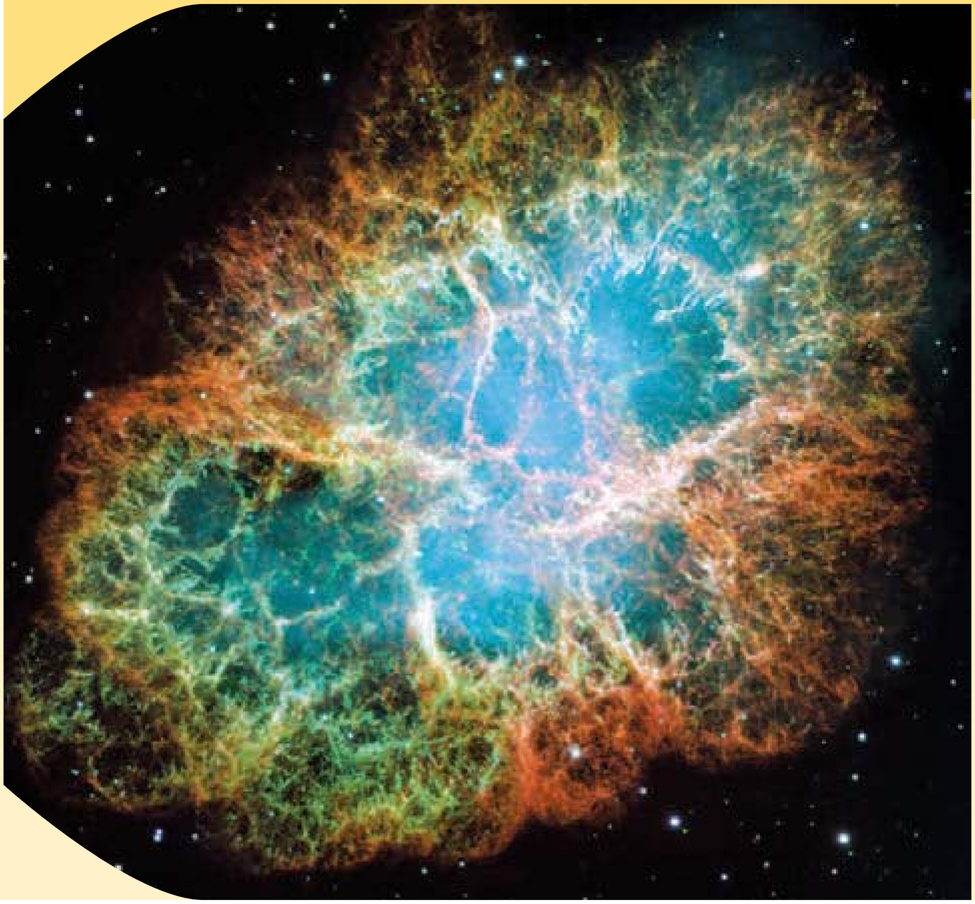
CHAPTER

34

Electromagnetic Waves



This image of the Crab Nebula taken with visible light shows a variety of colors, with each color representing a different wavelength of visible light. *(NASA, ESA, J. Hester, A. Loll (ASU))*

The waves described in Chapters 16, 17, and 18 are mechanical waves. By definition, the propagation of mechanical disturbances—such as sound waves, water waves, and waves on a string—requires the presence of a medium. This chapter is concerned with the properties of electromagnetic waves, which (unlike mechanical waves) can propagate through empty space.

We begin by considering Maxwell's contributions in modifying Ampère's law, which we studied in Chapter 30. We then discuss Maxwell's equations, which form the theoretical basis of all electromagnetic phenomena. These equations predict the existence of electromagnetic waves that propagate through space at the speed of light c according to the traveling wave analysis model. Heinrich Hertz confirmed Maxwell's prediction when he generated and detected electromagnetic waves in 1887. That discovery has led to many practical communication systems, including radio, television, cell phone systems, wireless Internet connectivity, and optoelectronics.

Next, we learn how electromagnetic waves are generated by oscillating electric charges. The waves radiated from the oscillating charges can be detected at great distances. Furthermore, because electromagnetic waves carry energy (T_{ER} in Eq. 8.2) and momentum, they can exert pressure on a surface. The chapter concludes with a description of the various frequency ranges in the electromagnetic spectrum.

34.1 Displacement Current and the General Form of Ampère's Law

In Chapter 30, we discussed using Ampère's law (Eq. 30.13) to analyze the magnetic fields created by currents:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I$$

In this equation, the line integral is over any closed path through which conduction current passes, where conduction current is defined by the expression $I = dq/dt$. (In this section, we use the term *conduction current* to refer to the current carried by charge carriers in the wire to distinguish it from a new type of current we shall introduce shortly.) We now show that Ampère's law in this form is valid only if any electric fields present are constant in time. James Clerk Maxwell recognized this limitation and modified Ampère's law to include time-varying electric fields.

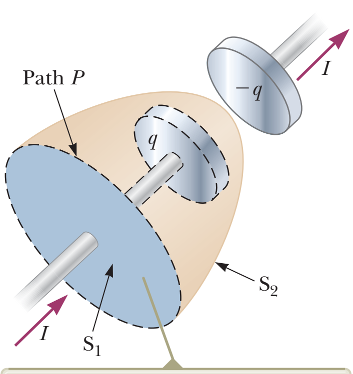
James Clerk Maxwell
Scottish Theoretical Physicist (1831–1879)
Maxwell developed the electromagnetic theory of light and the kinetic theory of gases, and explained the nature of Saturn's rings and color vision. Maxwell's successful interpretation of the electromagnetic field resulted in the field equations that bear his name. Formidable mathematical ability combined with great insight enabled him to lead the way in the study of electromagnetism and kinetic theory. He died of cancer before he was 50.

Consider a capacitor being charged as illustrated in Figure 34.1. When a conduction current is present, the charge on the positive plate changes, but no conduction current exists in the gap between the plates because there are no charge carriers in the gap. Now consider the two surfaces S_1 and S_2 in Figure 34.1, bounded by the same path P. Ampère's law states that $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ around this path must equal $\mu_0 I$, where I is the total current through *any* surface bounded by the path P.

When the path P is considered to be the boundary of S_1, $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I$ because the conduction current I passes through S_1. When the path is considered to be the boundary of S_2, however, $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = 0$ because no conduction current passes through S_2. Therefore, we have a contradictory situation that arises from the discontinuity of the current! Maxwell solved this problem by postulating an additional term on the right side of Ampère's law, which includes a factor called the **displacement current** I_d defined as[1]

$$I_d \equiv \epsilon_0 \frac{d\Phi_E}{dt} \tag{34.1}$$

◀ **Displacement current**



The conduction current I in the wire passes only through S_1, which leads to a contradiction in Ampère's law that is resolved only if one postulates a displacement current through S_2.

Figure 34.1 Two surfaces S_1 and S_2 near the plate of a capacitor are bounded by the same path P.

[1]*Displacement* in this context does not have the meaning it does in Chapter 2. Despite the inaccurate implications, the word is historically entrenched in the language of physics, so we continue to use it.

where ϵ_0 is the permittivity of free space (see Section 23.3) and $\Phi_E \equiv \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$ is the electric flux (see Eq. 24.3) through the surface bounded by the path of integration.

As the capacitor is being charged (or discharged), the changing electric field between the plates may be considered equivalent to a current that acts as a continuation of the conduction current in the wire. When the expression for the displacement current given by Equation 34.1 is added to the conduction current on the right side of Ampère's law, the difficulty represented in Figure 34.1 is resolved. No matter which surface bounded by the path P is chosen, either a conduction current or a displacement current passes through it. With this new term I_d, we can express the general form of Ampère's law (sometimes called the **Ampère–Maxwell law**) as

Ampère–Maxwell law ▶

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0(I + I_d) = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \qquad \textbf{(34.2)}$$

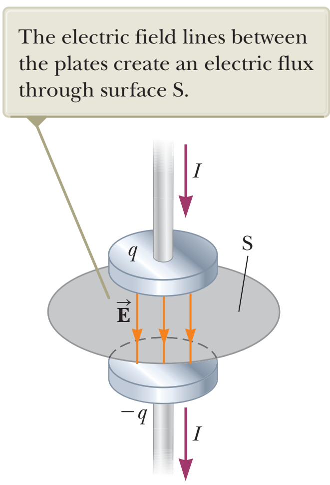


Figure 34.2 When a conduction current exists in the wires, a changing electric field $\vec{\mathbf{E}}$ exists between the plates of the capacitor.

We can understand the meaning of this expression by referring to Figure 34.2. The electric flux through surface S is $\Phi_E = \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = EA$, where A is the area of the capacitor plates and E is the magnitude of the uniform electric field between the plates. If q is the charge on the plates at any instant, then $E = q/(\epsilon_0 A)$ (see Section 26.2). Therefore, the electric flux through S is

$$\Phi_E = EA = \frac{q}{\epsilon_0}$$

Hence, the displacement current through S is

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt} = \frac{dq}{dt} \qquad \textbf{(34.3)}$$

That is, the displacement current I_d through S is precisely equal to the conduction current I in the wires connected to the capacitor!

By considering surface S, we can identify the displacement current as the source of the magnetic field on the surface boundary. The displacement current has its physical origin in the time-varying electric field. The central point of this formalism is that magnetic fields are produced *both* by conduction currents *and* by time-varying electric fields. This result was a remarkable example of theoretical work by Maxwell, and it contributed to major advances in the understanding of electromagnetism.

Quick Quiz 34.1 In an *RC* circuit, the capacitor begins to discharge. **(i)** During the discharge, in the region of space between the plates of the capacitor, is there (a) conduction current but no displacement current, (b) displacement current but no conduction current, (c) both conduction and displacement current, or (d) no current of any type? **(ii)** In the same region of space, is there (a) an electric field but no magnetic field, (b) a magnetic field but no electric field, (c) both electric and magnetic fields, or (d) no fields of any type?

Example 34.1 Displacement Current in a Capacitor

A sinusoidally varying voltage is applied across a capacitor as shown in Figure 34.3. The capacitance is $C = 8.00\ \mu\text{F}$, the frequency of the applied voltage is $f = 3.00$ kHz, and the voltage amplitude is $\Delta V_{\max} = 30.0$ V. Find the displacement current in the capacitor.

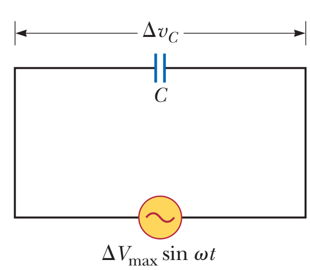


Figure 34.3 (Example 34.1)

SOLUTION

Conceptualize Figure 34.3 represents the circuit diagram for this situation. Figure 34.2 shows a close-up of the capacitor and the electric field between the plates.

Categorize We determine results using equations discussed in this section, so we categorize this example as a substitution problem.

34.1 continued

Evaluate the angular frequency of the source from Equation 15.12:

$$\omega = 2\pi f = 2\pi(3.00 \times 10^3 \text{ Hz}) = 1.88 \times 10^4 \text{ s}^{-1}$$

Use Equation 33.20 to express the potential difference in volts across the capacitor as a function of time in seconds:

$$\Delta v_C = \Delta V_{\max} \sin \omega t = 30.0 \sin (1.88 \times 10^4\ t)$$

Use Equation 34.3 to find the displacement current in amperes as a function of time. Note that the charge on the capacitor is $q = C\,\Delta v_C$:

$$i_d = \frac{dq}{dt} = \frac{d}{dt}(C\ \Delta v_C) = C\,\frac{d}{dt}(\Delta V_{\max} \sin \omega t)$$
$$= \omega C\, \Delta V_{\max} \cos \omega t$$

Substitute numerical values:

$$i_d = (1.88 \times 10^4 \text{ s}^{-1})(8.00 \times 10^{-6} \text{ C})(30.0 \text{ V}) \cos (1.88 \times 10^4\ t)$$
$$= \boxed{4.51 \cos (1.88 \times 10^4\ t)}$$

34.2 Maxwell's Equations and Hertz's Discoveries

We now present four equations that are regarded as the basis of all electrical and magnetic phenomena. These equations, developed by Maxwell, are as fundamental to electromagnetic phenomena as Newton's laws are to mechanical phenomena. In fact, the theory that Maxwell developed was more far-reaching than even he imagined because it turned out to be in agreement with the special theory of relativity, as Einstein showed in 1905.

Maxwell's equations represent the laws of electricity and magnetism that we have already discussed, but they have additional important consequences. For simplicity, we present **Maxwell's equations** as applied to free space, that is, in the absence of any dielectric or magnetic material. The four equations are

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q}{\epsilon_0} \qquad (34.4)$$

Gauss's law

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0 \qquad (34.5)$$

Gauss's law in magnetism

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt} \qquad (34.6)$$

Faraday's law

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I + \epsilon_0 \mu_0 \frac{d\Phi_E}{dt} \qquad (34.7)$$

Ampère–Maxwell law

Equation 34.4 is Gauss's law: the total electric flux through any closed surface equals the net charge inside that surface divided by ϵ_0. This law relates an electric field to the charge distribution that creates it.

Equation 34.5 is Gauss's law in magnetism, and it states that the net magnetic flux through a closed surface is zero. That is, the number of magnetic field lines that enter a closed volume must equal the number that leave that volume, which implies that magnetic field lines cannot begin or end at any point. If they did, it would mean that isolated magnetic monopoles existed at those points. That isolated magnetic monopoles have not been observed in nature can be taken as a confirmation of Equation 34.5.

Equation 34.6 is Faraday's law of induction, which describes the creation of an electric field by a changing magnetic flux. This law states that the emf, which is the line integral of the electric field around any closed path, equals the rate of change of magnetic flux through any surface bounded by that path. One consequence of Faraday's law is the current induced in a conducting loop placed in a time-varying magnetic field.

Equation 34.7 is the Ampère–Maxwell law, discussed in Section 34.1, and it describes the creation of a magnetic field by a changing electric field and by electric current: the line integral of the magnetic field around any closed path is the sum of μ_0 multiplied by the net current through that path and $\epsilon_0\mu_0$ multiplied by the rate of change of electric flux through any surface bounded by that path.

Once the electric and magnetic fields are known at some point in space, the force acting on a particle of charge q can be calculated from the electric and magnetic versions of the particle in a field model:

Lorentz force law ▶

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}} \tag{34.8}$$

This relationship is called the **Lorentz force law.** (We saw this relationship earlier as Eq. 29.6.) Maxwell's equations, together with this force law, completely describe all classical electromagnetic interactions in a vacuum.

Notice the symmetry of Maxwell's equations. Equations 34.4 and 34.5 are symmetric, apart from the absence of the term for magnetic monopoles in Equation 34.5. Furthermore, Equations 34.6 and 34.7 are symmetric in that the line integrals of $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ around a closed path are related to the rate of change of magnetic flux and electric flux, respectively. Maxwell's equations are of fundamental importance not only to electromagnetism, but to all science. Hertz once wrote, "One cannot escape the feeling that these mathematical formulas have an independent existence and an intelligence of their own, that they are wiser than we are, wiser even than their discoverers, that we get more out of them than we put into them."

In the next section, we show that Equations 34.6 and 34.7 can be combined to obtain a wave equation for both the electric field and the magnetic field. In empty space, where $q = 0$ and $I = 0$, the solution to these two equations shows that the speed at which electromagnetic waves travel equals the measured speed of light. This result led Maxwell to predict that light waves are a form of electromagnetic radiation.

Hertz performed experiments that verified Maxwell's prediction. The experimental apparatus Hertz used to generate and detect electromagnetic waves is shown schematically in Figure 34.4. An induction coil is connected to a transmitter made up of two spherical electrodes separated by a narrow gap. The coil provides short voltage surges to the electrodes, making one positive and the other negative. A spark is generated between the spheres when the electric field near either electrode surpasses the dielectric strength for air (3×10^6 V/m; see Table 26.1). Free electrons in a strong electric field are accelerated and gain enough energy to ionize any molecules they strike. This ionization provides more electrons, which can accelerate and cause further ionizations. As the air in the gap is ionized, it becomes a much better conductor and the discharge between the electrodes exhibits an oscillatory behavior at a very high frequency. From an electric-circuit viewpoint, this experimental apparatus is equivalent to an LC circuit in which the inductance is that of the coil and the capacitance is due to the spherical electrodes.

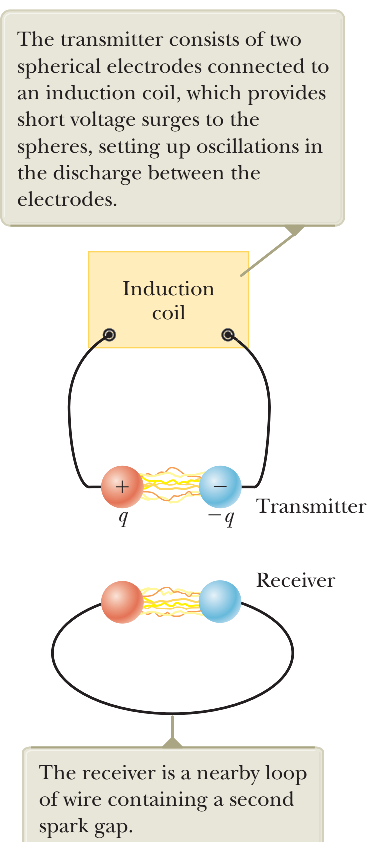


Figure 34.4 Schematic diagram of Hertz's apparatus for generating and detecting electromagnetic waves.

Because L and C are small in Hertz's apparatus, the frequency of oscillation is high, on the order of 100 MHz. (Recall from Eq. 32.22 that $\omega = 1/\sqrt{LC}$ for an LC circuit.) Electromagnetic waves are radiated at this frequency as a result of the oscillation of free charges in the transmitter circuit. Hertz was able to detect these waves using a single loop of wire with its own spark gap (the receiver). Such a receiver loop, placed several meters from the transmitter, has its own effective inductance, capacitance, and natural frequency of oscillation. In Hertz's experiment, sparks were induced across the gap of the receiving electrodes when the receiver's frequency was adjusted to match that of the transmitter. In this way, Hertz demonstrated that the oscillating current induced in the receiver was produced by electromagnetic waves radiated by the transmitter. His experiment is analogous to the mechanical phenomenon in which a tuning fork responds to acoustic vibrations from an identical tuning fork that is oscillating nearby.

In addition, Hertz showed in a series of experiments that the radiation generated by his spark-gap device exhibited the wave properties of interference, diffraction, reflection, refraction, and polarization, which are all properties exhibited by light as we shall see in Part 5. Therefore, it became evident that the radio-frequency waves Hertz was generating had properties similar to those of light waves and that they differed only in frequency and wavelength. Perhaps his most convincing experiment was the measurement of the speed of this radiation. Waves of known frequency were reflected from a metal sheet and created a standing-wave interference pattern whose nodal points could be detected. The measured distance between the nodal points enabled determination of the wavelength λ. Using the relationship $v = \lambda f$ (Eq. 16.12) from the traveling wave model, Hertz found that v was close to 3×10^8 m/s, the known speed c of visible light.

Heinrich Rudolf Hertz
German Physicist (1857–1894)
Hertz made his most important discovery of electromagnetic waves in 1887. After finding that the speed of an electromagnetic wave was the same as that of light, Hertz showed that electromagnetic waves, like light waves, could be reflected, refracted, and diffracted. The hertz, equal to one complete vibration or cycle per second, is named after him.

34.3 Plane Electromagnetic Waves

The properties of electromagnetic waves can be deduced from Maxwell's equations. One approach to deriving these properties is to solve the second-order differential equation obtained from Maxwell's third and fourth equations. A rigorous mathematical treatment of that sort is beyond the scope of this text. To circumvent this problem, let's assume the vectors for the electric field and magnetic field in an electromagnetic wave have a specific space–time behavior that is simple but consistent with Maxwell equations.

To understand the prediction of electromagnetic waves more fully, let's focus our attention on an electromagnetic wave that travels in the x direction (the *direction of propagation*). For this wave, the electric field $\vec{\mathbf{E}}$ is in the y direction and the magnetic field $\vec{\mathbf{B}}$ is in the z direction as shown in Figure 34.5. Such waves, in which the electric and magnetic fields are restricted to being parallel to a pair of perpendicular axes, are said to be **linearly polarized waves.** Furthermore, let's assume the field magnitudes E and B depend on x and t only, not on the y or z coordinate.

Let's also imagine that the source of the electromagnetic waves is such that a wave radiated from *any* position in the yz plane (not only from the origin as might be suggested by Fig. 34.5) propagates in the x direction and all such waves are emitted in phase. If we define a **ray** as the line along which the wave travels, all rays for these waves are parallel. This entire collection of waves is often called a **plane wave.** A surface connecting points of equal phase on all waves is a geometric plane called a **wave front,** introduced in Chapter 17. In comparison, a point source of radiation sends waves out radially in all directions. A surface connecting points of equal phase for this situation is a sphere, so this wave is called a **spherical wave.**

Pitfall Prevention 34.1
What Is "a" Wave? What do we mean by a *single* wave? The word *wave* represents both the emission from a *single point* ("wave radiated from *any* position in the yz plane" in the text) and the collection of waves from *all points* on the source ("**plane wave**" in the text). You should be able to use this term in both ways and understand its meaning from the context.

To generate the prediction of plane electromagnetic waves, we start with Faraday's law, Equation 34.6:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$

To apply this equation to the wave in Figure 34.5, consider a rectangle of width dx and height ℓ lying in the xy plane as shown in Figure 34.6 (page 1036). Let's first evaluate the line integral of $\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$ around this rectangle in the counterclockwise direction at an instant of time when the wave is passing through the rectangle. The contributions from the top and bottom of the rectangle are zero because $\vec{\mathbf{E}}$ is perpendicular to $d\vec{\mathbf{s}}$ for these paths. We can express the electric field on the right side of the rectangle as

$$E(x + dx) \approx E(x) + \left.\frac{dE}{dx}\right|_{t \text{ constant}} dx = E(x) + \frac{\partial E}{\partial x}\, dx$$

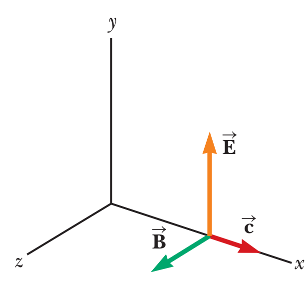


Figure 34.5 Electric and magnetic fields of an electromagnetic wave traveling at velocity $\vec{\mathbf{c}}$ in the positive x direction. The field vectors are shown at one instant of time and at one position in space. These fields depend on x and t.

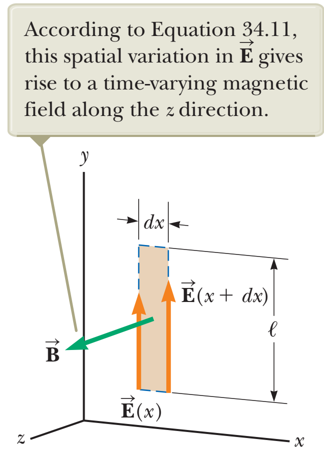


Figure 34.6 At an instant when a plane wave moving in the positive x direction passes through a rectangular path of width dx lying in the xy plane, the electric field in the y direction varies from $\vec{\mathbf{E}}(x)$ to $\vec{\mathbf{E}}(x + dx)$.

