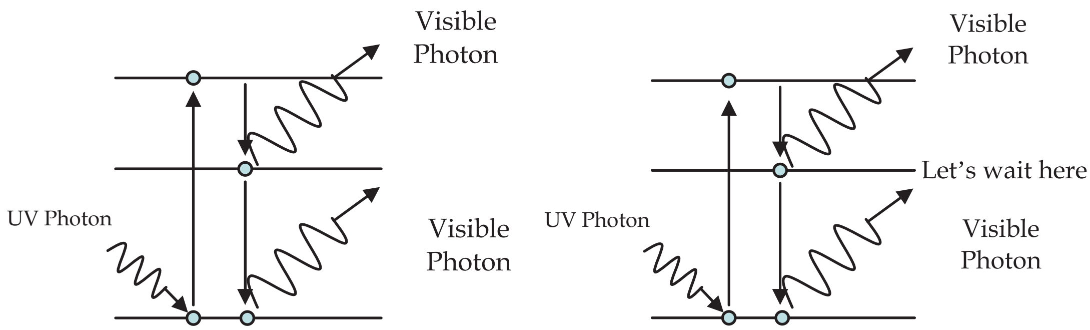


The upper and lower levels can be represented differently with the vertical direction representing energy. The emission and absorption of photons is shown below.



9. Fluorescence and phosphorence are two phenomena observed in some materials. When they are exposed to a source of light of a particular colour, they continue to emit light of a different colour even after the source has been turned off. So these materials can be

seen to glow even in the dark. In phosphorescence, a high-frequency photon raises an electron to an excited state. The electron jumps to an intermediate states, and then drops after a little while to the ground state. This is fluorescence. Phosphorescence is similar to fluorescence, except that phosphorescence materials continue to give off a secondary glow long (seconds to hours) after the initial illumination that excited the atoms.



10. One of the most important inventions of the 20th century is the laser which is short for:

LASER ≡ **L**ight **A**mplification by the **S**timulated **E**mission of **R**adiation

Lasers are important because they emit a very large number of photons all with one single frequency. How is this done? By some means - called optical pumping - atoms are excited to a high energy level. When one atom starts decaying to the lower state, it encourages all the others to decay as well. This is called spontaneous emission of radiation.

QUESTIONS AND EXERCISES – 34

Q.1 With reference to the diagram in point no. 3, suppose that light illuminates both the mirror and the darkened plate. On which will the force be greater? To answer this, ask on which plate is the change of momentum greater and then relate this to Newton's Law which states that force is rate of change of momentum.

Q.2 In order to remove an electron from a certain metal, it is necessary to give at least 2.2 eV energy to the electron. What should be the minimum frequency of a beam of light to ensure that photoelectrons are emitted? What if the frequency is larger than this? Where will the energy go?

Q.3 A laser pulse of green light at 550 nm puts out 2 joules of energy in 1 millisecond. How many photons does the pulse have? What if it is blue light?