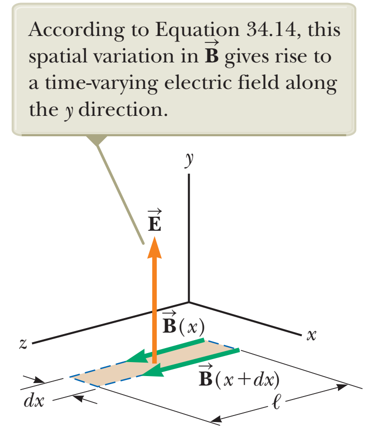


Figure 34.7 At an instant when a plane wave passes through a rectangular path of width dx lying in the xz plane, the magnetic field in the z direction varies from $\vec{\mathbf{B}}(x)$ to $\vec{\mathbf{B}}(x + dx)$.

where $E(x)$ is the field on the left side of the rectangle at this instant.[2] Therefore, the line integral over this rectangle is approximately

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = [E(x + dx)]\ell - [E(x)]\ell \approx \ell\left(\frac{\partial E}{\partial x}\right) dx \tag{34.9}$$

Because the magnetic field is in the z direction, the magnetic flux through the rectangle of area $\ell\, dx$ is approximately $\Phi_B = B\ell\, dx$ (assuming dx is very small compared with the wavelength of the wave). Taking the time derivative of the magnetic flux gives

$$\frac{d\Phi_B}{dt} = \ell\, dx \left.\frac{dB}{dt}\right|_{x\text{ constant}} = \ell\, dx \frac{\partial B}{\partial t} \tag{34.10}$$

Substituting Equations 34.9 and 34.10 into Equation 34.6 gives

$$\ell\left(\frac{\partial E}{\partial x}\right) dx = -\ell\, dx \frac{\partial B}{\partial t}$$

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} \tag{34.11}$$

In a similar manner, we can derive a second equation by starting with Maxwell's fourth equation in empty space (Eq. 34.7). In this case, the line integral of $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ is evaluated around a rectangle lying in the xz plane and having width dx and length ℓ as in Figure 34.7. Noting that the magnitude of the magnetic field changes from $B(x)$ to $B(x + dx)$ over the width dx and that the direction for taking the line integral is counterclockwise when viewed from above in Figure 34.7, the line integral over this rectangle is found to be approximately

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = [B(x)]\ell - [B(x + dx)]\ell \approx -\ell\left(\frac{\partial B}{\partial x}\right) dx \tag{34.12}$$

[2]Because dE/dx in this equation is expressed as the change in E with x at a given instant t, dE/dx is equivalent to the partial derivative $\partial E/\partial x$. Likewise, dB/dt means the change in B with time at a particular position x; therefore, in Equation 34.10, we can replace dB/dt with $\partial B/\partial t$.

The electric flux through the rectangle is $\Phi_E = E\ell\,dx$, which, when differentiated with respect to time, gives

$$\frac{\partial \Phi_E}{\partial t} = \ell\ dx \frac{\partial E}{\partial t} \tag{34.13}$$

Substituting Equations 34.12 and 34.13 into Equation 34.7 gives

$$-\ell\left(\frac{\partial B}{\partial x}\right) dx = \mu_0\,\epsilon_0\,\ell\ dx\left(\frac{\partial E}{\partial t}\right)$$

$$\frac{\partial B}{\partial x} = -\mu_0\epsilon_0 \frac{\partial E}{\partial t} \tag{34.14}$$

Taking the derivative of Equation 34.11 with respect to x and combining the result with Equation 34.14 gives

$$\frac{\partial^2 E}{\partial x^2} = -\frac{\partial}{\partial x}\left(\frac{\partial B}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\frac{\partial B}{\partial x}\right) = -\frac{\partial}{\partial t}\left(-\mu_0\epsilon_0 \frac{\partial E}{\partial t}\right)$$

$$\boxed{\frac{\partial^2 E}{\partial x^2} = \mu_0\epsilon_0 \frac{\partial^2 E}{\partial t^2}} \tag{34.15}$$

In the same manner, taking the derivative of Equation 34.14 with respect to x and combining it with Equation 34.11 gives

$$\boxed{\frac{\partial^2 B}{\partial x^2} = \mu_0\epsilon_0 \frac{\partial^2 B}{\partial t^2}} \tag{34.16}$$

Equations 34.15 and 34.16 both have the form of the linear wave equation[3] with the wave speed v replaced by c, where

$$\boxed{c = \frac{1}{\sqrt{\mu_0\epsilon_0}}} \tag{34.17}$$

◀ **Speed of electromagnetic waves**

Let's evaluate this speed numerically:

$$c = \frac{1}{\sqrt{(4\pi \times 10^{-7}\ \mathrm{T\cdot m/A})(8.854\ 19 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2})}}$$
$$= 2.997\ 92 \times 10^8\ \mathrm{m/s}$$

Because this speed is precisely the same as the speed of light in empty space, we are led to believe (correctly) that light is an electromagnetic wave.

The simplest solution to Equations 34.15 and 34.16 is a sinusoidal wave for which the field magnitudes E and B vary with x and t according to the expressions

$$E = E_{\max} \cos\,(kx - \omega t) \tag{34.18}$$

$$B = B_{\max} \cos\,(kx - \omega t) \tag{34.19}$$

◀ **Sinusoidal electric and magnetic fields**

where $E_{\max}$ and $B_{\max}$ are the maximum values of the fields. The angular wave number is $k = 2\pi/\lambda$, where λ is the wavelength. The angular frequency is $\omega = 2\pi f$, where f is the wave frequency. According to the traveling wave model, the ratio ω/k equals the speed of an electromagnetic wave, c:

$$\frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = \lambda f = c$$

[3]The linear wave equation is of the form $(\partial^2 y/\partial x^2) = (1/v^2)(\partial^2 y/\partial t^2)$, where v is the speed of the wave and y is the wave function. The linear wave equation was introduced as Equation 16.27, and we suggest you review Section 16.6.

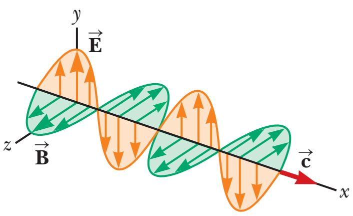


Figure 34.8 A sinusoidal electromagnetic wave moves in the positive x direction with a speed c.

where we have used Equation 16.12, $v = c = \lambda f$, which relates the speed, frequency, and wavelength of a sinusoidal wave. Therefore, for electromagnetic waves, the wavelength and frequency of these waves are related by

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{f} \tag{34.20}$$

Figure 34.8 is a pictorial representation, at one instant, of a sinusoidal, linearly polarized electromagnetic wave moving in the positive x direction.

We can generate other mathematical representations of the traveling wave model for electromagnetic waves. Taking partial derivatives of Equations 34.18 (with respect to x) and 34.19 (with respect to t) gives

$$\frac{\partial E}{\partial x} = -kE_{\text{max}} \sin (kx - \omega t)$$

$$\frac{\partial B}{\partial t} = \omega B_{\text{max}} \sin (kx - \omega t)$$

Substituting these results into Equation 34.11 shows that, at any instant,

$$kE_{\text{max}} = \omega B_{\text{max}}$$

$$\frac{E_{\text{max}}}{B_{\text{max}}} = \frac{\omega}{k} = c$$

Using these results together with Equations 34.18 and 34.19 gives

$$\boxed{\frac{E_{\text{max}}}{B_{\text{max}}} = \frac{E}{B} = c} \tag{34.21}$$

Pitfall Prevention 34.2

$\vec{E}$ Stronger Than $\vec{B}$? Because the value of c is so large, some students incorrectly interpret Equation 34.21 as meaning that the electric field is much stronger than the magnetic field. Electric and magnetic fields are measured in different units, however, so they cannot be directly compared. In Section 34.4, we find that the electric and magnetic fields contribute equally to the wave's energy.

That is, at every instant, the ratio of the magnitude of the electric field to the magnitude of the magnetic field in an electromagnetic wave equals the speed of light.

Finally, note that electromagnetic waves obey the superposition principle as described in the waves in interference analysis model (which we discussed in Section 18.1 with respect to mechanical waves) because the differential equations involving E and B are linear equations. For example, we can add two waves with the same frequency and polarization simply by adding the magnitudes of the two electric fields algebraically.

Quick Quiz 34.2 What is the phase difference between the sinusoidal oscillations of the electric and magnetic fields in Figure 34.8? **(a)** 180° **(b)** 90° **(c)** 0 **(d)** impossible to determine

Quick Quiz 34.3 An electromagnetic wave propagates in the negative y direction. The electric field at a point in space is momentarily oriented in the positive x direction. In which direction is the magnetic field at that point momentarily oriented? **(a)** the negative x direction **(b)** the positive y direction **(c)** the positive z direction **(d)** the negative z direction

Example 34.2 An Electromagnetic Wave AM

A sinusoidal electromagnetic wave of frequency 40.0 MHz travels in free space in the x direction as in Figure 34.9.

(A) Determine the wavelength and period of the wave.

34.2 continued

SOLUTION

Conceptualize Imagine the wave in Figure 34.9 moving to the right along the x axis, with the electric and magnetic fields oscillating in phase.

Categorize We use the mathematical representation of the *traveling wave* model for electromagnetic waves.

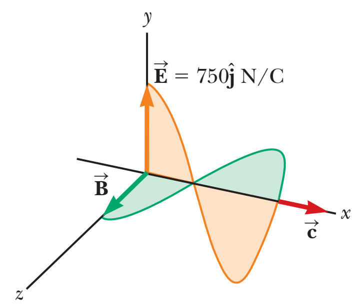


Figure 34.9 (Example 34.2) At some instant, a plane electromagnetic wave moving in the x direction has a maximum electric field of 750 N/C in the positive y direction.

Analyze

Use Equation 34.20 to find the wavelength of the wave:

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{40.0 \times 10^6 \text{ Hz}} = \boxed{7.50 \text{ m}}$$

Find the period T of the wave as the inverse of the frequency:

$$T = \frac{1}{f} = \frac{1}{40.0 \times 10^6 \text{ Hz}} = \boxed{2.50 \times 10^{-8} \text{ s}}$$

(B) At some point and at some instant, the electric field has its maximum value of 750 N/C and is directed along the y axis. Calculate the magnitude and direction of the magnetic field at this position and time.

SOLUTION

Use Equation 34.21 to find the magnitude of the magnetic field:

$$B_{max} = \frac{E_{max}}{c} = \frac{750 \text{ N/C}}{3.00 \times 10^8 \text{ m/s}} = \boxed{2.50 \times 10^{-6} \text{ T}}$$

Because $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ must be perpendicular to each other and perpendicular to the direction of wave propagation (x in this case), we conclude that $\vec{\mathbf{B}}$ is in the z direction.

Finalize Notice that the wavelength is several meters. This is relatively long for an electromagnetic wave. As we will see in Section 34.7, this wave belongs to the radio range of frequencies.

34.4 Energy Carried by Electromagnetic Waves

In our discussion of the nonisolated system model for energy in Section 8.1, we identified electromagnetic radiation as one method of energy transfer across the boundary of a system. The amount of energy transferred by electromagnetic waves is symbolized as T_{ER} in Equation 8.2. The rate of transfer of energy by an electromagnetic wave is described by a vector $\vec{\mathbf{S}}$, called the **Poynting vector,** which is defined by the expression

$$\boxed{\vec{\mathbf{S}} \equiv \frac{1}{\mu_0} \vec{\mathbf{E}} \times \vec{\mathbf{B}}} \qquad (34.22)$$

◀ Poynting vector

The magnitude of the Poynting vector represents the rate at which energy passes through a unit surface area perpendicular to the direction of wave propagation. Therefore, the magnitude of $\vec{\mathbf{S}}$ represents *power per unit area*. The direction of the vector is along the direction of wave propagation (Fig. 34.10, page 1040). The SI units of $\vec{\mathbf{S}}$ are J/s · m^2 = W/m^2.

As an example, let's evaluate the magnitude of $\vec{\mathbf{S}}$ for a plane electromagnetic wave where $|\vec{\mathbf{E}} \times \vec{\mathbf{B}}| = EB$. In this case,

$$S = \frac{EB}{\mu_0} \qquad (34.23)$$

Pitfall Prevention 34.3

An Instantaneous Value The Poynting vector given by Equation 34.22 is time dependent. Its magnitude varies in time, reaching a maximum value at the same instant the magnitudes of $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ do. The *average* rate of energy transfer is given by Equation 34.24 on the next page.

Pitfall Prevention 34.4

Irradiance In this discussion, intensity is defined in the same way as in Chapter 17 (as power per unit area). In the optics industry, however, power per unit area is called the *irradiance*. Radiant intensity is defined as the power in watts per solid angle (measured in steradians).

Because $B = E/c$, we can also express this result as

$$S = \frac{E^2}{\mu_0 c} = \frac{cB^2}{\mu_0}$$

These equations for S apply at any instant of time and represent the *instantaneous* rate at which energy is passing through a unit area in terms of the instantaneous values of E and B.

What is of greater interest for a sinusoidal plane electromagnetic wave is the time average of S over one or more cycles, which is called the *wave intensity* I. (We discussed the intensity of sound waves in Chapter 17.) When this average is taken, we obtain an expression involving the time average of $\cos^2(kx - \omega t)$, which equals $\frac{1}{2}$. Hence, the average value of S (in other words, the intensity of the wave) is

Wave intensity ▶

$$I = S_{\text{avg}} = \frac{E_{\max} B_{\max}}{2\mu_0} = \frac{E_{\max}^2}{2\mu_0 c} = \frac{cB_{\max}^2}{2\mu_0} \tag{34.24}$$

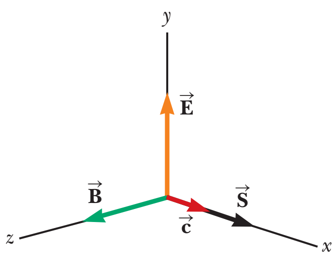


Figure 34.10 The Poynting vector $\vec{\mathbf{S}}$ for a plane electromagnetic wave is along the direction of wave propagation.

Recall that the energy per unit volume associated with an electric field, which is the instantaneous energy density u_E, is given by Equation 26.13:

$$u_E = \tfrac{1}{2}\epsilon_0 E^2$$

Also recall that the instantaneous energy density u_B associated with a magnetic field is given by Equation 32.14:

$$u_B = \frac{B^2}{2\mu_0}$$

Because E and B vary with time for an electromagnetic wave, the energy densities also vary with time. Using the relationships $B = E/c$ and $c = 1/\sqrt{\mu_0\epsilon_0}$, the expression for u_B becomes

$$u_B = \frac{(E/c)^2}{2\mu_0} = \frac{\mu_0\epsilon_0}{2\mu_0}E^2 = \tfrac{1}{2}\epsilon_0 E^2$$

Comparing this result with the expression for u_E, we see that

$$u_B = u_E = \tfrac{1}{2}\epsilon_0 E^2 = \frac{B^2}{2\mu_0}$$

That is, the instantaneous energy density associated with the magnetic field of an electromagnetic wave equals the instantaneous energy density associated with the electric field. Hence, in a given volume, the energy is equally shared by the two fields.

The **total instantaneous energy density** u is equal to the sum of the energy densities associated with the electric and magnetic fields:

Total instantaneous energy density of an electromagnetic wave ▶

$$u = u_E + u_B = \epsilon_0 E^2 = \frac{B^2}{\mu_0}$$

When this total instantaneous energy density is averaged over one or more cycles of an electromagnetic wave, we again obtain a factor of $\frac{1}{2}$. Hence, for any electromagnetic wave, the total average energy per unit volume is

Average energy density of an electromagnetic wave ▶

$$u_{\text{avg}} = \epsilon_0 (E^2)_{\text{avg}} = \tfrac{1}{2}\epsilon_0 E_{\max}^2 = \frac{B_{\max}^2}{2\mu_0} \tag{34.25}$$

Comparing this result with Equation 34.24 for the average value of S, we see that

$$I = S_{\text{avg}} = c u_{\text{avg}} \tag{34.26}$$

In other words, the intensity of an electromagnetic wave equals the average energy density multiplied by the speed of light.

The Sun delivers about 10^3 W/m^2 of energy to the Earth's surface via electromagnetic radiation. Let's calculate the total power that is incident on the roof of a home. The roof's dimensions are 8.00 m $\times$ 20.0 m. We assume the average magnitude of the Poynting vector for solar radiation at the surface of the Earth is $S_{avg} = 1\,000$ W/m^2. This average value represents the power per unit area, or the light intensity. Assuming the radiation is incident normal to the roof, we obtain

$$P_{avg} = S_{avg}A = (1\,000\ \text{W/m}^2)(8.00\ \text{m} \times 20.0\ \text{m}) = 1.60 \times 10^5\ \text{W}$$

This power is large compared with the power requirements of a typical home. If this power could be absorbed and made available to electrical devices, it would provide more than enough energy for the average home. Solar energy is not easily harnessed, however, and the prospects for large-scale conversion are not as bright as may appear from this calculation. For example, the efficiency of conversion from solar energy is typically 12–18% for photovoltaic cells, reducing the available power by an order of magnitude. Other considerations reduce the power even further. Depending on location, the radiation is most likely not incident normal to the roof and, even if it is, this situation exists for only a short time near the middle of the day. No energy is available for about half of each day during the nighttime hours, and cloudy days further reduce the available energy. Finally, while energy is arriving at a large rate during the middle of the day, some of it must be stored for later use, requiring batteries or other storage devices. All in all, complete solar operation of homes is not currently cost effective for most homes.

Example 34.3 Fields on the Page

Estimate the maximum magnitudes of the electric and magnetic fields of the light that is incident on this page because of the visible light coming from your desk lamp. Treat the lightbulb as a point source of electromagnetic radiation that is 5% efficient at transforming energy coming in by electrical transmission to energy leaving by visible light.

SOLUTION

Conceptualize The filament in your lightbulb emits electromagnetic radiation. The brighter the light, the larger the magnitudes of the electric and magnetic fields.

Categorize Because the lightbulb is to be treated as a point source, it emits equally in all directions, so the outgoing electromagnetic radiation can be modeled as a spherical wave.

Analyze Recall from Equation 17.13 that the wave intensity I a distance r from a point source is $I = P_{avg}/4\pi r^2$, where P_{avg} is the average power output of the source and $4\pi r^2$ is the area of a sphere of radius r centered on the source.

Set this expression for I equal to the intensity of an electromagnetic wave given by Equation 34.24:

$$I = \frac{P_{avg}}{4\pi r^2} = \frac{E_{max}^2}{2\mu_0 c}$$

Solve for the electric field magnitude:

$$E_{max} = \sqrt{\frac{\mu_0 c P_{avg}}{2\pi r^2}}$$

Let's make some assumptions about numbers to enter in this equation. The visible light output of a 60-W lightbulb operating at 5% efficiency is approximately 3.0 W by visible light. (The remaining energy transfers out of the lightbulb by thermal conduction and invisible radiation.) A reasonable distance from the lightbulb to the page might be 0.30 m.

Substitute these values:

$$E_{max} = \sqrt{\frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(3.00 \times 10^8\ \text{m/s})(3.0\ \text{W})}{2\pi(0.30\ \text{m})^2}}$$

$$= \boxed{45\ \text{V/m}}$$

continued

34.3 continued

Use Equation 34.21 to find the magnetic field magnitude:

$$B_{max} = \frac{E_{max}}{c} = \frac{45 \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = \boxed{1.5 \times 10^{-7} \text{ T}}$$

Finalize This value of the magnetic field magnitude is two orders of magnitude smaller than the Earth's magnetic field.

34.5 Momentum and Radiation Pressure

Electromagnetic waves transport linear momentum as well as energy. As this momentum is absorbed by some surface, pressure is exerted on the surface. Therefore, the surface is a nonisolated system for momentum. In this discussion, let's assume the electromagnetic wave strikes the surface at normal incidence and transports a total energy T_{ER} to the surface in a time interval Δt. Maxwell showed that if the surface absorbs all the incident energy T_{ER} in this time interval (as does a black body, introduced in Section 20.7), the total momentum $\vec{\mathbf{p}}$ transported to the surface has a magnitude

Momentum transported to a perfectly absorbing surface

$$p = \frac{T_{ER}}{c} \quad \text{(complete absorption)} \qquad \textbf{(34.27)}$$

The pressure P exerted on the surface is defined as force per unit area F/A, which when combined with Newton's second law gives

$$P = \frac{F}{A} = \frac{1}{A}\frac{dp}{dt}$$

Substituting Equation 34.27 into this expression for pressure P gives

$$P = \frac{1}{A}\frac{dp}{dt} = \frac{1}{A}\frac{d}{dt}\left(\frac{T_{ER}}{c}\right) = \frac{1}{c}\frac{(dT_{ER}/dt)}{A}$$

We recognize $(dT_{ER}/dt)/A$ as the rate at which energy is arriving at the surface per unit area, which is the magnitude of the Poynting vector. Therefore, the radiation pressure P exerted on the perfectly absorbing surface is

Radiation pressure exerted on a perfectly absorbing surface

$$P = \frac{S}{c} \quad \text{(complete absorption)} \qquad \textbf{(34.28)}$$

If the surface is a perfect reflector (such as a mirror) and incidence is normal, the momentum transported to the surface in a time interval Δt is twice that given by Equation 34.27. That is, the momentum transferred to the surface by the incoming light is $p = T_{ER}/c$ and that transferred by the reflected light is also $p = T_{ER}/c$. Therefore,

Pitfall Prevention 34.5

So Many p's We have p for momentum and P for pressure, and they are both related to P for power! Be sure to keep all these symbols straight.

$$p = \frac{2T_{ER}}{c} \quad \text{(complete reflection)} \qquad \textbf{(34.29)}$$

The radiation pressure exerted on a perfectly reflecting surface for normal incidence of the wave is

Radiation pressure exerted on a perfectly reflecting surface

$$P = \frac{2S}{c} \quad \text{(complete reflection)} \qquad \textbf{(34.30)}$$

The pressure on a surface having a reflectivity somewhere between these two extremes has a value between S/c and $2S/c$, depending on the properties of the surface.

Although radiation pressures are very small (about 5×10^{-6} N/m^2 for direct sunlight), *solar sailing* is a low-cost means of sending spacecraft to the planets. Large sheets experience radiation pressure from sunlight and are used in much the way canvas sheets are used on earthbound sailboats. In 2010, the Japan Aerospace Exploration Agency (JAXA) launched the first spacecraft to use solar sailing as its primary propulsion, *IKAROS* (Interplanetary Kite-craft Accelerated by Radiation of the Sun). Successful testing of this spacecraft would lead to a larger effort to send a spacecraft to Jupiter by radiation pressure later in the present decade.

Quick Quiz 34.4 To maximize the radiation pressure on the sails of a spacecraft using solar sailing, should the sheets be **(a)** very black to absorb as much sunlight as possible or **(b)** very shiny to reflect as much sunlight as possible?

Conceptual Example 34.4 Sweeping the Solar System

A great amount of dust exists in interplanetary space. Although in theory these dust particles can vary in size from molecular size to a much larger size, very little of the dust in our solar system is smaller than about 0.2 μm. Why?

SOLUTION

The dust particles are subject to two significant forces: the gravitational force that draws them toward the Sun and the radiation-pressure force that pushes them away from the Sun. The gravitational force is proportional to the cube of the radius of a spherical dust particle because it is proportional to the mass and therefore to the volume $4\pi r^3/3$ of the particle. The radiation pressure is proportional to the square of the radius because it depends on the planar cross section of the particle. For large particles, the gravitational force is greater than the force from radiation pressure. For particles having radii less than about 0.2 μm, the radiation-pressure force is greater than the gravitational force. As a result, these particles are swept out of our solar system by sunlight.

Example 34.5 Pressure from a Laser Pointer

When giving presentations, many people use a laser pointer to direct the attention of the audience to information on a screen. If a 3.0-mW pointer creates a spot on a screen that is 2.0 mm in diameter, determine the radiation pressure on a screen that reflects 70% of the light that strikes it. The power 3.0 mW is a time-averaged value.

SOLUTION

Conceptualize Imagine the waves striking the screen and exerting a radiation pressure on it. The pressure should not be very large.

Categorize This problem involves a calculation of radiation pressure using an approach like that leading to Equation 34.28 or Equation 34.30, but it is complicated by the 70% reflection.

Analyze We begin by determining the magnitude of the beam's Poynting vector.

Divide the time-averaged power delivered via the electromagnetic wave by the cross-sectional area of the beam:

$$S_{\text{avg}} = \frac{(Power)_{\text{avg}}}{A} = \frac{(Power)_{\text{avg}}}{\pi r^2} = \frac{3.0 \times 10^{-3}\ \text{W}}{\pi\left(\dfrac{2.0 \times 10^{-3}\ \text{m}}{2}\right)^2} = 955\ \text{W/m}^2$$

Now let's determine the radiation pressure from the laser beam. Equation 34.30 indicates that a completely reflected beam would apply an average pressure of $P_{\text{avg}} = 2S_{\text{avg}}/c$. We can model the actual reflection as follows. Imagine that the surface absorbs the beam, resulting in pressure $P_{\text{avg}} = S_{\text{avg}}/c$. Then the surface emits the beam, resulting in additional pressure $P_{\text{avg}} = S_{\text{avg}}/c$. If the surface emits only a fraction f of the beam (so that f is the amount of the incident beam reflected), the pressure due to the emitted beam is $P_{\text{avg}} = fS_{\text{avg}}/c$.

Use this model to find the total pressure on the surface due to absorption and re-emission (reflection):

$$P_{\text{avg}} = \frac{S_{\text{avg}}}{c} + f\,\frac{S_{\text{avg}}}{c} = (1 + f)\,\frac{S_{\text{avg}}}{c}$$

continued

34.5 continued

Evaluate this pressure for a beam that is 70% reflected: $P_{avg} = (1 + 0.70)\dfrac{955\ \text{W/m}^2}{3.0 \times 10^8\ \text{m/s}} = \boxed{5.4 \times 10^{-6}\ \text{N/m}^2}$

Finalize The pressure has an extremely small value, as expected. (Recall from Section 14.2 that atmospheric pressure is approximately $10^5\ \text{N/m}^2$.) Consider the magnitude of the Poynting vector, $S_{avg} = 955\ \text{W/m}^2$. It is about the same as the intensity of sunlight at the Earth's surface. For this reason, it is not safe to shine the beam of a laser pointer into a person's eyes, which may be more dangerous than looking directly at the Sun.

WHAT IF? What if the laser pointer is moved twice as far away from the screen? Does that affect the radiation pressure on the screen?

Answer Because a laser beam is popularly represented as a beam of light with constant cross section, you might think that the intensity of radiation, and therefore the radiation pressure, is independent of distance from the screen. A laser beam, however, does not have a constant cross section at all distances from the source; rather, there is a small but measurable divergence of the beam. If the laser is moved farther away from the screen, the area of illumination on the screen increases, decreasing the intensity. In turn, the radiation pressure is reduced.

In addition, the doubled distance from the screen results in more loss of energy from the beam due to scattering from air molecules and dust particles as the light travels from the laser to the screen. This energy loss further reduces the radiation pressure on the screen.

34.6 Production of Electromagnetic Waves by an Antenna

Stationary charges and steady currents cannot produce electromagnetic waves. If the current in a wire changes with time, however, the wire emits electromagnetic waves. The fundamental mechanism responsible for this radiation is the acceleration of a charged particle. **Whenever a charged particle accelerates, energy is transferred away from the particle by electromagnetic radiation.**

Let's consider the production of electromagnetic waves by a *half-wave antenna.* In this arrangement, two conducting rods are connected to a source of alternating voltage (such as an *LC* oscillator) as shown in Figure 34.11. The length of each rod is equal to one-quarter the wavelength of the radiation emitted when the oscillator operates at frequency f. The oscillator forces charges to accelerate back and forth between the two rods. Figure 34.11 shows the configuration of the electric and magnetic fields at some instant when the current is upward. The separation of charges in the upper and lower portions of the antenna make the electric field lines resemble those of an electric dipole. (As a result, this type of antenna is sometimes called a *dipole antenna.*) Because these charges are continuously oscillating between the two rods, the antenna can be approximated by an oscillating electric dipole. The current representing the movement of charges between the ends of the antenna produces magnetic field lines forming concentric circles around the antenna that are perpendicular to the electric field lines at all points. The magnetic field is zero at all points along the axis of the antenna. Furthermore, $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are 90° out of phase in time; for example, the current is zero when the charges at the outer ends of the rods are at a maximum.

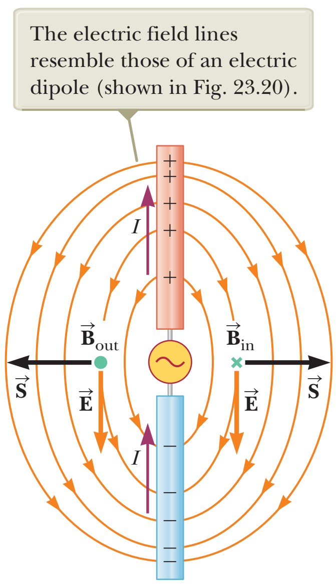


Figure 34.11 A half-wave antenna consists of two metal rods connected to an alternating voltage source. This diagram shows $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ at an arbitrary instant when the current is upward.

At the two points where the magnetic field is shown in Figure 34.11, the Poynting vector $\vec{\mathbf{S}}$ is directed radially outward, indicating that energy is flowing away from the antenna at this instant. At later times, the fields and the Poynting vector reverse direction as the current alternates. Because $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are 90° out of phase at points near the dipole, the net energy flow is zero. From this fact, you might conclude (incorrectly) that no energy is radiated by the dipole.

Energy is indeed radiated, however. Because the dipole fields fall off as $1/r^3$ (as shown in Example 23.6 for the electric field of a static dipole), they are negligible at great distances from the antenna. At these great distances, something else causes a type of radiation different from that close to the antenna. The source of this radiation is the continuous induction of an electric field by the time-varying magnetic field and the induction of a magnetic field by the time-varying electric field, predicted by Equations 34.6 and 34.7. The electric and magnetic fields produced in this manner are in phase with each other and vary as $1/r$. The result is an outward flow of energy at all times.

The angular dependence of the radiation intensity produced by a dipole antenna is shown in Figure 34.12. Notice that the intensity and the power radiated are a maximum in a plane that is perpendicular to the antenna and passing through its midpoint. Furthermore, the power radiated is zero along the antenna's axis. A mathematical solution to Maxwell's equations for the dipole antenna shows that the intensity of the radiation varies as $(\sin^2 \theta)/r^2$, where θ is measured from the axis of the antenna.

Electromagnetic waves can also induce currents in a receiving antenna. The response of a dipole receiving antenna at a given position is a maximum when the antenna axis is parallel to the electric field at that point and zero when the axis is perpendicular to the electric field.

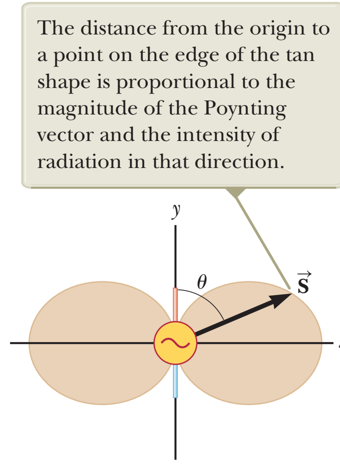


Figure 34.12 Angular dependence of the intensity of radiation produced by an oscillating electric dipole.

Quick Quiz 34.5 If the antenna in Figure 34.11 represents the source of a distant radio station, what would be the best orientation for your portable radio antenna located to the right of the figure? **(a)** up-down along the page **(b)** left-right along the page **(c)** perpendicular to the page

34.7 The Spectrum of Electromagnetic Waves

The various types of electromagnetic waves are listed in Figure 34.13 (page 1046), which shows the **electromagnetic spectrum.** Notice the wide ranges of frequencies and wavelengths. No sharp dividing point exists between one type of wave and the next. Remember that all forms of the various types of radiation are produced by the same phenomenon: acceleration of electric charges. The names given to the types of waves are simply a convenient way to describe the region of the spectrum in which they lie.

Radio waves, whose wavelengths range from more than 10^4 m to about 0.1 m, are the result of charges accelerating through conducting wires. They are generated by such electronic devices as LC oscillators and are used in radio and television communication systems.

Microwaves have wavelengths ranging from approximately 0.3 m to 10^{-4} m and are also generated by electronic devices. Because of their short wavelengths, they are well suited for radar systems and for studying the atomic and molecular properties of matter. Microwave ovens are an interesting domestic application of these waves. It has been suggested that solar energy could be harnessed by beaming microwaves to the Earth from a solar collector in space.

Infrared waves have wavelengths ranging from approximately 10^{-3} m to the longest wavelength of visible light, 7×10^{-7} m. These waves, produced by molecules and room-temperature objects, are readily absorbed by most materials. The infrared (IR) energy absorbed by a substance appears as internal energy because the energy agitates the object's atoms, increasing their vibrational or translational motion, which results in a temperature increase. Infrared radiation has practical and scientific applications in many areas, including physical therapy, IR photography, and vibrational spectroscopy.

Visible light, the most familiar form of electromagnetic waves, is the part of the electromagnetic spectrum the human eye can detect. Light is produced by the rearrangement of electrons in atoms and molecules. The various wavelengths of visible light, which correspond to different colors, range from red ($\lambda \approx 7 \times 10^{-7}$ m) to violet ($\lambda \approx 4 \times 10^{-7}$ m). The sensitivity of the human eye is a function of wavelength, being a maximum at a wavelength of about 5.5×10^{-7} m. With that in mind, why do you suppose tennis balls often have a yellow-green color? Table 34.1 provides

Pitfall Prevention 34.6

"Heat Rays" Infrared rays are often called "heat rays," but this terminology is a misnomer. Although infrared radiation is used to raise or maintain temperature as in the case of keeping food warm with "heat lamps" at a fast-food restaurant, all wavelengths of electromagnetic radiation carry energy that can cause the temperature of a system to increase. As an example, consider a potato baking in your microwave oven.

Table 34.1 Approximate Correspondence Between Wavelengths of Visible Light and Color

Wavelength Range (nm)	Color Description
400–430	Violet
430–485	Blue
485–560	Green
560–590	Yellow
590–625	Orange
625–700	Red

Note: The wavelength ranges here are approximate. Different people will describe colors differently.

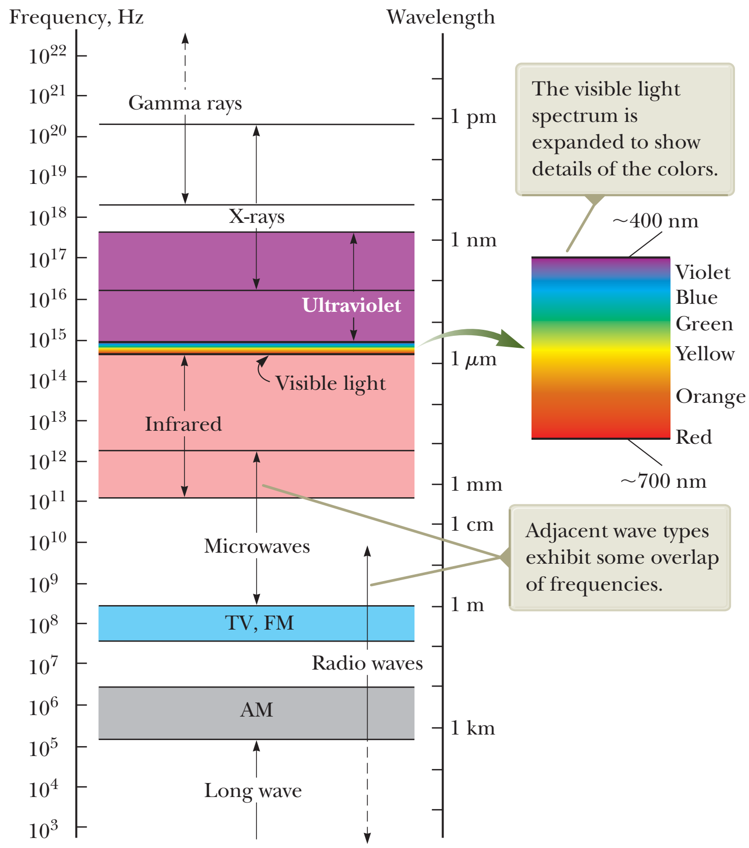


Figure 34.13 The electromagnetic spectrum.

approximate correspondences between the wavelength of visible light and the color assigned to it by humans. Light is the basis of the science of optics and optical instruments, to be discussed in Chapters 35 through 38.

Ultraviolet waves cover wavelengths ranging from approximately 4×10^{-7} m to 6×10^{-10} m. The Sun is an important source of ultraviolet (UV) light, which is the main cause of sunburn. Sunscreen lotions are transparent to visible light but absorb most UV light. The higher a sunscreen's solar protection factor, or SPF, the greater the percentage of UV light absorbed. Ultraviolet rays have also been implicated in the formation of cataracts, a clouding of the lens inside the eye.

Most of the UV light from the Sun is absorbed by ozone (O_3) molecules in the Earth's upper atmosphere, in a layer called the stratosphere. This ozone shield converts lethal high-energy UV radiation to IR radiation, which in turn warms the stratosphere.

Raymond A. Serway

Wearing sunglasses that do not block ultraviolet (UV) light is worse for your eyes than wearing no sunglasses at all. The lenses of any sunglasses absorb some visible light, thereby causing the wearer's pupils to dilate. If the glasses do not also block UV light, more damage may be done to the lens of the eye because of the dilated pupils. If you wear no sunglasses at all, your pupils are contracted, you squint, and much less UV light enters your eyes. High-quality sunglasses block nearly all the eye-damaging UV light.

X-rays have wavelengths in the range from approximately 10^{-8} m to 10^{-12} m. The most common source of x-rays is the stopping of high-energy electrons upon bombarding a metal target. X-rays are used as a diagnostic tool in medicine and as a treatment for certain forms of cancer. Because x-rays can damage or destroy living tissues and organisms, care must be taken to avoid unnecessary exposure or overexposure. X-rays are also used in the study of crystal structure because x-ray wavelengths are comparable to the atomic separation distances in solids (about 0.1 nm).

Gamma rays are electromagnetic waves emitted by radioactive nuclei and during certain nuclear reactions. High-energy gamma rays are a component of cosmic rays that enter the Earth's atmosphere from space. They have wavelengths ranging from approximately 10^{-10} m to less than 10^{-14} m. Gamma rays are highly penetrating and produce serious damage when absorbed by living tissues. Consequently, those working near such dangerous radiation must be protected with heavily absorbing materials such as thick layers of lead.

Quick Quiz 34.6 In many kitchens, a microwave oven is used to cook food. The frequency of the microwaves is on the order of 10^{10} Hz. Are the wavelengths of these microwaves on the order of **(a)** kilometers, **(b)** meters, **(c)** centimeters, or **(d)** micrometers?

Quick Quiz 34.7 A radio wave of frequency on the order of 10^5 Hz is used to carry a sound wave with a frequency on the order of 10^3 Hz. Is the wavelength of this radio wave on the order of **(a)** kilometers, **(b)** meters, **(c)** centimeters, or **(d)** micrometers?

Summary

Definitions

In a region of space in which there is a changing electric field, there is a **displacement current** defined as

$$I_d \equiv \epsilon_0 \frac{d\Phi_E}{dt} \quad \textbf{(34.1)}$$

where ϵ_0 is the permittivity of free space (see Section 23.3) and $\Phi_E = \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$ is the electric flux.

The rate at which energy passes through a unit area by electromagnetic radiation is described by the **Poynting vector $\vec{\mathbf{S}}$,** where

$$\vec{\mathbf{S}} \equiv \frac{1}{\mu_0} \vec{\mathbf{E}} \times \vec{\mathbf{B}} \quad \textbf{(34.22)}$$

Concepts and Principles

When used with the **Lorentz force law, $\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$, Maxwell's equations** describe all electromagnetic phenomena:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q}{\epsilon_0} \quad \textbf{(34.4)}$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0 \quad \textbf{(34.5)}$$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt} \quad \textbf{(34.6)}$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I + \epsilon_0\mu_0 \frac{d\Phi_E}{dt} \quad \textbf{(34.7)}$$

Electromagnetic waves, which are predicted by Maxwell's equations, have the following properties and are described by the following mathematical representations of the traveling wave model for electromagnetic waves.

- The electric field and the magnetic field each satisfy a wave equation. These two wave equations, which can be obtained from Maxwell's third and fourth equations, are

$$\frac{\partial^2 E}{\partial x^2} = \mu_0\epsilon_0 \frac{\partial^2 E}{\partial t^2} \quad \textbf{(34.15)}$$

$$\frac{\partial^2 B}{\partial x^2} = \mu_0\epsilon_0 \frac{\partial^2 B}{\partial t^2} \quad \textbf{(34.16)}$$

- The waves travel through a vacuum with the speed of light c, where

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}} \quad \textbf{(34.17)}$$

- Numerically, the speed of electromagnetic waves in a vacuum is 3.00×10^8 m/s.
- The wavelength and frequency of electromagnetic waves are related by

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{f} \quad \textbf{(34.20)}$$

- The electric and magnetic fields are perpendicular to each other and perpendicular to the direction of wave propagation.
- The instantaneous magnitudes of $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ in an electromagnetic wave are related by the expression

$$\frac{E}{B} = c \quad \textbf{(34.21)}$$

- Electromagnetic waves carry energy.
- Electromagnetic waves carry momentum.

continued

Because electromagnetic waves carry momentum, they exert pressure on surfaces. If an electromagnetic wave whose Poynting vector is $\vec{\mathbf{S}}$ is completely absorbed by a surface upon which it is normally incident, the radiation pressure on that surface is

$$P = \frac{S}{c} \quad \text{(complete absorption)} \tag{34.28}$$

If the surface totally reflects a normally incident wave, the pressure is doubled.

The electric and magnetic fields of a sinusoidal plane electromagnetic wave propagating in the positive x direction can be written as

$$E = E_{\max} \cos (kx - \omega t) \tag{34.18}$$

$$B = B_{\max} \cos (kx - \omega t) \tag{34.19}$$

where k is the angular wave number and ω is the angular frequency of the wave. These equations represent special solutions to the wave equations for E and B.

The average value of the Poynting vector for a plane electromagnetic wave has a magnitude

$$S_{\text{avg}} = \frac{E_{\max} B_{\max}}{2\mu_0} = \frac{E_{\max}^2}{2\mu_0 c} = \frac{cB_{\max}^2}{2\mu_0} \tag{34.24}$$

The intensity of a sinusoidal plane electromagnetic wave equals the average value of the Poynting vector taken over one or more cycles.

The electromagnetic spectrum includes waves covering a broad range of wavelengths, from long radio waves at more than 10^4 m to gamma rays at less than 10^{-14} m.

Objective Questions

1. denotes answer available in *Student Solutions Manual/Study Guide*

1. A spherical interplanetary grain of dust of radius 0.2 μm is at a distance r_1 from the Sun. The gravitational force exerted by the Sun on the grain just balances the force due to radiation pressure from the Sun's light. **(i)** Assume the grain is moved to a distance $2r_1$ from the Sun and released. At this location, what is the net force exerted on the grain? (a) toward the Sun (b) away from the Sun (c) zero (d) impossible to determine without knowing the mass of the grain **(ii)** Now assume the grain is moved back to its original location at r_1, compressed so that it crystallizes into a sphere with significantly higher density, and then released. In this situation, what is the net force exerted on the grain? Choose from the same possibilities as in part (i).

2. A small source radiates an electromagnetic wave with a single frequency into vacuum, equally in all directions. **(i)** As the wave moves, does its frequency (a) increase, (b) decrease, or (c) stay constant? Using the same choices, answer the same question about **(ii)** its wavelength, **(iii)** its speed, **(iv)** its intensity, and **(v)** the amplitude of its electric field.

3. A typical microwave oven operates at a frequency of 2.45 GHz. What is the wavelength associated with the electromagnetic waves in the oven? (a) 8.20 m (b) 12.2 cm (c) 1.20×10^8 m (d) 8.20×10^{-9} m (e) none of those answers

4. A student working with a transmitting apparatus like Heinrich Hertz's wishes to adjust the electrodes to generate electromagnetic waves with a frequency half as large as before. **(i)** How large should she make the effective capacitance of the pair of electrodes? (a) four times larger than before (b) two times larger than before (c) one-half as large as before (d) one-fourth as large as before (e) none of those answers **(ii)** After she makes the required adjustment, what will the wavelength of the transmitted wave be? Choose from the same possibilities as in part (i).

5. Assume you charge a comb by running it through your hair and then hold the comb next to a bar magnet. Do the electric and magnetic fields produced constitute an electromagnetic wave? (a) Yes they do, necessarily. (b) Yes they do because charged particles are moving inside the bar magnet. (c) They can, but only if the electric field of the comb and the magnetic field of the magnet are perpendicular. (d) They can, but only if both the comb and the magnet are moving. (e) They can, if either the comb or the magnet or both are accelerating.

6. Which of the following statements are true regarding electromagnetic waves traveling through a vacuum? More than one statement may be correct. (a) All waves have the same wavelength. (b) All waves have the same frequency. (c) All waves travel at 3.00×10^8 m/s. (d) The electric and magnetic fields associated with the waves are perpendicular to each other and to the direction of wave propagation. (e) The speed of the waves depends on their frequency.

7. A plane electromagnetic wave with a single frequency moves in vacuum in the positive x direction. Its amplitude is uniform over the yz plane. **(i)** As the wave moves, does its frequency (a) increase, (b) decrease, or (c) stay constant? Using the same choices, answer the same question about **(ii)** its wavelength, **(iii)** its speed, **(iv)** its intensity, and **(v)** the amplitude of its magnetic field.

8. Assume the amplitude of the electric field in a plane electromagnetic wave is E_1 and the amplitude of the magnetic field is B_1. The source of the wave is then adjusted so that the amplitude of the electric field doubles to become $2E_1$. **(i)** What happens to the amplitude of the magnetic field in this process? (a) It becomes four times larger. (b) It becomes two times larger. (c) It can stay constant. (d) It becomes one-half as large. (e) It becomes one-fourth as large. **(ii)** What happens to the intensity of the wave? Choose from the same possibilities as in part (i).

9. An electromagnetic wave with a peak magnetic field magnitude of 1.50×10^{-7} T has an associated peak electric field of what magnitude? (a) 0.500×10^{-15} N/C (b) 2.00×10^{-5} N/C (c) 2.20×10^{4} N/C (d) 45.0 N/C (e) 22.0 N/C

10. **(i)** Rank the following kinds of waves according to their wavelength ranges from those with the largest typical or average wavelength to the smallest, noting any cases of equality: (a) gamma rays (b) microwaves (c) radio waves (d) visible light (e) x-rays **(ii)** Rank the kinds of waves according to their frequencies from highest to lowest. **(iii)** Rank the kinds of waves according to their speeds in vacuum from fastest to slowest.

11. Consider an electromagnetic wave traveling in the positive y direction. The magnetic field associated with the wave at some location at some instant points in the negative x direction as shown in Figure OQ34.11. What is the direction of the electric field at this position and at this instant? (a) the positive x direction (b) the positive y direction (c) the positive z direction (d) the negative z direction (e) the negative y direction

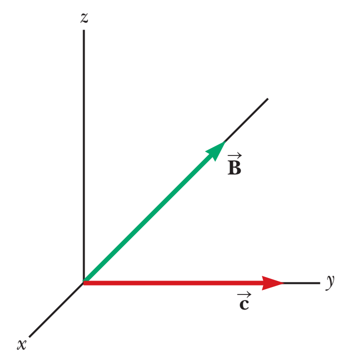


Figure OQ34.11

Conceptual Questions

$\boxed{1.}$ denotes answer available in *Student Solutions Manual/Study Guide*

$\boxed{1.}$ Suppose a creature from another planet has eyes that are sensitive to infrared radiation. Describe what the alien would see if it looked around your library. In particular, what would appear bright and what would appear dim?

2. For a given incident energy of an electromagnetic wave, why is the radiation pressure on a perfectly reflecting surface twice as great as that on a perfectly absorbing surface?

$\boxed{3.}$ Radio stations often advertise "instant news." If that means you can hear the news the instant the radio announcer speaks it, is the claim true? What approximate time interval is required for a message to travel from Maine to California by radio waves? (Assume the waves can be detected at this range.)

4. List at least three differences between sound waves and light waves.

5. If a high-frequency current exists in a solenoid containing a metallic core, the core becomes warm due to induction. Explain why the material rises in temperature in this situation.

6. When light (or other electromagnetic radiation) travels across a given region, (a) what is it that oscillates? (b) What is it that is transported?

7. Why should an infrared photograph of a person look different from a photograph taken with visible light?

8. Do Maxwell's equations allow for the existence of magnetic monopoles? Explain.

9. Despite the advent of digital television, some viewers still use "rabbit ears" atop their sets (Fig. CQ34.9) instead of purchasing cable television service or satellite dishes. Certain orientations of the receiving antenna on a television set give better reception than others. Furthermore, the best orientation varies from station to station. Explain.

Figure CQ34.9 Conceptual Question 9 and Problem 78.

$\boxed{10.}$ What does a radio wave do to the charges in the receiving antenna to provide a signal for your car radio?

11. Describe the physical significance of the Poynting vector.

12. An empty plastic or glass dish being removed from a microwave oven can be cool to the touch, even when food on an adjoining dish is hot. How is this phenomenon possible?

13. What new concept did Maxwell's generalized form of Ampère's law include?

Problems

ENHANCED WebAssign The problems found in this chapter may be assigned online in Enhanced WebAssign

1. straightforward; **2.** intermediate; **3.** challenging

1. full solution available in the *Student Solutions Manual/Study Guide*

AMT Analysis Model tutorial available in Enhanced WebAssign

GP Guided Problem

M Master It tutorial available in Enhanced WebAssign

W Watch It video solution available in Enhanced WebAssign

Section 34.1 Displacement Current and the General Form of Ampère's Law

1. Consider the situation shown in Figure P34.1. An electric field of 300 V/m is confined to a circular area d = 10.0 cm in diameter and directed outward perpendicular to the plane of the figure. If the field is increasing at a rate of 20.0 V/m · s, what are (a) the direction and (b) the magnitude of the magnetic field at the point P, r = 15.0 cm from the center of the circle?

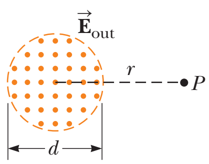


Figure P34.1

2. W A 0.200-A current is charging a capacitor that has circular plates 10.0 cm in radius. If the plate separation is 4.00 mm, (a) what is the time rate of increase of electric field between the plates? (b) What is the magnetic field between the plates 5.00 cm from the center?

3. M A 0.100-A current is charging a capacitor that has square plates 5.00 cm on each side. The plate separation is 4.00 mm. Find (a) the time rate of change of electric flux between the plates and (b) the displacement current between the plates.

Section 34.2 Maxwell's Equations and Hertz's Discoveries

4. AMT W An electron moves through a uniform electric field $\vec{\mathbf{E}} = (2.50\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}})$ V/m and a uniform magnetic field $\vec{\mathbf{B}} = 0.400\hat{\mathbf{k}}$ T. Determine the acceleration of the electron when it has a velocity $\vec{\mathbf{v}} = 10.0\hat{\mathbf{i}}$ m/s.

5. M A proton moves through a region containing a uniform electric field given by $\vec{\mathbf{E}} = 50.0\hat{\mathbf{j}}$ V/m and a uniform magnetic field $\vec{\mathbf{B}} = (0.200\hat{\mathbf{i}} + 0.300\hat{\mathbf{j}} + 0.400\hat{\mathbf{k}})$ T. Determine the acceleration of the proton when it has a velocity $\vec{\mathbf{v}} = 200\hat{\mathbf{i}}$ m/s.

6. A very long, thin rod carries electric charge with the linear density 35.0 nC/m. It lies along the x axis and moves in the x direction at a speed of 1.50×10^7 m/s. (a) Find the electric field the rod creates at the point (x = 0, y = 20.0 cm, z = 0). (b) Find the magnetic field it creates at the same point. (c) Find the force exerted on an electron at this point, moving with a velocity of $(2.40 \times 10^8)\hat{\mathbf{i}}$ m/s.

Section 34.3 Plane Electromagnetic Waves

Note: Assume the medium is vacuum unless specified otherwise.

7. Suppose you are located 180 m from a radio transmitter. (a) How many wavelengths are you from the transmitter if the station calls itself 1150 AM? (The AM band frequencies are in kilohertz.) (b) What if this station is 98.1 FM? (The FM band frequencies are in megahertz.)

8. A diathermy machine, used in physiotherapy, generates electromagnetic radiation that gives the effect of "deep heat" when absorbed in tissue. One assigned frequency for diathermy is 27.33 MHz. What is the wavelength of this radiation?

9. The distance to the North Star, Polaris, is approximately 6.44×10^{18} m. (a) If Polaris were to burn out today, how many years from now would we see it disappear? (b) What time interval is required for sunlight to reach the Earth? (c) What time interval is required for a microwave signal to travel from the Earth to the Moon and back?

10. The red light emitted by a helium–neon laser has a wavelength of 632.8 nm. What is the frequency of the light waves?

11. **Review.** A standing-wave pattern is set up by radio waves between two metal sheets 2.00 m apart, which is the shortest distance between the plates that produces a standing-wave pattern. What is the frequency of the radio waves?

12. W An electromagnetic wave in vacuum has an electric field amplitude of 220 V/m. Calculate the amplitude of the corresponding magnetic field.

13. The speed of an electromagnetic wave traveling in a transparent nonmagnetic substance is $v = 1/\sqrt{\kappa\mu_0\epsilon_0}$, where κ is the dielectric constant of the substance. Determine the speed of light in water, which has a dielectric constant of 1.78 at optical frequencies.

14. A radar pulse returns to the transmitter–receiver after a total travel time of 4.00×10^{-4} s. How far away is the object that reflected the wave?

15. M Figure P34.15 shows a plane electromagnetic sinusoidal wave propagating in the x direction. Suppose the wavelength is 50.0 m and the electric field vibrates in the xy plane with an amplitude of 22.0 V/m. Calculate

(a) the frequency of the wave and (b) the magnetic field $\vec{\mathbf{B}}$ when the electric field has its maximum value in the negative y direction. (c) Write an expression for $\vec{\mathbf{B}}$ with the correct unit vector, with numerical values for B_{max}, k, and ω, and with its magnitude in the form

$$B = B_{max} \cos (kx - \omega t)$$

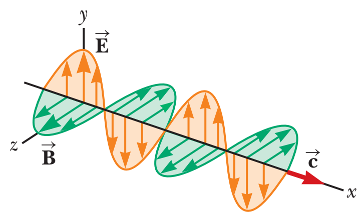


Figure P34.15 Problems 15 and 70.

16. Verify by substitution that the following equations are solutions to Equations 34.15 and 34.16, respectively:

$$E = E_{max} \cos (kx - \omega t)$$

$$B = B_{max} \cos (kx - \omega t)$$

17. AMT **Review.** A microwave oven is powered by a magnetron, an electronic device that generates electromagnetic waves of frequency 2.45 GHz. The microwaves enter the oven and are reflected by the walls. The standing-wave pattern produced in the oven can cook food unevenly, with hot spots in the food at antinodes and cool spots at nodes, so a turntable is often used to rotate the food and distribute the energy. If a microwave oven intended for use with a turntable is instead used with a cooking dish in a fixed position, the antinodes can appear as burn marks on foods such as carrot strips or cheese. The separation distance between the burns is measured to be 6 cm $\pm$ 5%. From these data, calculate the speed of the microwaves.

18. *Why is the following situation impossible?* An electromagnetic wave travels through empty space with electric and magnetic fields described by

$$E = 9.00 \times 10^3 \cos [(9.00 \times 10^6)x - (3.00 \times 10^{15})t]$$

$$B = 3.00 \times 10^{-5} \cos [(9.00 \times 10^6)x - (3.00 \times 10^{15})t]$$

where all numerical values and variables are in SI units.

19. M In SI units, the electric field in an electromagnetic wave is described by

$$E_y = 100 \sin (1.00 \times 10^7 x - \omega t)$$

Find (a) the amplitude of the corresponding magnetic field oscillations, (b) the wavelength λ, and (c) the frequency f.

Section 34.4 Energy Carried by Electromagnetic Waves

20. At what distance from the Sun is the intensity of sunlight three times the value at the Earth? (The average Earth–Sun separation is 1.496×10^{11} m.)

21. W If the intensity of sunlight at the Earth's surface under a fairly clear sky is 1 000 W/m^2, how much electromagnetic energy per cubic meter is contained in sunlight?

22. The power of sunlight reaching each square meter of the Earth's surface on a clear day in the tropics is close to 1 000 W. On a winter day in Manitoba, the power concentration of sunlight can be 100 W/m^2. Many human activities are described by a power per unit area on the order of 10^2 W/m^2 or less. (a) Consider, for example, a family of four paying $66 to the electric company every 30 days for 600 kWh of energy carried by electrical transmission to their house, which has floor dimensions of 13.0 m by 9.50 m. Compute the power per unit area used by the family. (b) Consider a car 2.10 m wide and 4.90 m long traveling at 55.0 mi/h using gasoline having "heat of combustion" 44.0 MJ/kg with fuel economy 25.0 mi/gal. One gallon of gasoline has a mass of 2.54 kg. Find the power per unit area used by the car. (c) Explain why direct use of solar energy is not practical for running a conventional automobile. (d) What are some uses of solar energy that are more practical?

23. M A community plans to build a facility to convert solar radiation to electrical power. The community requires 1.00 MW of power, and the system to be installed has an efficiency of 30.0% (that is, 30.0% of the solar energy incident on the surface is converted to useful energy that can power the community). Assuming sunlight has a constant intensity of 1 000 W/m^2, what must be the effective area of a perfectly absorbing surface used in such an installation?

24. In a region of free space, the electric field at an instant of time is $\vec{\mathbf{E}} = (80.0\hat{\mathbf{i}} + 32.0\hat{\mathbf{j}} - 64.0\hat{\mathbf{k}})$ N/C and the magnetic field is $\vec{\mathbf{B}} = (0.200\hat{\mathbf{i}} + 0.080\,0\hat{\mathbf{j}} + 0.290\hat{\mathbf{k}})\,\mu$T. (a) Show that the two fields are perpendicular to each other. (b) Determine the Poynting vector for these fields.

25. When a high-power laser is used in the Earth's atmosphere, the electric field associated with the laser beam can ionize the air, turning it into a conducting plasma that reflects the laser light. In dry air at 0°C and 1 atm, electric breakdown occurs for fields with amplitudes above about 3.00 MV/m. (a) What laser beam intensity will produce such a field? (b) At this maximum intensity, what power can be delivered in a cylindrical beam of diameter 5.00 mm?

26. **Review.** Model the electromagnetic wave in a microwave oven as a plane traveling wave moving to the left, with an intensity of 25.0 kW/m^2. An oven contains two cubical containers of small mass, each full of water. One has an edge length of 6.00 cm, and the other, 12.0 cm. Energy falls perpendicularly on one face of each container. The water in the smaller container absorbs 70.0% of the energy that falls on it. The water in the larger container absorbs 91.0%. That is, the fraction 0.300 of the incoming microwave energy passes through a 6.00-cm thickness of water, and the fraction (0.300)(0.300) = 0.090 passes through a 12.0-cm thickness. Assume a negligible amount of energy leaves either container by heat. Find the temperature change of the water in each container over a time interval of 480 s.

27. W High-power lasers in factories are used to cut through cloth and metal (Fig. P34.27). One such laser has a beam diameter of 1.00 mm and generates an electric field having an amplitude of 0.700 MV/m at the target. Find (a) the amplitude of the magnetic field produced, (b) the intensity of the laser, and (c) the power delivered by the laser.

Philippe Plailly/SPL/Photo Researchers, Inc.

Figure P34.27

28. Consider a bright star in our night sky. Assume its distance from the Earth is 20.0 light-years (ly) and its power output is 4.00×10^{28} W, about 100 times that of the Sun. (a) Find the intensity of the starlight at the Earth. (b) Find the power of the starlight the Earth intercepts. One light-year is the distance traveled by light through a vacuum in one year.

29. M What is the average magnitude of the Poynting vector 5.00 mi from a radio transmitter broadcasting isotropically (equally in all directions) with an average power of 250 kW?

30. Assuming the antenna of a 10.0-kW radio station radiates spherical electromagnetic waves, (a) compute the maximum value of the magnetic field 5.00 km from the antenna and (b) state how this value compares with the surface magnetic field of the Earth.

31. W **Review.** An AM radio station broadcasts isotropically (equally in all directions) with an average power of 4.00 kW. A receiving antenna 65.0 cm long is at a location 4.00 mi from the transmitter. Compute the amplitude of the emf that is induced by this signal between the ends of the receiving antenna.

32. At what distance from a 100-W electromagnetic wave point source does $E_{max} = 15.0$ V/m?

33. The filament of an incandescent lamp has a 150-Ω resistance and carries a direct current of 1.00 A. The filament is 8.00 cm long and 0.900 mm in radius. (a) Calculate the Poynting vector at the surface of the filament, associated with the static electric field producing the current and the current's static magnetic field. (b) Find the magnitude of the static electric and magnetic fields at the surface of the filament.

34. At one location on the Earth, the rms value of the magnetic field caused by solar radiation is 1.80 μT. From this value, calculate (a) the rms electric field due to solar radiation, (b) the average energy density of the solar component of electromagnetic radiation at this location, and (c) the average magnitude of the Poynting vector for the Sun's radiation.

Section 34.5 Momentum and Radiation Pressure

35. A 25.0-mW laser beam of diameter 2.00 mm is reflected at normal incidence by a perfectly reflecting mirror. Calculate the radiation pressure on the mirror.

36. A radio wave transmits 25.0 W/m^2 of power per unit area. A flat surface of area A is perpendicular to the direction of propagation of the wave. Assuming the surface is a perfect absorber, calculate the radiation pressure on it.

37. M A 15.0-mW helium–neon laser emits a beam of circular cross section with a diameter of 2.00 mm. (a) Find the maximum electric field in the beam. (b) What total energy is contained in a 1.00-m length of the beam? (c) Find the momentum carried by a 1.00-m length of the beam.

38. A helium–neon laser emits a beam of circular cross section with a radius r and a power P. (a) Find the maximum electric field in the beam. (b) What total energy is contained in a length ℓ of the beam? (c) Find the momentum carried by a length ℓ of the beam.

39. AMT A uniform circular disk of mass $m = 24.0$ g and radius $r = 40.0$ cm hangs vertically from a fixed, frictionless, horizontal hinge at a point on its circumference as shown in Figure P34.39a. A beam of electromagnetic radiation with intensity 10.0 MW/m^2 is incident on the disk in a direction perpendicular to its surface. The disk is perfectly absorbing, and the resulting radiation pressure makes the disk rotate. Assuming the radiation is *always* perpendicular to the surface of the disk, find the angle θ through which the disk rotates from the vertical as it reaches its new equilibrium position shown in Figure 34.39b.

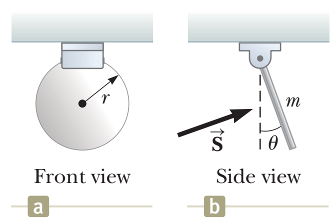


Figure P34.39

40. The intensity of sunlight at the Earth's distance from the Sun is 1 370 W/m^2. Assume the Earth absorbs all the sunlight incident upon it. (a) Find the total force the Sun exerts on the Earth due to radiation pressure. (b) Explain how this force compares with the Sun's gravitational attraction.

41. A plane electromagnetic wave of intensity 6.00 W/m^2, moving in the x direction, strikes a small perfectly reflecting pocket mirror, of area 40.0 cm^2, held in the yz plane. (a) What momentum does the wave transfer to the mirror each second? (b) Find the force the wave exerts on the mirror. (c) Explain the relationship between the answers to parts (a) and (b).

42. Assume the intensity of solar radiation incident on the upper atmosphere of the Earth is 1 370 W/m^2 and use data from Table 13.2 as necessary. Determine (a) the intensity of solar radiation incident on Mars, (b) the total power incident on Mars, and (c) the radiation force that acts on that planet if it absorbs nearly all the light. (d) State how this force compares with the gravitational attraction exerted by the Sun on Mars. (e) Compare the ratio of the gravitational force to the light-pressure force exerted on the Earth and the ratio of these forces exerted on Mars, found in part (d).

43. AMT W A possible means of space flight is to place a perfectly reflecting aluminized sheet into orbit around the Earth and then use the light from the Sun to push this "solar sail." Suppose a sail of area $A = 6.00 \times 10^5$ m^2 and mass $m = 6.00 \times 10^3$ kg is placed in orbit facing the Sun. Ignore all gravitational effects and assume a solar intensity of 1 370 W/m^2. (a) What force is exerted on the sail? (b) What is the sail's acceleration? (c) Assuming the acceleration calculated in part (b) remains constant, find the time interval required for the sail to reach the Moon, 3.84×10^8 m away, starting from rest at the Earth.

Section 34.6 Production of Electromagnetic Waves by an Antenna

44. Extremely low-frequency (ELF) waves that can penetrate the oceans are the only practical means of communicating with distant submarines. (a) Calculate the length of a quarter-wavelength antenna for a transmitter generating ELF waves of frequency 75.0 Hz into air. (b) How practical is this means of communication?

45. A Marconi antenna, used by most AM radio stations, consists of the top half of a Hertz antenna (also known as a half-wave antenna because its length is $\lambda/2$). The lower end of this Marconi (quarter-wave) antenna is connected to Earth ground, and the ground itself serves as the missing lower half. What are the heights of the Marconi antennas for radio stations broadcasting at (a) 560 kHz and (b) 1 600 kHz?

46. A large, flat sheet carries a uniformly distributed electric current with current per unit width J_s. This current creates a magnetic field on both sides of the sheet, parallel to the sheet and perpendicular to the current, with magnitude $B = \frac{1}{2}\mu_0 J_s$. If the current is in the y direction and oscillates in time according to

$$J_{\max}(\cos \omega t)\hat{\mathbf{j}} = J_{\max}[\cos(-\omega t)]\hat{\mathbf{j}}$$

the sheet radiates an electromagnetic wave. Figure P34.46 shows such a wave emitted from one point on the sheet chosen to be the origin. Such electromagnetic waves are emitted from all points on the sheet. The magnetic field of the wave to the right of the sheet is described by the wave function

$$\vec{\mathbf{B}} = \tfrac{1}{2}\mu_0 J_{\max}[\cos(kx - \omega t)]\hat{\mathbf{k}}$$

(a) Find the wave function for the electric field of the wave to the right of the sheet. (b) Find the Poynting vector as a function of x and t. (c) Find the intensity of the wave. (d) **What If?** If the sheet is to emit radiation in each direction (normal to the plane of the sheet) with intensity 570 W/m^2, what maximum value of sinusoidal current density is required?

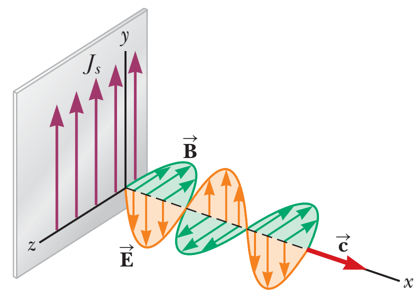


Figure P34.46

47. **Review.** Accelerating charges radiate electromagnetic waves. Calculate the wavelength of radiation produced by a proton in a cyclotron with a magnetic field of 0.350 T.

48. **Review.** Accelerating charges radiate electromagnetic waves. Calculate the wavelength of radiation produced by a proton of mass m_p moving in a circular path perpendicular to a magnetic field of magnitude B.

49. Two vertical radio-transmitting antennas are separated by half the broadcast wavelength and are driven in phase with each other. In what horizontal directions are (a) the strongest and (b) the weakest signals radiated?

Section 34.7 The Spectrum of Electromagnetic Waves

50. W Compute an order-of-magnitude estimate for the frequency of an electromagnetic wave with wavelength equal to (a) your height and (b) the thickness of a sheet of paper. How is each wave classified on the electromagnetic spectrum?

51. What are the wavelengths of electromagnetic waves in free space that have frequencies of (a) 5.00×10^{19} Hz and (b) 4.00×10^9 Hz?

52. An important news announcement is transmitted by radio waves to people sitting next to their radios 100 km from the station and by sound waves to people sitting across the newsroom 3.00 m from the newscaster. Taking the speed of sound in air to be 343 m/s, who receives the news first? Explain.

53. In addition to cable and satellite broadcasts, television stations still use VHF and UHF bands for digitally broadcasting their signals. Twelve VHF television channels (channels 2 through 13) lie in the range of frequencies between 54.0 MHz and 216 MHz. Each channel is assigned a width of 6.00 MHz, with the two ranges 72.0–76.0 MHz and 88.0–174 MHz reserved for non-TV purposes. (Channel 2, for example, lies between 54.0 and 60.0 MHz.) Calculate the broadcast wavelength range for (a) channel 4, (b) channel 6, and (c) channel 8.

Additional Problems

54. Classify waves with frequencies of 2 Hz, 2 kHz, 2 MHz, 2 GHz, 2 THz, 2 PHz, 2 EHz, 2 ZHz, and 2 YHz on the electromagnetic spectrum. Classify waves with wavelengths of 2 km, 2 m, 2 mm, 2 μm, 2 nm, 2 pm, 2 fm, and 2 am.

55. Assume the intensity of solar radiation incident on the cloud tops of the Earth is 1 370 W/m^2. (a) Taking the average Earth–Sun separation to be 1.496×10^{11} m, calculate the total power radiated by the Sun. Determine the maximum values of (b) the electric field and (c) the magnetic field in the sunlight at the Earth's location.

56. In 1965, Arno Penzias and Robert Wilson discovered the cosmic microwave radiation left over from the big bang expansion of the Universe. Suppose the energy density of this background radiation is 4.00×10^{-14} J/m^3. Determine the corresponding electric field amplitude.

57. The eye is most sensitive to light having a frequency of 5.45×10^{14} Hz, which is in the green-yellow region of the visible electromagnetic spectrum. What is the wavelength of this light?

58. Write expressions for the electric and magnetic fields of a sinusoidal plane electromagnetic wave having an electric field amplitude of 300 V/m and a frequency of 3.00 GHz and traveling in the positive x direction.

59. One goal of the Russian space program is to illuminate dark northern cities with sunlight reflected to the Earth from a 200-m diameter mirrored surface in orbit. Several smaller prototypes have already been constructed and put into orbit. (a) Assume that sunlight with intensity 1 370 W/m^2 falls on the mirror nearly perpendicularly and that the atmosphere of the Earth allows 74.6% of the energy of sunlight to pass though it in clear weather. What is the power received by a city when the space mirror is reflecting light to it? (b) The plan is for the reflected sunlight to cover a circle of diameter 8.00 km. What is the intensity of light (the average magnitude of the Poynting vector) received by the city? (c) This intensity is what percentage of the vertical component of sunlight at St. Petersburg in January, when the sun reaches an angle of 7.00° above the horizon at noon?

60. A microwave source produces pulses of 20.0-GHz radiation, with each pulse lasting 1.00 ns. A parabolic reflector with a face area of radius 6.00 cm is used to focus the microwaves into a parallel beam of radiation as shown in Figure P34.60. The average power during each pulse is 25.0 kW. (a) What is the wavelength of these microwaves? (b) What is the total energy contained in each pulse? (c) Compute the average energy density inside each pulse. (d) Determine the amplitude of the electric and magnetic fields in these microwaves. (e) Assuming that this pulsed beam strikes an absorbing surface, compute the force exerted on the surface during the 1.00-ns duration of each pulse.

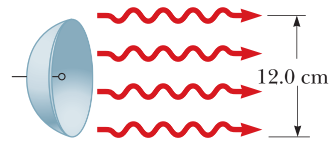


Figure P34.60

61. The intensity of solar radiation at the top of the Earth's atmosphere is 1 370 W/m^2. Assuming 60% of the incoming solar energy reaches the Earth's surface and you absorb 50% of the incident energy, make an order-of-magnitude estimate of the amount of solar energy you absorb if you sunbathe for 60 minutes.

62. Two handheld radio transceivers with dipole antennas are separated by a large, fixed distance. If the transmitting antenna is vertical, what fraction of the maximum received power will appear in the receiving antenna when it is inclined from the vertical (a) by 15.0°? (b) By 45.0°? (c) By 90.0°?

63. AMT Consider a small, spherical particle of radius r located in space a distance $R = 3.75 \times 10^{11}$ m from the Sun. Assume the particle has a perfectly absorbing surface and a mass density of $\rho = 1.50$ g/cm^3. Use $S = 214$ W/m^2 as the value of the solar intensity at the location of the particle. Calculate the value of r for which the particle is in equilibrium between the gravitational force and the force exerted by solar radiation.

64. Consider a small, spherical particle of radius r located in space a distance R from the Sun, of mass M_S. Assume the particle has a perfectly absorbing surface and a mass density ρ. The value of the solar intensity at the particle's location is S. Calculate the value of r for which the particle is in equilibrium between the gravitational force and the force exerted by solar radiation. Your answer should be in terms of S, R, ρ, and other constants.

65. M A dish antenna having a diameter of 20.0 m receives (at normal incidence) a radio signal from a distant source as shown in Figure P34.65. The radio signal is a continuous sinusoidal wave with amplitude $E_{max} = 0.200$ μV/m.

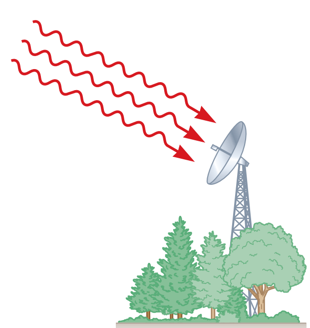


Figure P34.65

Assume the antenna absorbs all the radiation that falls on the dish. (a) What is the amplitude of the magnetic field in this wave? (b) What is the intensity of the radiation received by this antenna? (c) What is the power received by the antenna? (d) What force is exerted by the radio waves on the antenna?

66. The Earth reflects approximately 38.0% of the incident sunlight from its clouds and surface. (a) Given that the intensity of solar radiation at the top of the atmosphere is 1 370 W/m^2, find the radiation pressure on the Earth, in pascals, at the location where the Sun is straight overhead. (b) State how this quantity compares with normal atmospheric pressure at the Earth's surface, which is 101 kPa.

67. **Review.** A 1.00-m-diameter circular mirror focuses the Sun's rays onto a circular absorbing plate 2.00 cm in radius, which holds a can containing 1.00 L of water at 20.0°C. (a) If the solar intensity is 1.00 kW/m^2, what is the intensity on the absorbing plate? At the plate, what are the maximum magnitudes of the fields (b) $\vec{\mathbf{E}}$ and (c) $\vec{\mathbf{B}}$? (d) If 40.0% of the energy is absorbed, what time interval is required to bring the water to its boiling point?

68. (a) A stationary charged particle at the origin creates an electric flux of 487 N · m^2/C through any closed surface surrounding the charge. Find the electric field it creates in the empty space around it as a function of radial distance r away from the particle. (b) A small source at the origin emits an electromagnetic wave with a single frequency into vacuum, equally in all directions, with power 25.0 W. Find the electric field amplitude as a function of radial distance away from the source. (c) At what distance is the amplitude of the electric field in the wave equal to 3.00 MV/m, representing the dielectric strength of air? (d) As the distance from the source doubles, what happens to the field amplitude? (e) State how the behavior shown in part (d) compares with the behavior of the field in part (a).

69. **Review.** (a) A homeowner has a solar water heater installed on the roof of his house (Fig. P34.69). The heater is a flat, closed box with excellent thermal insulation. Its interior is painted black, and its front face is made of insulating glass. Its emissivity for visible light is 0.900, and its emissivity for infrared light is 0.700. Light from the noontime Sun is incident perpendicular to the glass with an intensity of 1 000 W/m^2, and no water enters or leaves the box. Find the steady-state temperature of the box's interior. (b) **What If?** The homeowner builds an identical box with no water tubes. It lies flat on the ground in front of the house. He uses it as a cold frame, where he plants seeds in early spring. Assuming the same noontime Sun is at an elevation angle of 50.0°, find the steady-state temperature of the interior of the box when its ventilation slots are tightly closed.

easyshoot/Shutterstock.com

Figure P34.69

70. GP You may wish to review Sections 16.5 and 17.3 on the transport of energy by string waves and sound. Figure P34.15 is a graphical representation of an electromagnetic wave moving in the x direction. We wish to find an expression for the intensity of this wave by means of a different process from that by which Equation 34.24 was generated. (a) Sketch a graph of the electric field in this wave at the instant $t = 0$, letting your flat paper represent the xy plane. (b) Compute the energy density u_E in the electric field as a function of x at the instant $t = 0$. (c) Compute the energy density in the magnetic field u_B as a function of x at that instant. (d) Find the total energy density u as a function of x, expressed in terms of only the electric field amplitude. (e) The energy in a "shoebox" of length λ and frontal area A is $E_\lambda = \int_0^\lambda uA\,dx$. (The symbol E_λ for energy in a wavelength imitates the notation of Section 16.5.) Perform the integration to compute the amount of this energy in terms of A, λ, $E_{\max}$, and universal constants. (f) We may think of the energy transport by the whole wave as a series of these shoeboxes going past as if carried on a conveyor belt. Each shoebox passes by a point in a time interval defined as the period $T = 1/f$ of the wave. Find the power the wave carries through area A. (g) The intensity of the wave is the power per unit area through which the wave passes. Compute this intensity in terms of $E_{\max}$ and universal constants. (h) Explain how your result compares with that given in Equation 34.24.

71. Lasers have been used to suspend spherical glass beads in the Earth's gravitational field. (a) A black bead has a radius of 0.500 mm and a density of 0.200 g/cm^3. Determine the radiation intensity needed to support the bead. (b) What is the minimum power required for this laser?

72. Lasers have been used to suspend spherical glass beads in the Earth's gravitational field. (a) A black bead has radius r and density ρ. Determine the radiation intensity needed to support the bead. (b) What is the minimum power required for this laser?

73. **Review.** A 5.50-kg black cat and her four black kittens, each with mass 0.800 kg, sleep snuggled together on a mat on a cool night, with their bodies forming a hemisphere. Assume the hemisphere has a surface temperature of 31.0°C, an emissivity of 0.970, and a uniform density of 990 kg/m^3. Find (a) the radius of the hemisphere, (b) the area of its curved surface, (c) the radiated power emitted by the cats at their curved surface, and (d) the intensity of radiation at this surface. You may think of the emitted electromagnetic wave as having a single predominant frequency. Find (e) the amplitude of the electric field in the electromagnetic wave just outside the surface of the cozy pile and (f) the amplitude of the magnetic field. (g) **What If?** The next night, the kittens all sleep alone, curling up into separate hemispheres like their mother. Find the total radiated power of the family. (For simplicity, ignore the cats' absorption of radiation from the environment.)

74. The electromagnetic power radiated by a nonrelativistic particle with charge q moving with acceleration a is

$$P = \frac{q^2 a^2}{6\pi\epsilon_0 c^3}$$

where ϵ_0 is the permittivity of free space (also called the permittivity of vacuum) and c is the speed of light in vacuum. (a) Show that the right side of this equation has units of watts. An electron is placed in a constant electric field of magnitude 100 N/C. Determine (b) the acceleration of the electron and (c) the electromagnetic power radiated by this electron. (d) **What If?** If a proton is placed in a cyclotron with a radius of 0.500 m and a magnetic field of magnitude 0.350 T, what electromagnetic power does this proton radiate just before leaving the cyclotron?

75. **Review.** Gliese 581c is the first Earth-like extrasolar terrestrial planet discovered. Its parent star, Gliese 581, is a red dwarf that radiates electromagnetic waves with power 5.00×10^{24} W, which is only 1.30% of the power of the Sun. Assume the emissivity of the planet is equal for infrared and for visible light and the planet has a uniform surface temperature. Identify (a) the projected area over which the planet absorbs light from Gliese 581 and (b) the radiating area of the planet. (c) If an average temperature of 287 K is necessary for life to exist on Gliese 581c, what should the radius of the planet's orbit be?

Challenge Problems

76. A plane electromagnetic wave varies sinusoidally at 90.0 MHz as it travels through vacuum along the positive x direction. The peak value of the electric field is 2.00 mV/m, and it is directed along the positive y direction. Find (a) the wavelength, (b) the period, and (c) the maximum value of the magnetic field. (d) Write expressions in SI units for the space and time variations of the electric field and of the magnetic field. Include both numerical values and unit vectors to indicate directions. (e) Find the average power per unit area this wave carries through space. (f) Find the average energy density in the radiation (in joules per cubic meter). (g) What radiation pressure would this wave exert upon a perfectly reflecting surface at normal incidence?

77. A linearly polarized microwave of wavelength 1.50 cm is directed along the positive x axis. The electric field vector has a maximum value of 175 V/m and vibrates in the xy plane. Assuming the magnetic field component of the wave can be written in the form $B = B_{max} \sin (kx - \omega t)$, give values for (a) B_{max}, (b) k, and (c) ω. (d) Determine in which plane the magnetic field vector vibrates. (e) Calculate the average value of the Poynting vector for this wave. (f) If this wave were directed at normal incidence onto a perfectly reflecting sheet, what radiation pressure would it exert? (g) What acceleration would be imparted to a 500-g sheet (perfectly reflecting and at normal incidence) with dimensions of 1.00 m × 0.750 m?

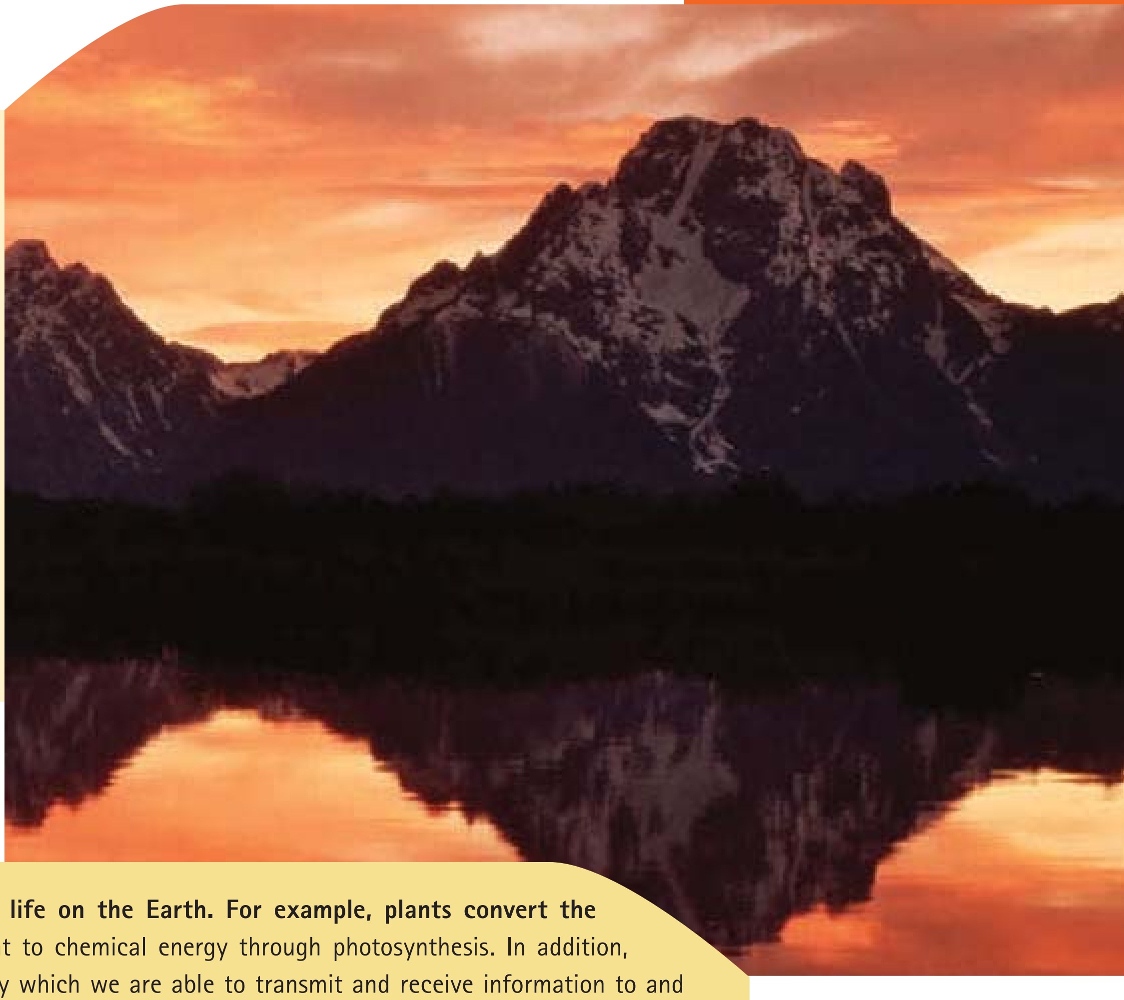
78. **Review.** In the absence of cable input or a satellite dish, a television set can use a dipole-receiving antenna for VHF channels and a loop antenna for UHF channels. In Figure CQ34.9, the "rabbit ears" form the VHF antenna and the smaller loop of wire is the UHF antenna. The UHF antenna produces an emf from the changing magnetic flux through the loop. The television station broadcasts a signal with a frequency f, and the signal has an electric field amplitude E_{max} and a magnetic field amplitude B_{max} at the location of the receiving antenna. (a) Using Faraday's law, derive an expression for the amplitude of the emf that appears in a single-turn, circular loop antenna with a radius r that is small compared with the wavelength of the wave. (b) If the electric field in the signal points vertically, what orientation of the loop gives the best reception?

79. AMT **Review.** An astronaut, stranded in space 10.0 m from her spacecraft and at rest relative to it, has a mass (including equipment) of 110 kg. Because she has a 100-W flashlight that forms a directed beam, she considers using the beam as a photon rocket to propel herself continuously toward the spacecraft. (a) Calculate the time interval required for her to reach the spacecraft by this method. (b) **What If?** Suppose she throws the 3.00-kg flashlight in the direction away from the spacecraft instead. After being thrown, the flashlight moves at 12.0 m/s relative to the recoiling astronaut. After what time interval will the astronaut reach the spacecraft?

Light and Optics

PART

5



The Grand Tetons in western Wyoming are reflected in a smooth lake at sunset. The optical principles we study in this part of the book will explain the nature of the reflected image of the mountains and why the sky appears red. *(David Muench/Terra/Corbis)*

Light is basic to almost all life on the Earth. For example, plants convert the energy transferred by sunlight to chemical energy through photosynthesis. In addition, light is the principal means by which we are able to transmit and receive information to and from objects around us and throughout the Universe. Light is a form of electromagnetic radiation and represents energy transfer from the source to the observer.

Many phenomena in our everyday life depend on the properties of light. When you watch a television or view photos on a computer monitor, you are seeing millions of colors formed from combinations of only three colors that are physically on the screen: red, blue, and green. The blue color of the daytime sky is a result of the optical phenomenon of *scattering* of light by air molecules, as are the red and orange colors of sunrises and sunsets. You see your image in your bathroom mirror in the morning or the images of other cars in your rearview mirror when you are driving. These images result from *reflection* of light. If you wear glasses or contact lenses, you are depending on *refraction* of light for clear vision. The colors of a rainbow result from *dispersion* of light as it passes through raindrops hovering in the sky after a rainstorm. If you have ever seen the colored circles of the glory surrounding the shadow of your airplane on clouds as you fly above them, you are seeing an effect that results from *interference* of light. The phenomena mentioned here have been studied by scientists and are well understood.

In the introduction to Chapter 35, we discuss the dual nature of light. In some cases, it is best to model light as a stream of particles; in others, a wave model works better. Chapters 35 through 38 concentrate on those aspects of light that are best understood through the wave model of light. In Part 6, we will investigate the particle nature of light. ■

CHAPTER

35

The Nature of Light and the Principles of Ray Optics

This photograph of a rainbow shows the range of colors from red on the top to violet on the bottom. The appearance of the rainbow depends on three optical phenomena discussed in this chapter: reflection, refraction, and dispersion. The faint pastel-colored bows beneath the main rainbow are called supernumerary bows. They are formed by interference between rays of light leaving raindrops below those causing the main rainbow. *(John W. Jewett, Jr.)*

This first chapter on optics begins by introducing two historical models for light and discussing early methods for measuring the speed of light. Next we study the fundamental phenomena of geometric optics: reflection of light from a surface and refraction as the light crosses the boundary between two media. We also study the dispersion of light as it refracts into materials, resulting in visual displays such as the rainbow. Finally, we investigate the phenomenon of total internal reflection, which is the basis for the operation of optical fibers and the technology of fiber optics.

35.1 The Nature of Light

Before the beginning of the 19th century, light was considered to be a stream of particles that either was emitted by the object being viewed or emanated from the eyes of the viewer. Newton, the chief architect of the particle model of light, held that particles were emitted from a light source and that these particles stimulated the sense of sight upon entering the eye. Using this idea, he was able to explain reflection and refraction.

Most scientists accepted Newton's particle model. During Newton's lifetime, however, another model was proposed, one that argued that light might be some sort of wave motion. In 1678, Dutch physicist and astronomer Christian Huygens showed that a wave model of light could also explain reflection and refraction.

In 1801, Thomas Young (1773–1829) provided the first clear experimental demonstration of the wave nature of light. Young showed that under appropriate conditions light rays interfere with one another according to the waves in interference model, just like mechanical waves (Chapter 18). Such behavior could not be explained at that time by a particle model because there was no conceivable way in which two or more particles could come together and cancel one another. Additional developments during the 19th century led to the general acceptance of the wave model of light, the most important resulting from the work of Maxwell, who in 1873 asserted that light was a form of high-frequency electromagnetic wave. As discussed in Chapter 34, Hertz provided experimental confirmation of Maxwell's theory in 1887 by producing and detecting electromagnetic waves.

Christian Huygens
Dutch Physicist and Astronomer (1629–1695)
Huygens is best known for his contributions to the fields of optics and dynamics. To Huygens, light was a type of vibratory motion, spreading out and producing the sensation of light when impinging on the eye. On the basis of this theory, he deduced the laws of reflection and refraction and explained the phenomenon of double refraction.

Although the wave model and the classical theory of electricity and magnetism were able to explain most known properties of light, they could not explain some subsequent experiments. The most striking phenomenon is the photoelectric effect, also discovered by Hertz: when light strikes a metal surface, electrons are sometimes ejected from the surface. As one example of the difficulties that arose, experiments showed that the kinetic energy of an ejected electron is independent of the light intensity. This finding contradicted the wave model, which held that a more intense beam of light should add more energy to the electron. Einstein proposed an explanation of the photoelectric effect in 1905 using a model based on the concept of quantization developed by Max Planck (1858–1947) in 1900. The quantization model assumes the energy of a light wave is present in particles called *photons;* hence, the energy is said to be quantized. According to Einstein's theory, the energy of a photon is proportional to the frequency of the electromagnetic wave:

$$E = hf \qquad (35.1)$$

◀ **Energy of a photon**

where the constant of proportionality $h = 6.63 \times 10^{-34}$ J · s is called *Planck's constant.* We study this theory in Chapter 40.

In view of these developments, light must be regarded as having a dual nature. Light exhibits the characteristics of a wave in some situations and the characteristics of a particle in other situations. Light is light, to be sure. The question "Is light a wave or a particle?" is inappropriate, however. Sometimes light acts like a wave, and other times it acts like a particle. In the next few chapters, we investigate the wave nature of light.

35.2 Measurements of the Speed of Light

Light travels at such a high speed (to three digits, $c = 3.00 \times 10^8$ m/s) that early attempts to measure its speed were unsuccessful. Galileo attempted to measure the speed of light by positioning two observers in towers separated by approximately 10 km. Each observer carried a shuttered lantern. One observer would open his lantern first, and then the other would open his lantern at the moment he saw the light from the first lantern. Galileo reasoned that by knowing the transit time of the light beams from one lantern to the other and the distance between the two lanterns, he could obtain the speed. His results were inconclusive. Today, we realize (as Galileo concluded) that it is impossible to measure the speed of light in this manner because the transit time for the light is so much less than the reaction time of the observers.

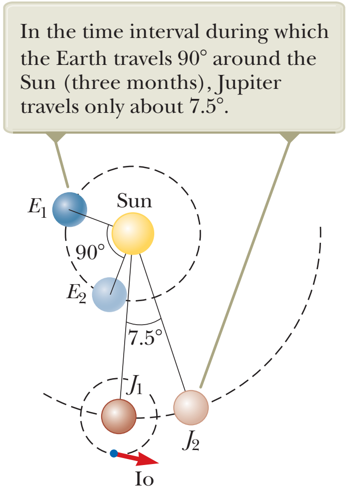


Figure 35.1 Roemer's method for measuring the speed of light (drawing not to scale).

Roemer's Method

In 1675, Danish astronomer Ole Roemer (1644–1710) made the first successful estimate of the speed of light. Roemer's technique involved astronomical observations of Io, one of the moons of Jupiter. Io has a period of revolution around Jupiter of approximately 42.5 h. The period of revolution of Jupiter around the Sun is about 12 yr; therefore, as the Earth moves through 90° around the Sun, Jupiter revolves through only $(\frac{1}{12})90° = 7.5°$ (Fig. 35.1).

An observer using the orbital motion of Io as a clock would expect the orbit to have a constant period. After collecting data for more than a year, however, Roemer observed a systematic variation in Io's period. He found that the periods were longer than average when the Earth was receding from Jupiter and shorter than average when the Earth was approaching Jupiter. Roemer attributed this variation in period to the distance between the Earth and Jupiter changing from one observation to the next.

Using Roemer's data, Huygens estimated the lower limit for the speed of light to be approximately 2.3×10^8 m/s. This experiment is important historically because it demonstrated that light does have a finite speed and gave an estimate of this speed.

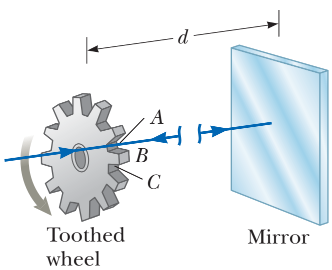


Figure 35.2 Fizeau's method for measuring the speed of light using a rotating toothed wheel. The light source is considered to be at the location of the wheel; therefore, the distance d is known.

Fizeau's Method

The first successful method for measuring the speed of light by means of purely terrestrial techniques was developed in 1849 by French physicist Armand H. L. Fizeau (1819–1896). Figure 35.2 represents a simplified diagram of Fizeau's apparatus. The basic procedure is to measure the total time interval during which light travels from some point to a distant mirror and back. If d is the distance between the light source (considered to be at the location of the wheel) and the mirror and if the time interval for one round trip is Δt, the speed of light is $c = 2d/\Delta t$.

To measure the transit time, Fizeau used a rotating toothed wheel, which converts a continuous beam of light into a series of light pulses. The rotation of such a wheel controls what an observer at the light source sees. For example, if the pulse traveling toward the mirror and passing the opening at point A in Figure 35.2 should return to the wheel at the instant tooth B had rotated into position to cover the return path, the pulse would not reach the observer. At a greater rate of rotation, the opening at point C could move into position to allow the reflected pulse to reach the observer. Knowing the distance d, the number of teeth in the wheel, and the angular speed of the wheel, Fizeau arrived at a value of 3.1×10^8 m/s. Similar measurements made by subsequent investigators yielded more precise values for c, which led to the currently accepted value of $2.997\,924\,58 \times 10^8$ m/s.

Example 35.1 Measuring the Speed of Light with Fizeau's Wheel AM

Assume Fizeau's wheel has 360 teeth and rotates at 27.5 rev/s when a pulse of light passing through opening A in Figure 35.2 is blocked by tooth B on its return. If the distance to the mirror is 7 500 m, what is the speed of light?

SOLUTION

Conceptualize Imagine a pulse of light passing through opening A in Figure 35.2 and reflecting from the mirror. By the time the pulse arrives back at the wheel, tooth B has rotated into the position previously occupied by opening A.

Categorize The wheel is a rigid object rotating at constant angular speed. We model the pulse of light as a *particle under constant speed*.

Analyze The wheel has 360 teeth, so it must have 360 openings. Therefore, because the light passes through opening A but is blocked by the tooth immediately adjacent to A, the wheel must rotate through an angular displacement of $\frac{1}{720}$ rev in the time interval during which the light pulse makes its round trip.

Use Equation 10.2, with the angular speed constant, to find the time interval for the pulse's round trip:

$$\Delta t = \frac{\Delta\theta}{\omega} = \frac{\frac{1}{720}\text{ rev}}{27.5\text{ rev/s}} = 5.05 \times 10^{-5}\text{ s}$$

35.1 continued

From the particle under constant speed model, find the speed of the pulse of light:

$$c = \frac{2d}{\Delta t} = \frac{2(7\,500\text{ m})}{5.05 \times 10^{-5}\text{ s}} = \boxed{2.97 \times 10^{8}\text{ m/s}}$$

Finalize This result is very close to the actual value of the speed of light.

35.3 The Ray Approximation in Ray Optics

The field of **ray optics** (sometimes called *geometric optics*) involves the study of the propagation of light. Ray optics assumes light travels in a fixed direction in a straight line as it passes through a uniform medium and changes its direction when it meets the surface of a different medium or if the optical properties of the medium are nonuniform in either space or time. In our study of ray optics here and in Chapter 36, we use what is called the **ray approximation.** To understand this approximation, first notice that the rays of a given wave are straight lines perpendicular to the wave fronts as illustrated in Figure 35.3 for a plane wave. In the ray approximation, a wave moving through a medium travels in a straight line in the direction of its rays.

If the wave meets a barrier in which there is a circular opening whose diameter is much larger than the wavelength as in Figure 35.4a, the wave emerging from the opening continues to move in a straight line (apart from some small edge effects); hence, the ray approximation is valid. If the diameter of the opening is on the order of the wavelength as in Figure 35.4b, the waves spread out from the opening in all directions. This effect, called *diffraction*, will be studied in Chapter 37. Finally, if the opening is much smaller than the wavelength, the opening can be approximated as a point source of waves as shown in Fig. 35.4c.

Similar effects are seen when waves encounter an opaque object of dimension d. In that case, when $\lambda << d$, the object casts a sharp shadow.

The ray approximation and the assumption that $\lambda << d$ are used in this chapter and in Chapter 36, both of which deal with ray optics. This approximation is very good for the study of mirrors, lenses, prisms, and associated optical instruments such as telescopes, cameras, and eyeglasses.

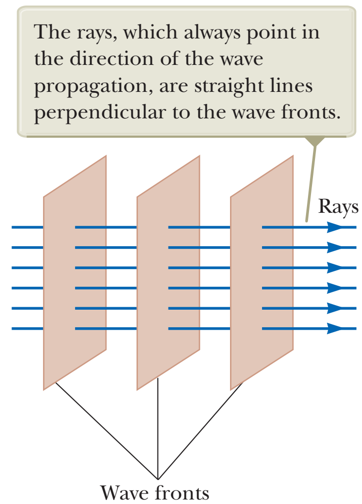


Figure 35.3 A plane wave propagating to the right.

35.4 Analysis Model: Wave Under Reflection

We introduced the concept of reflection of waves in a discussion of waves on strings in Section 16.4. As with waves on strings, when a light ray traveling in one medium encounters a boundary with another medium, part of the incident light

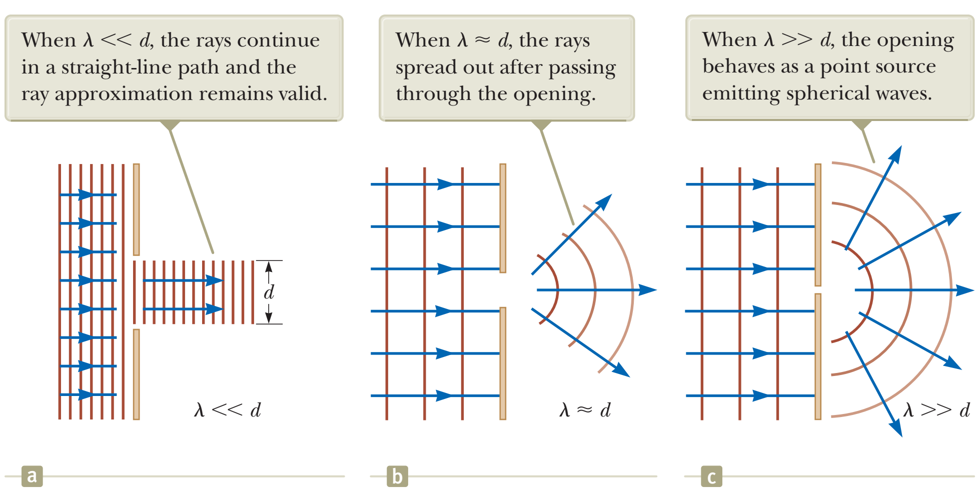


Figure 35.4 A plane wave of wavelength λ is incident on a barrier in which there is an opening of diameter d.

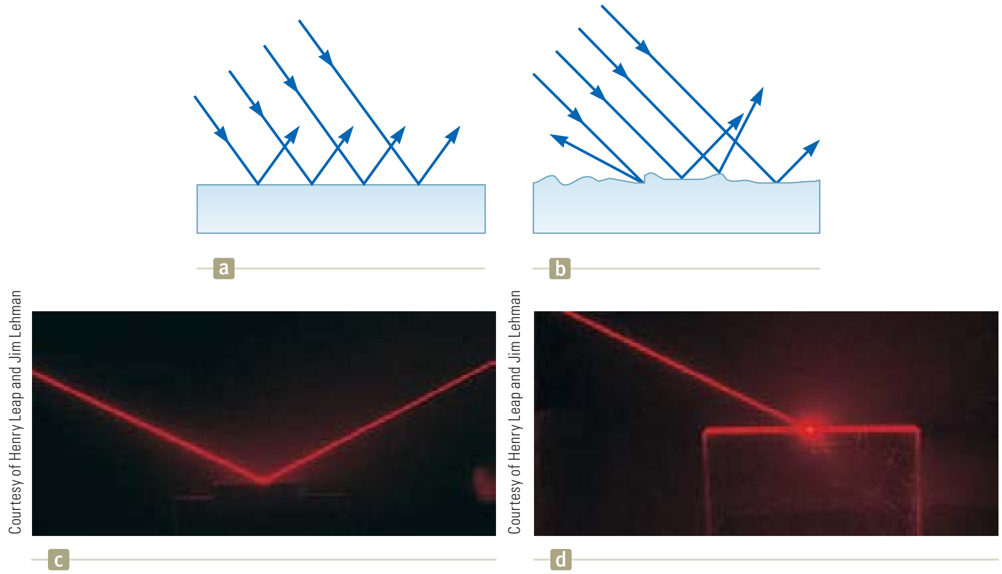


Figure 35.5 Schematic representation of (a) specular reflection, where the reflected rays are all parallel to one another, and (b) diffuse reflection, where the reflected rays travel in random directions. (c) and (d) Photographs of specular and diffuse reflection using laser light.

is reflected. For waves on a one-dimensional string, the reflected wave must necessarily be restricted to a direction along the string. For light waves traveling in three-dimensional space, no such restriction applies and the reflected light waves can be in directions different from the direction of the incident waves. Figure 35.5a shows several rays of a beam of light incident on a smooth, mirror-like, reflecting surface. The reflected rays are parallel to one another as indicated in the figure. The direction of a reflected ray is in the plane perpendicular to the reflecting surface that contains the incident ray. Reflection of light from such a smooth surface is called **specular reflection.** If the reflecting surface is rough as in Figure 35.5b, the surface reflects the rays not as a parallel set but in various directions. Reflection from any rough surface is known as **diffuse reflection.** A surface behaves as a smooth surface as long as the surface variations are much smaller than the wavelength of the incident light.

The difference between these two kinds of reflection explains why it is more difficult to see while driving on a rainy night than on a dry night. If the road is wet, the smooth surface of the water specularly reflects most of your headlight beams away from your car (and perhaps into the eyes of oncoming drivers). When the road is dry, its rough surface diffusely reflects part of your headlight beam back toward you, allowing you to see the road more clearly. Your bathroom mirror exhibits specular reflection, whereas light reflecting from this page experiences diffuse reflection. In this book, we restrict our study to specular reflection and use the term *reflection* to mean specular reflection.

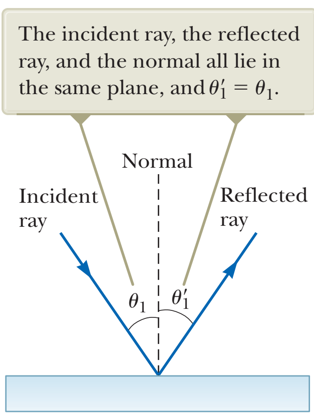


Figure 35.6 The wave under reflection model.

Pitfall Prevention 35.1

Subscript Notation The subscript 1 refers to parameters for the light in the initial medium. When light travels from one medium to another, we use the subscript 2 for the parameters associated with the light in the new medium. In this discussion, the light stays in the same medium, so we only have to use the subscript 1.

Consider a light ray traveling in air and incident at an angle on a flat, smooth surface as shown in Figure 35.6. The incident and reflected rays make angles θ_1 and θ_1', respectively, where the angles are measured between the normal and the rays. (The normal is a line drawn perpendicular to the surface at the point where the incident ray strikes the surface.) Experiments and theory show that the angle of reflection equals the angle of incidence:

Law of reflection ▶

$$\theta_1' = \theta_1 \quad (35.2)$$

This relationship is called the **law of reflection.** Because reflection of waves from an interface between two media is a common phenomenon, we identify an analysis model for this situation: the **wave under reflection.** Equation 35.2 is the mathematical representation of this model.

Quick Quiz 35.1 In the movies, you sometimes see an actor looking in a mirror and you can see his face in the mirror. It can be said with certainty that during the filming of such a scene, the actor sees in the mirror: **(a)** his face **(b)** your face **(c)** the director's face **(d)** the movie camera **(e)** impossible to determine

Example 35.2 The Double-Reflected Light Ray AM

Two mirrors make an angle of 120° with each other as illustrated in Figure 35.7a. A ray is incident on mirror M_1 at an angle of 65° to the normal. Find the direction of the ray after it is reflected from mirror M_2.

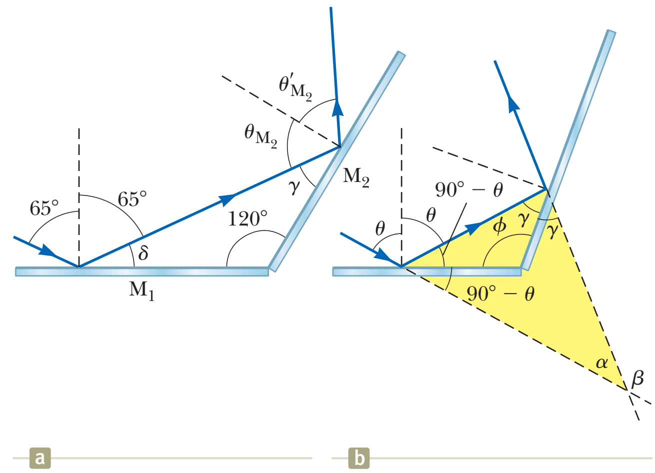


Figure 35.7 (Example 35.2) (a) Mirrors M_1 and M_2 make an angle of 120° with each other. (b) The geometry for an arbitrary mirror angle.

SOLUTION

Conceptualize Figure 35.7a helps conceptualize this situation. The incoming ray reflects from the first mirror, and the reflected ray is directed toward the second mirror. Therefore, there is a second reflection from the second mirror.

Categorize Because the interactions with both mirrors are simple reflections, we apply the *wave under reflection* model and some geometry.

Analyze From the law of reflection, the first reflected ray makes an angle of 65° with the normal.

Find the angle the first reflected ray makes with the horizontal: $\delta = 90° - 65° = 25°$

From the triangle made by the first reflected ray and the two mirrors, find the angle the reflected ray makes with M_2: $\gamma = 180° - 25° - 120° = 35°$

Find the angle the first reflected ray makes with the normal to M_2: $\theta_{M_2} = 90° - 35° = 55°$

From the law of reflection, find the angle the second reflected ray makes with the normal to M_2: $\theta'_{M_2} = \theta_{M_2} = \boxed{55°}$

Finalize Let's explore variations in the angle between the mirrors as follows.

WHAT IF? If the incoming and outgoing rays in Figure 35.7a are extended behind the mirror, they cross at an angle of 60° and the overall change in direction of the light ray is 120°. This angle is the same as that between the mirrors. What if the angle between the mirrors is changed? Is the overall change in the direction of the light ray always equal to the angle between the mirrors?

Answer Making a general statement based on one data point or one observation is always a dangerous practice! Let's investigate the change in direction for a general situation. Figure 35.7b shows the mirrors at an arbitrary angle ϕ and the incoming light ray striking the mirror at an arbitrary angle θ with respect to the normal to the mirror surface. In accordance with the law of reflection and the sum of the interior angles of a triangle, the angle γ is given by $\gamma = 180° - (90° - \theta) - \phi = 90° + \theta - \phi$.

Consider the triangle highlighted in yellow in Figure 35.7b and determine α: $\alpha + 2\gamma + 2(90° - \theta) = 180° \rightarrow \alpha = 2(\theta - \gamma)$

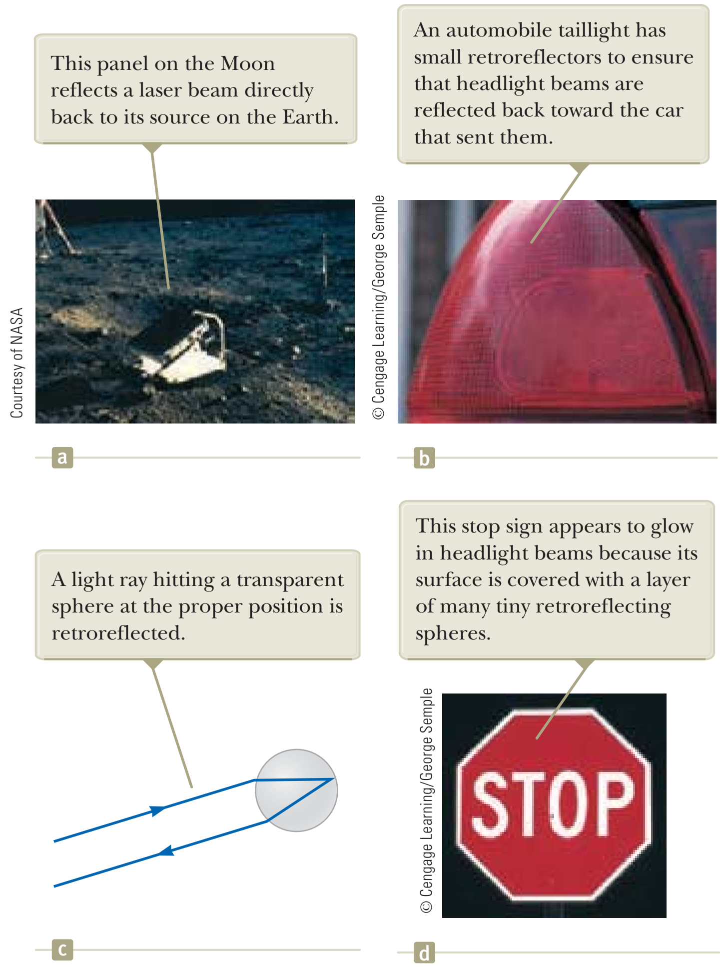
Notice from Figure 35.7b that the change in direction of the light ray is angle β. Use the geometry in the figure to solve for β:

$$\beta = 180° - \alpha = 180° - 2(\theta - \gamma)$$
$$= 180° - 2[\theta - (90° + \theta - \phi)] = 360° - 2\phi$$

Notice that β is not equal to ϕ. For $\phi = 120°$, we obtain $\beta = 120°$, which happens to be the same as the mirror angle; that is true only for this special angle between the mirrors, however. For example, if $\phi = 90°$, we obtain $\beta = 180°$. In that case, the light is reflected straight back to its origin.

If the angle between two mirrors is 90°, the reflected beam returns to the source parallel to its original path as discussed in the What If? section of the preceding example. This phenomenon, called *retroreflection,* has many practical applications. If a third mirror is placed perpendicular to the first two so that the three form the

Figure 35.8 Applications of retroreflection.



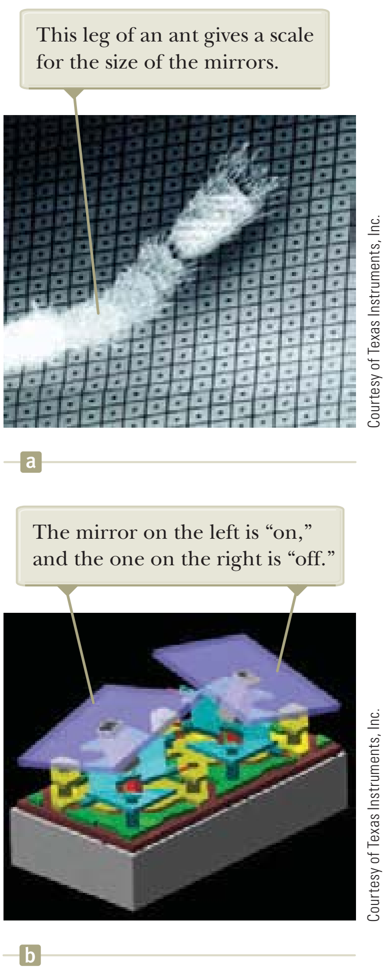


Figure 35.9 (a) An array of mirrors on the surface of a digital micromirror device. Each mirror has an area of approximately 16 μm^2. (b) A close-up view of two single micromirrors.

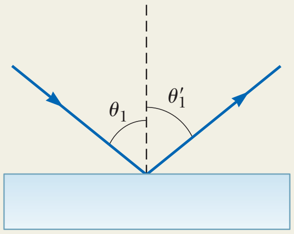
corner of a cube, retroreflection works in three dimensions. In 1969, a panel of many small reflectors was placed on the Moon by the *Apollo 11* astronauts (Fig. 35.8a). A laser beam from the Earth is reflected directly back on itself, and its transit time is measured. This information is used to determine the distance to the Moon with an uncertainty of 15 cm. (Imagine how difficult it would be to align a regular flat mirror on the Moon so that the reflected laser beam would hit a particular location on the Earth!) A more everyday application is found in automobile taillights. Part of the plastic making up the taillight is formed into many tiny cube corners (Fig. 35.8b) so that headlight beams from cars approaching from the rear are reflected back to the drivers. Instead of cube corners, small spherical bumps are sometimes used (Fig. 35.8c). Tiny clear spheres are used in a coating material found on many road signs. Due to retroreflection from these spheres, the stop sign in Figure 35.8d appears much brighter than it would if it were simply a flat, shiny surface. Retroreflectors are also used for reflective panels on running shoes and running clothing to allow joggers to be seen at night.

Another practical application of the law of reflection is the digital projection of movies, television shows, and computer presentations. A digital projector uses an optical semiconductor chip called a *digital micromirror device.* This device contains an array of tiny mirrors (Fig. 35.9a) that can be individually tilted by means of signals to an address electrode underneath the edge of the mirror. Each mirror corresponds to a pixel in the projected image. When the pixel corresponding to a given mirror is to be bright, the mirror is in the "on" position and is oriented so as to reflect light from a source illuminating the array to the screen (Fig. 35.9b). When the pixel for this mirror is to be dark, the mirror is "off" and is tilted so that the light is reflected away from the screen. The brightness of the pixel is determined by the total time interval during which the mirror is in the "on" position during the display of one image.

Digital movie projectors use three micromirror devices, one for each of the primary colors red, blue, and green, so that movies can be displayed with up to 35 trillion colors. Because information is stored as binary data, a digital movie does not degrade with time as does film. Furthermore, because the movie is entirely in the form of computer software, it can be delivered to theaters by means of satellites, optical discs, or optical fiber networks.

Analysis Model Wave Under Reflection

Imagine a wave (electromagnetic or mechanical) traveling through space and striking a flat surface at an angle θ_1 with respect to the normal to the surface. The wave will reflect from the surface in a direction described by the **law of reflection—**the angle of reflection θ_1' equals the angle of incidence θ_1:



$$\theta_1' = \theta_1 \tag{35.2}$$

Examples:

- sound waves from an orchestra reflect from a bandshell out to the audience
- a mirror is used to deflect a laser beam in a laser light show
- your bathroom mirror reflects light from your face back to you to form an image of your face (Chapter 36)
- x-rays reflected from a crystalline material create an optical pattern that can be used to understand the structure of the solid (Chapter 38)

35.5 Analysis Model: Wave Under Refraction

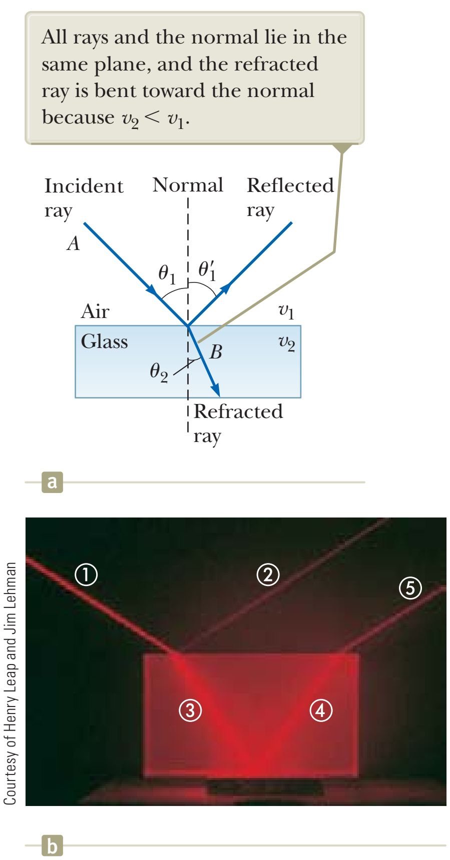
In addition to the phenomenon of reflection discussed for waves on strings in Section 16.4, we also found that some of the energy of the incident wave transmits into the new medium. For example, consider Figures 16.15 and 16.16, in which a pulse on a string approaching a junction with another string both reflects from and transmits past the junction and into the second string. Similarly, when a ray of light traveling through a transparent medium encounters a boundary leading into another transparent medium as shown in Figure 35.10, part of the energy is reflected and part enters the second medium. As with reflection, the direction of the transmitted wave exhibits an interesting behavior because of the three-dimensional nature of the light waves. The ray that enters the second medium changes its direction of propagation at the boundary and is said to be **refracted.** The incident ray, the reflected ray, and the refracted ray all lie in the same plane. The **angle of refraction,** θ_2 in Figure 35.10a, depends on the properties of the two media and on the angle of incidence θ_1 through the relationship

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1} \tag{35.3}$$

where v_1 is the speed of light in the first medium and v_2 is the speed of light in the second medium.

The path of a light ray through a refracting surface is reversible. For example, the ray shown in Figure 35.10a travels from point *A* to point *B*. If the ray originated at *B*, it would travel along line *BA* to reach point *A* and the reflected ray would point downward and to the left in the glass.

Quick Quiz 35.2 If beam ① is the incoming beam in Figure 35.10b, which of the other four red lines are reflected beams and which are refracted beams?



Courtesy of Henry Leap and Jim Lehman

Figure 35.10 (a) The wave under refraction model. (b) Light incident on the Lucite block refracts both when it enters the block and when it leaves the block.

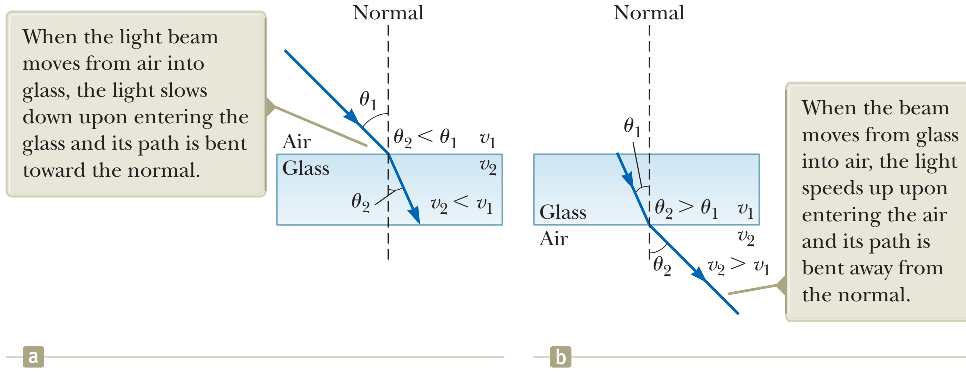


Figure 35.11 The refraction of light as it (a) moves from air into glass and (b) moves from glass into air.

From Equation 35.3, we can infer that when light moves from a material in which its speed is high to a material in which its speed is lower as shown in Figure 35.11a, the angle of refraction θ_2 is less than the angle of incidence θ_1 and the ray is bent *toward* the normal. If the ray moves from a material in which light moves slowly to a material in which it moves more rapidly as illustrated in Figure 35.11b, then θ_2 is greater than θ_1 and the ray is bent *away* from the normal.

The behavior of light as it passes from air into another substance and then re-emerges into air is often a source of confusion to students. When light travels in air, its speed is 3.00×10^8 m/s, but this speed is reduced to approximately 2×10^8 m/s when the light enters a block of glass. When the light re-emerges into air, its speed instantaneously increases to its original value of 3.00×10^8 m/s. This effect is far different from what happens, for example, when a bullet is fired through a block of wood. In that case, the speed of the bullet decreases as it moves through the wood because some of its original energy is used to tear apart the wood fibers. When the bullet enters the air once again, it emerges at a speed lower than it had when it entered the wood.

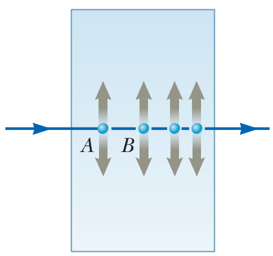


Figure 35.12 Light passing from one atom to another in a medium. The blue spheres are electrons, and the vertical arrows represent their oscillations.

To see why light behaves as it does, consider Figure 35.12, which represents a beam of light entering a piece of glass from the left. Once inside the glass, the light may encounter an electron bound to an atom, indicated as point *A*. Let's assume light is absorbed by the atom, which causes the electron to oscillate (a detail represented by the double-headed vertical arrows). The oscillating electron then acts as an antenna and radiates the beam of light toward an atom at *B*, where the light is again absorbed. The details of these absorptions and radiations are best explained in terms of quantum mechanics (Chapter 42). For now, it is sufficient to think of light passing from one atom to another through the glass. Although light travels from one atom to another at 3.00×10^8 m/s, the absorption and radiation that take place cause the *average* light speed through the material to fall to approximately 2×10^8 m/s. Once the light emerges into the air, absorption and radiation cease and the light travels at a constant speed of 3.00×10^8 m/s.

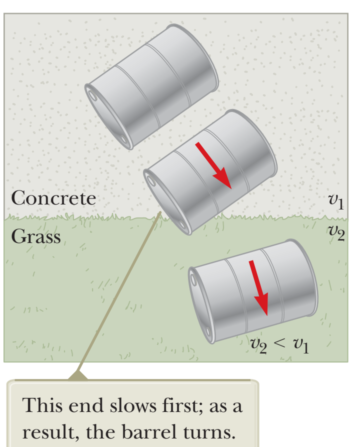


Figure 35.13 Overhead view of a barrel rolling from concrete onto grass.

A mechanical analog of refraction is shown in Figure 35.13. When the left end of the rolling barrel reaches the grass, it slows down, whereas the right end remains on the concrete and moves at its original speed. This difference in speeds causes the barrel to pivot, which changes the direction of travel.

Index of Refraction

In general, the speed of light in any material is *less* than its speed in vacuum. In fact, *light travels at its maximum speed c in vacuum.* It is convenient to define the **index of refraction** n of a medium to be the ratio

Index of refraction ▶

$$n \equiv \frac{\text{speed of light in vacuum}}{\text{speed of light in a medium}} \equiv \frac{c}{v} \tag{35.4}$$

This definition shows that the index of refraction is a dimensionless number greater than unity because v is always less than c. Furthermore, n is equal to unity for vacuum. The indices of refraction for various substances are listed in Table 35.1.

As light travels from one medium to another, its frequency does not change but its wavelength does. To see why that is true, consider Figure 35.14. Waves pass an observer at point A in medium 1 with a certain frequency and are incident on the boundary between medium 1 and medium 2. The frequency with which the waves pass an observer at point B in medium 2 must equal the frequency at which they pass point A. If that were not the case, energy would be piling up or disappearing at the boundary. Because there is no mechanism for that to occur, the frequency must be a constant as a light ray passes from one medium into another. Therefore, because the relationship $v = \lambda f$ (Eq. 16.12) from the traveling wave model must be valid in both media and because $f_1 = f_2 = f$, we see that

$$v_1 = \lambda_1 f \quad \text{and} \quad v_2 = \lambda_2 f \tag{35.5}$$

Because $v_1 \neq v_2$, it follows that $\lambda_1 \neq \lambda_2$ as shown in Figure 35.14.

We can obtain a relationship between index of refraction and wavelength by dividing the first Equation 35.5 by the second and then using Equation 35.4:

$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1} \tag{35.6}$$

This expression gives

$$\lambda_1 n_1 = \lambda_2 n_2$$

If medium 1 is vacuum or, for all practical purposes, air, then $n_1 = 1$. Hence, it follows from Equation 35.6 that the index of refraction of any medium can be expressed as the ratio

$$n = \frac{\lambda}{\lambda_n} \tag{35.7}$$

where λ is the wavelength of light in vacuum and λ_n is the wavelength of light in the medium whose index of refraction is n. From Equation 35.7, we see that because $n > 1$, $\lambda_n < \lambda$.

We are now in a position to express Equation 35.3 in an alternative form. Replacing the v_2/v_1 term in Equation 35.3 with n_1/n_2 from Equation 35.6 gives

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2} \tag{35.8}$$

◀ **Snell's law of refraction**

The experimental discovery of this relationship is usually credited to Willebrord Snell (1591–1626) and it is therefore known as **Snell's law of refraction.** We shall

As a wave moves between the media, its wavelength changes but its frequency remains constant.

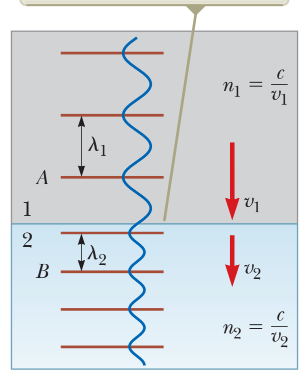


Figure 35.14 A wave travels from medium 1 to medium 2, in which it moves with lower speed.

Pitfall Prevention 35.2

An Inverse Relationship The index of refraction is *inversely* proportional to the wave speed. As the wave speed v decreases, the index of refraction n increases. Therefore, the higher the index of refraction of a material, the more it *slows down* light from its speed in vacuum. The more the light slows down, the more θ_2 differs from θ_1 in Equation 35.8.

Pitfall Prevention 35.3

***n* Is Not an Integer Here** The symbol n has been used several times as an integer, such as in Chapter 18 to indicate the standing wave mode on a string or in an air column. The index of refraction n is *not* an integer.

Table 35.1 **Indices of Refraction**

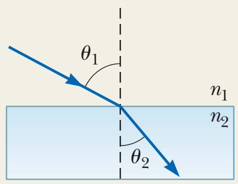
Substance	Index of Refraction	Substance	Index of Refraction
Solids at 20°C		*Liquids at 20°C*	
Cubic zirconia	2.20	Benzene	1.501
Diamond (C)	2.419	Carbon disulfide	1.628
Fluorite (CaF_2)	1.434	Carbon tetrachloride	1.461
Fused quartz (SiO_2)	1.458	Ethyl alcohol	1.361
Gallium phosphide	3.50	Glycerin	1.473
Glass, crown	1.52	Water	1.333
Glass, flint	1.66		
Ice (H_2O)	1.309	*Gases at 0°C, 1 atm*	
Polystyrene	1.49	Air	1.000 293
Sodium chloride (NaCl)	1.544	Carbon dioxide	1.000 45

Note: All values are for light having a wavelength of 589 nm in vacuum.

examine this equation further in Section 35.6. Refraction of waves at an interface between two media is a common phenomenon, so we identify an analysis model for this situation: the **wave under refraction.** Equation 35.8 is the mathematical representation of this model for electromagnetic radiation. Other waves, such as seismic waves and sound waves, also exhibit refraction according to this model, and the mathematical representation of the model for these waves is Equation 35.3.

Quick Quiz 35.3 Light passes from a material with index of refraction 1.3 into one with index of refraction 1.2. Compared to the incident ray, what happens to the refracted ray? **(a)** It bends toward the normal. **(b)** It is undeflected. **(c)** It bends away from the normal.

Analysis Model Wave Under Refraction



Imagine a wave (electromagnetic or mechanical) traveling through space and striking a flat surface at an angle θ_1 with respect to the normal to the surface. Some of the energy of the wave refracts into the medium below the surface in a direction θ_2 described by the **law of refraction—**

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1} \quad \textbf{(35.3)}$$

where v_1 and v_2 are the speeds of the wave in medium 1 and medium 2, respectively.

For light waves, **Snell's law of refraction** states that

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \textbf{(35.8)}$$

where n_1 and n_2 are the indices of refraction in the two media.

Examples:

- sound waves moving upward from the shore of a lake refract in warmer layers of air higher above the lake and travel downward to a listener in a boat, making sounds from the shore louder than expected
- light from the sky approaches a hot roadway at a grazing angle and refracts upward so as to miss the roadway and enter a driver's eye, giving the illusion of a pool of water on the distant roadway
- light is sent over long distances in an optical fiber because of a difference in index of refraction between the fiber and the surrounding material (Section 35.8)
- a magnifying glass forms an enlarged image of a postage stamp due to refraction of light through the lens (Chapter 36)

Example 35.3 Angle of Refraction for Glass AM

A light ray of wavelength 589 nm traveling through air is incident on a smooth, flat slab of crown glass at an angle of 30.0° to the normal.

(A) Find the angle of refraction.

SOLUTION

Conceptualize Study Figure 35.11a, which illustrates the refraction process occurring in this problem. We expect that $\theta_2 < \theta_1$ because the speed of light is lower in the glass.

Categorize This is a typical problem in which we apply the *wave under refraction* model.

Analyze Rearrange Snell's law of refraction to find $\sin\theta_2$:

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$$

Solve for θ_2:

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right)$$

Substitute indices of refraction from Table 35.1 and the incident angle:

$$\theta_2 = \sin^{-1}\left(\frac{1.00}{1.52}\sin 30.0°\right) = \boxed{19.2°}$$

(B) Find the speed of this light once it enters the glass.

35.3 continued

SOLUTION

Solve Equation 35.4 for the speed of light in the glass: $v = \dfrac{c}{n}$

Substitute numerical values: $v = \dfrac{3.00 \times 10^8 \text{ m/s}}{1.52} = \boxed{1.97 \times 10^8 \text{ m/s}}$

(C) What is the wavelength of this light in the glass?

SOLUTION

Use Equation 35.7 to find the wavelength in the glass: $\lambda_n = \dfrac{\lambda}{n} = \dfrac{589 \text{ nm}}{1.52} = \boxed{388 \text{ nm}}$

Finalize In part (A), note that $\theta_2 < \theta_1$, consistent with the slower speed of the light found in part (B). In part (C), we see that the wavelength of the light is shorter in the glass than in the air.

Example 35.4 Light Passing Through a Slab AM

A light beam passes from medium 1 to medium 2, with the latter medium being a thick slab of material whose index of refraction is n_2 (Fig. 35.15). Show that the beam emerging into medium 1 from the other side is parallel to the incident beam.

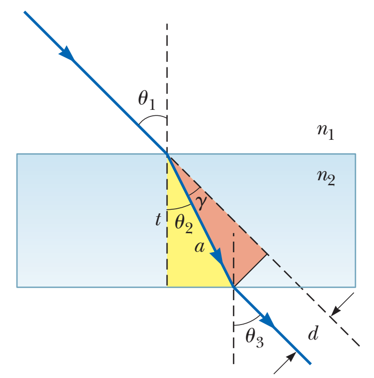


Figure 35.15 (Example 35.4) The dashed line drawn parallel to the ray coming out the bottom of the slab represents the path the light would take were the slab not there.

SOLUTION

Conceptualize Follow the path of the light beam as it enters and exits the slab of material in Figure 35.15, where we have assumed that $n_2 > n_1$. The ray bends toward the normal upon entering and away from the normal upon leaving.

Categorize Like Example 35.3, this is another typical problem in which we apply the *wave under refraction* model.

Analyze Apply Snell's law of refraction to the upper surface: (1) $\sin\theta_2 = \dfrac{n_1}{n_2}\sin\theta_1$

Apply Snell's law to the lower surface: (2) $\sin\theta_3 = \dfrac{n_2}{n_1}\sin\theta_2$

Substitute Equation (1) into Equation (2): $\sin\theta_3 = \dfrac{n_2}{n_1}\left(\dfrac{n_1}{n_2}\sin\theta_1\right) = \sin\theta_1$

Finalize Therefore, $\theta_3 = \theta_1$ and the slab does not alter the direction of the beam. It does, however, offset the beam parallel to itself by the distance d shown in Figure 35.15.

WHAT IF? What if the thickness t of the slab is doubled? Does the offset distance d also double?

Answer Consider the region of the light path within the slab in Figure 35.15. The distance a is the hypotenuse of two right triangles.

Find an expression for a from the yellow triangle: $a = \dfrac{t}{\cos\theta_2}$

Find an expression for d from the red triangle: $d = a\sin\gamma = a\sin(\theta_1 - \theta_2)$

Combine these equations: $d = \dfrac{t}{\cos\theta_2}\sin(\theta_1 - \theta_2)$

For a given incident angle θ_1, the refracted angle θ_2 is determined solely by the index of refraction, so the offset distance d is proportional to t. If the thickness doubles, so does the offset distance.

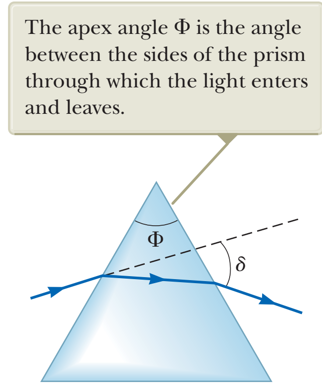


Figure 35.16 A prism refracts a single-wavelength light ray through an angle of deviation δ.

In Example 35.4, the light passes through a slab of material with parallel sides. What happens when light strikes a prism with nonparallel sides as shown in Figure 35.16? In this case, the outgoing ray does not propagate in the same direction as the incoming ray. A ray of single-wavelength light incident on the prism from the left emerges at angle δ from its original direction of travel. This angle δ is called the **angle of deviation.** The **apex angle** Φ of the prism, shown in the figure, is defined as the angle between the surface at which the light enters the prism and the second surface that the light encounters.

Example 35.5 Measuring *n* Using a Prism AM

Although we do not prove it here, the minimum angle of deviation δ_{min} for a prism occurs when the angle of incidence θ_1 is such that the refracted ray inside the prism makes the same angle with the normal to the two prism faces[1] as shown in Figure 35.17. Obtain an expression for the index of refraction of the prism material in terms of the minimum angle of deviation and the apex angle Φ.

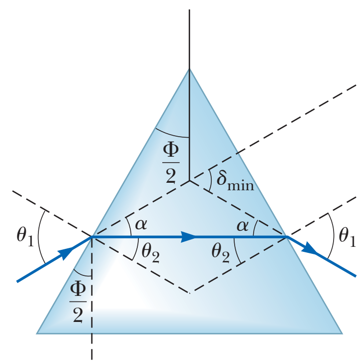


Figure 35.17 (Example 35.5) A light ray passing through a prism at the minimum angle of deviation δ_{min}.

SOLUTION

Conceptualize Study Figure 35.17 carefully and be sure you understand why the light ray comes out of the prism traveling in a different direction.

Categorize In this example, light enters a material through one surface and leaves the material at another surface. Let's apply the *wave under refraction* model to the light passing through the prism.

Analyze Consider the geometry in Figure 35.17, where we have used symmetry to label several angles. The reproduction of the angle $\Phi/2$ at the location of the incoming light ray shows that $\theta_2 = \Phi/2$. The theorem that an exterior angle of any triangle equals the sum of the two opposite interior angles shows that $\delta_{min} = 2\alpha$. The geometry also shows that $\theta_1 = \theta_2 + \alpha$.

Combine these three geometric results:

$$\theta_1 = \theta_2 + \alpha = \frac{\Phi}{2} + \frac{\delta_{min}}{2} = \frac{\Phi + \delta_{min}}{2}$$

Apply the wave under refraction model at the left surface and solve for n:

$$(1.00)\sin\theta_1 = n\sin\theta_2 \;\rightarrow\; n = \frac{\sin\theta_1}{\sin\theta_2}$$

Substitute for the incident and refracted angles:

$$n = \frac{\sin\left(\dfrac{\Phi + \delta_{min}}{2}\right)}{\sin(\Phi/2)} \qquad \textbf{(35.9)}$$

[1]The details of this proof are available in texts on optics.

35.5 continued

Finalize Knowing the apex angle Φ of the prism and measuring δ_{min}, you can calculate the index of refraction of the prism material. Furthermore, a hollow prism can be used to determine the values of n for various liquids filling the prism.

35.6 Huygens's Principle

The laws of reflection and refraction were stated earlier in this chapter without proof. In this section, we develop these laws by using a geometric method proposed by Huygens in 1678. **Huygens's principle** is a geometric construction for using knowledge of an earlier wave front to determine the position of a new wave front at some instant:

> All points on a given wave front are taken as point sources for the production of spherical secondary waves, called wavelets, that propagate outward through a medium with speeds characteristic of waves in that medium. After some time interval has passed, the new position of the wave front is the surface tangent to the wavelets.

First, consider a plane wave moving through free space as shown in Figure 35.18a. At $t = 0$, the wave front is indicated by the plane labeled AA'. In Huygens's construction, each point on this wave front is considered a point source. For clarity, only three point sources on AA' are shown. With these sources for the wavelets, we draw circular arcs, each of radius $c\,\Delta t$, where c is the speed of light in vacuum and Δt is some time interval during which the wave propagates. The surface drawn tangent to these wavelets is the plane BB', which is the wave front at a later time, and is parallel to AA'. In a similar manner, Figure 35.18b shows Huygens's construction for a spherical wave.

Pitfall Prevention 35.4

Of What Use Is Huygens's Principle? At this point, the importance of Huygens's principle may not be evident. Predicting the position of a future wave front may not seem to be very critical. We will use Huygens's principle here to generate the laws of reflection and refraction and in later chapters to explain additional wave phenomena for light.

Huygens's Principle Applied to Reflection and Refraction

We now derive the laws of reflection and refraction, using Huygens's principle.

For the law of reflection, refer to Figure 35.19. The line AB represents a plane wave front of the incident light just as ray 1 strikes the surface. At this instant, the wave at A sends out a Huygens wavelet (appearing at a later time as the light brown circular arc passing through D); the reflected light makes an angle γ' with the surface. At the

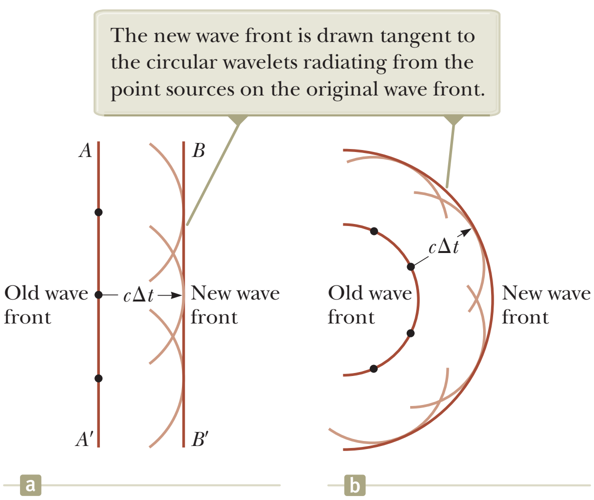


Figure 35.18 Huygens's construction for (a) a plane wave propagating to the right and (b) a spherical wave propagating to the right.

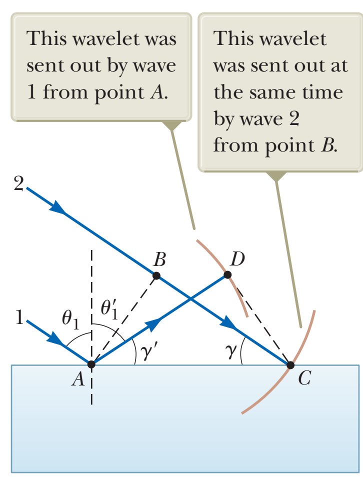


Figure 35.19 Huygens's construction for proving the law of reflection.

same time, the wave at B emits a Huygens wavelet (the light brown circular arc passing through C) with the incident light making an angle γ with the surface. Figure 35.19 shows these wavelets after a time interval Δt, after which ray 2 strikes the surface. Because both rays 1 and 2 move with the same speed, we must have $AD = BC = c\,\Delta t$.

The remainder of our analysis depends on geometry. Notice that the two triangles ABC and ADC are congruent because they have the same hypotenuse AC and because $AD = BC$. Figure 35.19 shows that

$$\cos\gamma = \frac{BC}{AC} \quad \text{and} \quad \cos\gamma' = \frac{AD}{AC}$$

where $\gamma = 90° - \theta_1$ and $\gamma' = 90° - \theta_1'$. Because $AD = BC$,

$$\cos\gamma = \cos\gamma'$$

Therefore,

$$\gamma = \gamma'$$
$$90° - \theta_1 = 90° - \theta_1'$$

and

$$\theta_1 = \theta_1'$$

which is the law of reflection.

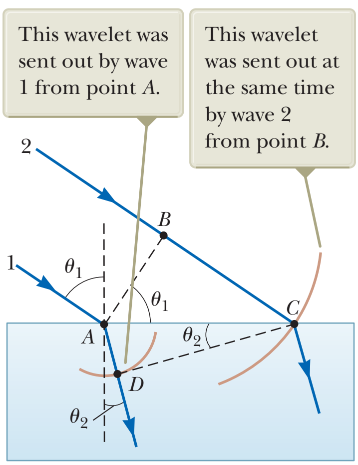


Figure 35.20 Huygens's construction for proving Snell's law of refraction.

Now let's use Huygens's principle to derive Snell's law of refraction. We focus our attention on the instant ray 1 strikes the surface and the subsequent time interval until ray 2 strikes the surface as in Figure 35.20. During this time interval, the wave at A sends out a Huygens wavelet (the light brown arc passing through D) and the light refracts into the material, making an angle θ_2 with the normal to the surface. In the same time interval, the wave at B sends out a Huygens wavelet (the light brown arc passing through C) and the light continues to propagate in the same direction. Because these two wavelets travel through different media, the radii of the wavelets are different. The radius of the wavelet from A is $AD = v_2\,\Delta t$, where v_2 is the wave speed in the second medium. The radius of the wavelet from B is $BC = v_1\,\Delta t$, where v_1 is the wave speed in the original medium.

From triangles ABC and ADC, we find that

$$\sin\theta_1 = \frac{BC}{AC} = \frac{v_1\,\Delta t}{AC} \quad \text{and} \quad \sin\theta_2 = \frac{AD}{AC} = \frac{v_2\,\Delta t}{AC}$$

Dividing the first equation by the second gives

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$$

From Equation 35.4, however, we know that $v_1 = c/n_1$ and $v_2 = c/n_2$. Therefore,

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}$$

and

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

which is Snell's law of refraction.

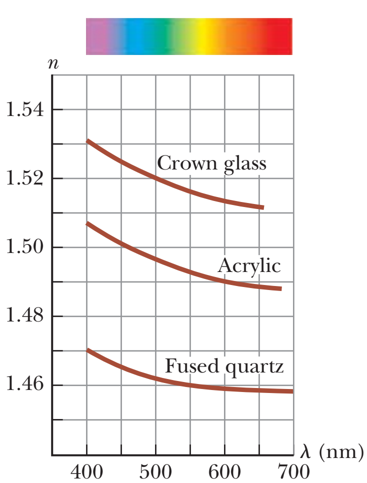


Figure 35.21 Variation of index of refraction with vacuum wavelength for three materials.

35.7 Dispersion

An important property of the index of refraction n is that, for a given material, the index varies with the wavelength of the light passing through the material as Figure 35.21 shows. This behavior is called **dispersion.** Because n is a function of wavelength, Snell's law of refraction indicates that light of different wavelengths is refracted at different angles when incident on a material.

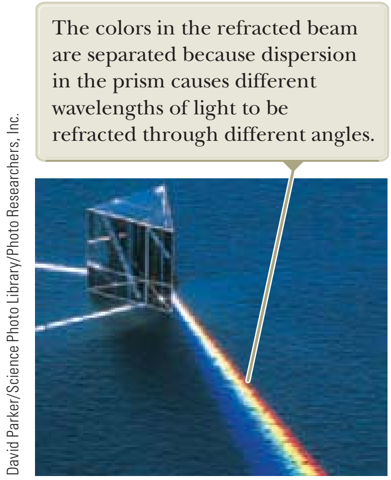


Figure 35.22 White light enters a glass prism at the upper left.

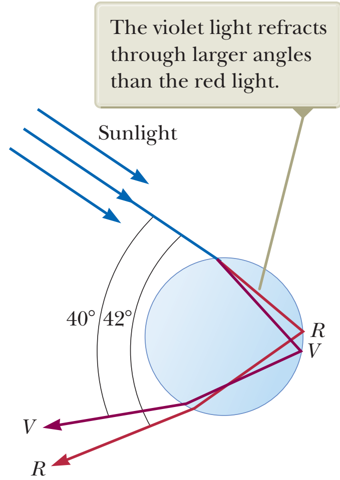


Figure 35.23 Path of sunlight through a spherical raindrop. Light following this path contributes to the visible rainbow.

Pitfall Prevention 35.5

A Rainbow of Many Light Rays

Pictorial representations such as Figure 35.23 are subject to misinterpretation. The figure shows one ray of light entering the raindrop and undergoing reflection and refraction, exiting the raindrop in a range of 40° to 42° from the entering ray. This illustration might be interpreted incorrectly as meaning that *all* light entering the raindrop exits in this small range of angles. In reality, light exits the raindrop over a much larger range of angles, from 0° to 42°. A careful analysis of the reflection and refraction from the spherical raindrop shows that the range of 40° to 42° is where the *highest-intensity light* exits the raindrop.

Figure 35.21 shows that the index of refraction generally decreases with increasing wavelength. For example, violet light refracts more than red light does when passing into a material.

Now suppose a beam of *white light* (a combination of all visible wavelengths) is incident on a prism as illustrated in Figure 35.22. Clearly, the angle of deviation δ depends on wavelength. The rays that emerge spread out in a series of colors known as the **visible spectrum.** These colors, in order of decreasing wavelength, are red, orange, yellow, green, blue, and violet. Newton showed that each color has a particular angle of deviation and that the colors can be recombined to form the original white light.

The dispersion of light into a spectrum is demonstrated most vividly in nature by the formation of a rainbow, which is often seen by an observer positioned between the Sun and a rain shower. To understand how a rainbow is formed, consider Figure 35.23. We will need to apply both the wave under reflection and wave under refraction models. A ray of sunlight (which is white light) passing overhead strikes a drop of water in the atmosphere and is refracted and reflected as follows. It is first refracted at the front surface of the drop, with the violet light deviating the most and the red light the least. At the back surface of the drop, the light is reflected and returns to the front surface, where it again undergoes refraction as it moves from water into air. The rays leave the drop such that the angle between the incident white light and the most intense returning violet ray is 40° and the angle between the incident white light and the most intense returning red ray is 42°. This small angular difference between the returning rays causes us to see a colored bow.

Now suppose an observer is viewing a rainbow as shown in Figure 35.24. If a raindrop high in the sky is being observed, the most intense red light returning from the drop reaches the observer because it is deviated the least; the most intense violet light, however, passes over the observer because it is deviated the most. Hence, the observer sees red light coming from this drop. Similarly, a drop lower in the sky directs the most intense violet light toward the observer and appears violet to the observer. (The most intense red light from this drop passes below the observer's eye and is not seen.) The most intense light from other colors of the spectrum reaches the observer from raindrops lying between these two extreme positions.

Figure 35.25 (page 1074) shows a *double rainbow.* The secondary rainbow is fainter than the primary rainbow, and the colors are reversed. The secondary rainbow arises from light that makes two reflections from the interior surface before exiting

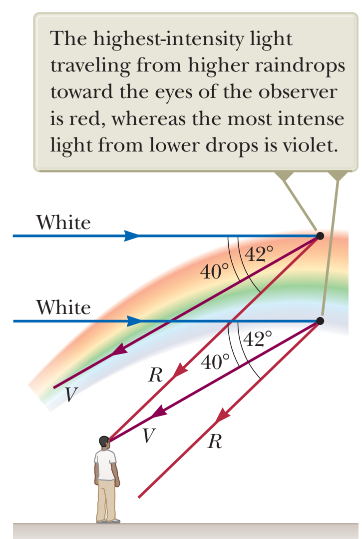


Figure 35.24 The formation of a rainbow seen by an observer standing with the Sun behind his back.

Mark D. Phillips/Photo Researchers, Inc.

Figure 35.25 This photograph of a rainbow shows a distinct secondary rainbow with the colors reversed.

the raindrop. In the laboratory, rainbows have been observed in which the light makes more than 30 reflections before exiting the water drop. Because each reflection involves some loss of light due to refraction of part of the incident light out of the water drop, the intensity of these higher-order rainbows is small compared with that of the primary rainbow.

Quick Quiz 35.4 In photography, lenses in a camera use refraction to form an image on a light-sensitive surface. Ideally, you want all the colors in the light from the object being photographed to be refracted by the same amount. Of the materials shown in Figure 35.21, which would you choose for a single-element camera lens? **(a)** crown glass **(b)** acrylic **(c)** fused quartz **(d)** impossible to determine

35.8 Total Internal Reflection

An interesting effect called **total internal reflection** can occur when light is directed from a medium having a given index of refraction toward one having a lower index of refraction. Consider Figure 35.26a, in which a light ray travels in medium 1 and meets the boundary between medium 1 and medium 2, where n_1 is greater than n_2. In the figure, labels 1 through 5 indicate various possible directions of the ray consistent with the wave under refraction model. The refracted rays are bent away from the normal because n_1 is greater than n_2. At some particular angle of incidence θ_c, called the **critical angle,** the refracted light ray moves parallel to the boundary so that $\theta_2 = 90°$ (Fig. 35.26b). For angles of incidence greater than θ_c, the ray is entirely reflected at the boundary as shown by ray 5 in Figure 35.26a.

We can use Snell's law of refraction to find the critical angle. When $\theta_1 = \theta_c$, $\theta_2 = 90°$ and Equation 35.8 gives

$$n_1 \sin \theta_c = n_2 \sin 90° = n_2$$

Critical angle for total internal reflection

$$\sin \theta_c = \frac{n_2}{n_1} \quad (\text{for } n_1 > n_2) \tag{35.10}$$

This equation can be used only when n_1 is greater than n_2. That is, total internal reflection occurs only when light is directed from a medium of a given index of refraction toward a medium of lower index of refraction. If n_1 were less than n_2,

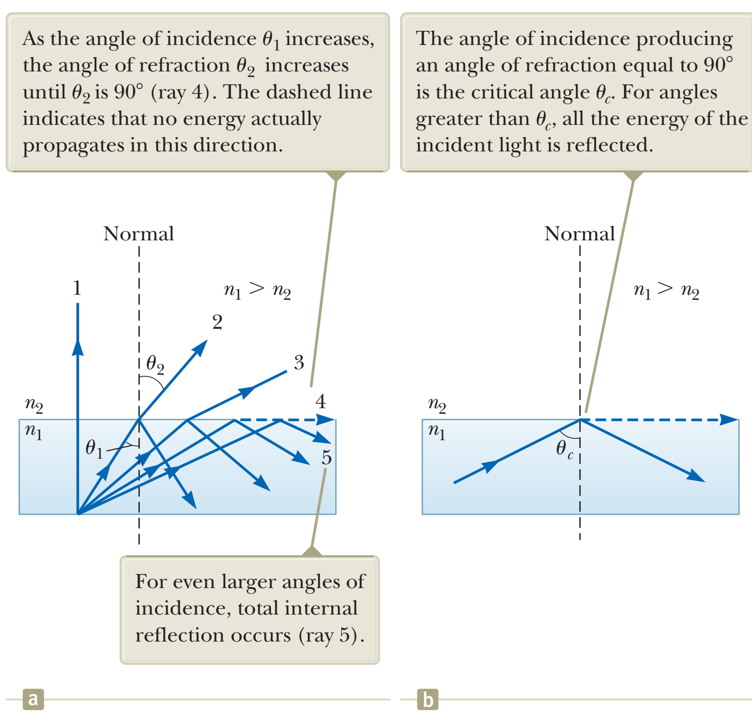


Figure 35.26 (a) Rays travel from a medium of index of refraction n_1 into a medium of index of refraction n_2, where $n_2 < n_1$. (b) Ray 4 is singled out.

Equation 35.10 would give $\sin \theta_c > 1$, which is a meaningless result because the sine of an angle can never be greater than unity.

The critical angle for total internal reflection is small when n_1 is considerably greater than n_2. For example, the critical angle for a diamond in air is 24°. Any ray inside the diamond that approaches the surface at an angle greater than 24° is completely reflected back into the crystal. This property, combined with proper faceting, causes diamonds to sparkle. The angles of the facets are cut so that light is "caught" inside the crystal through multiple internal reflections. These multiple reflections give the light a long path through the medium, and substantial dispersion of colors occurs. By the time the light exits through the top surface of the crystal, the rays associated with different colors have been fairly widely separated from one another.

Cubic zirconia also has a high index of refraction and can be made to sparkle very much like a diamond. If a suspect jewel is immersed in corn syrup, the difference in n for the cubic zirconia and that for the corn syrup is small and the critical angle is therefore great. Hence, more rays escape sooner; as a result, the sparkle completely disappears. A real diamond does not lose all its sparkle when placed in corn syrup.

Quick Quiz 35.5 In Figure 35.27, five light rays enter a glass prism from the left. **(i)** How many of these rays undergo total internal reflection at the slanted surface of the prism? (a) one (b) two (c) three (d) four (e) five **(ii)** Suppose the prism in Figure 35.27 can be rotated in the plane of the paper. For *all five* rays to experience total internal reflection from the slanted surface, should the prism be rotated (a) clockwise or (b) counterclockwise?

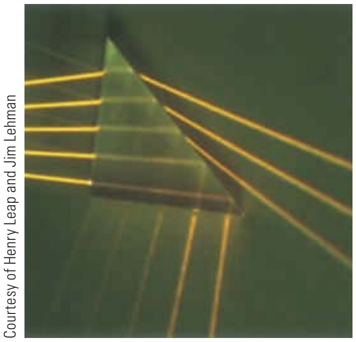


Figure 35.27 (Quick Quiz 35.5) Five nonparallel light rays enter a glass prism from the left.

Example 35.6 A View from the Fish's Eye

Find the critical angle for an air–water boundary. (Assume the index of refraction of water is 1.33.)

SOLUTION

Conceptualize Study Figure 35.26 to understand the concept of total internal reflection and the significance of the critical angle.

Categorize We use concepts developed in this section, so we categorize this example as a substitution problem.

Apply Equation 35.10 to the air–water interface:

$$\sin \theta_c = \frac{n_2}{n_1} = \frac{1.00}{1.33} = 0.752$$

$$\theta_c = \boxed{48.8°}$$

WHAT IF? What if a fish in a still pond looks upward toward the water's surface at different angles relative to the surface as in Figure 35.28? What does it see?

Answer Because the path of a light ray is reversible, light traveling from medium 2 into medium 1 in Figure 35.26a follows the paths shown, but in the *opposite* direction. A fish looking upward toward the water surface as in Figure 35.28 can see out of the water if it looks toward the surface at an angle less than the critical angle. Therefore, when the fish's line of vision makes an angle of $\theta = 40°$ with the normal to the surface, for example, light from above the water reaches the fish's eye. At $\theta = 48.8°$, the critical angle for water, the light has to skim along the water's surface before being refracted to the fish's eye; at this angle, the fish can, in principle, see the entire shore of the pond. At angles greater than the critical angle, the light reaching the fish comes by means of total internal reflection at the surface. Therefore, at $\theta = 60°$, the fish sees a reflection of the bottom of the pond.

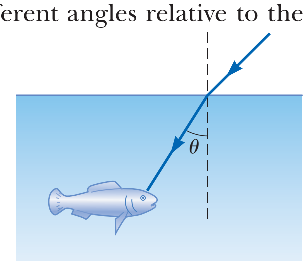


Figure 35.28 (Example 35.6) **What If?** A fish looks upward toward the water surface.

Optical Fibers

Another interesting application of total internal reflection is the use of glass or transparent plastic rods to "pipe" light from one place to another. As indicated in Figure 35.29 (page 1076), light is confined to traveling within a rod, even around curves, as the result of successive total internal reflections. Such a light pipe is flexible if thin fibers are used rather than thick rods. A flexible light pipe is called an **optical fiber.** If a bundle of parallel fibers is used to construct an optical transmission line, images can be transferred from one point to another. Part of the 2009 Nobel Prize in Physics was awarded to Charles K. Kao (b. 1933) for his discovery of how to transmit light signals over long distances through thin glass fibers. This discovery has led to the development of a sizable industry known as *fiber optics*.

A practical optical fiber consists of a transparent core surrounded by a *cladding*, a material that has a lower index of refraction than the core. The combination may be surrounded by a plastic *jacket* to prevent mechanical damage. Figure 35.30 shows a cutaway view of this construction. Because the index of refraction of the cladding is less than that of the core, light traveling in the core experiences total internal reflection if it arrives at the interface between the core and the cladding at an angle of incidence that exceeds the critical angle. In this case, light "bounces" along the core of the optical fiber, losing very little of its intensity as it travels.

Any loss in intensity in an optical fiber is essentially due to reflections from the two ends and absorption by the fiber material. Optical fiber devices are particularly useful for viewing an object at an inaccessible location. For example, physicians often use such devices to examine internal organs of the body or to perform surgery without making large incisions. Optical fiber cables are replacing copper wiring and coaxial cables for telecommunications because the fibers can carry a much greater volume of telephone calls or other forms of communication than electrical wires can.

Figure 35.31a shows a bundle of optical fibers gathered into an optical cable that can be used to carry communication signals. Figure 35.31b shows laser light following the curves of a coiled bundle by total internal reflection. Many computers and other electronic equipment now have optical ports as well as electrical ports for transferring information.

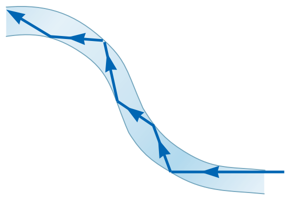


Figure 35.29 Light travels in a curved transparent rod by multiple internal reflections.

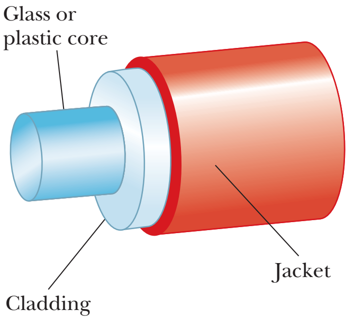


Figure 35.30 The construction of an optical fiber. Light travels in the core, which is surrounded by a cladding and a protective jacket.

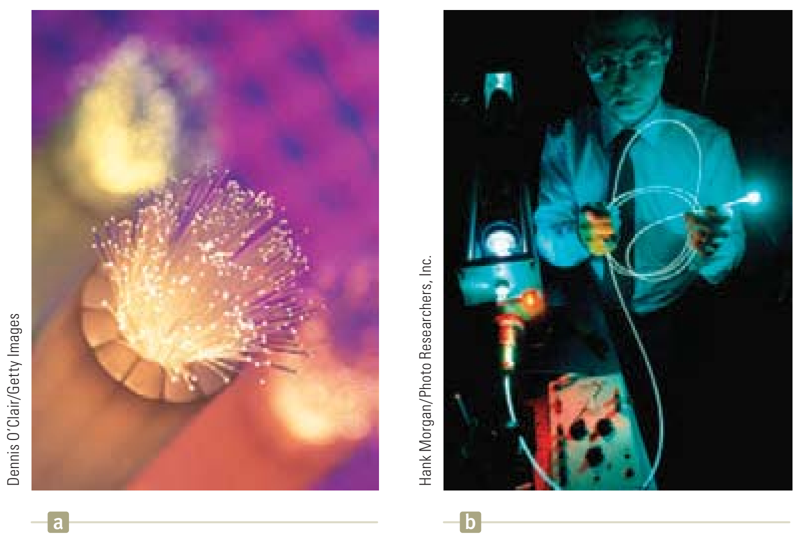


Figure 35.31 (a) Strands of glass optical fibers are used to carry voice, video, and data signals in telecommunication networks. (b) A bundle of optical fibers is illuminated by a laser.

Summary

Definition

The **index of refraction** n of a medium is defined by the ratio

$$n \equiv \frac{c}{v} \qquad (35.4)$$

where c is the speed of light in vacuum and v is the speed of light in the medium.

Concepts and Principles

In geometric optics, we use the **ray approximation,** in which a wave travels through a uniform medium in straight lines in the direction of the rays.

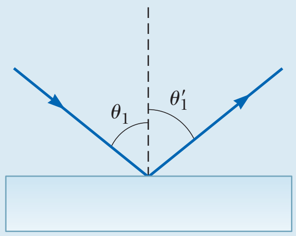
Total internal reflection occurs when light travels from a medium of high index of refraction to one of lower index of refraction. The **critical angle** θ_c for which total internal reflection occurs at an interface is given by

$$\sin \theta_c = \frac{n_2}{n_1} \quad (\text{for } n_1 > n_2) \tag{35.10}$$

Analysis Models for Problem Solving

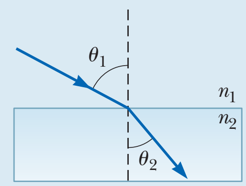
Wave Under Reflection. The **law of reflection** states that for a light ray (or other type of wave) incident on a smooth surface, the angle of reflection θ_1' equals the angle of incidence θ_1:

$$\theta_1' = \theta_1 \tag{35.2}$$



Wave Under Refraction. A wave crossing a boundary as it travels from medium 1 to medium 2 is **refracted.** The angle of refraction θ_2 is related to the incident angle θ_1 by the relationship

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1} \tag{35.3}$$



where v_1 and v_2 are the speeds of the wave in medium 1 and medium 2, respectively. The incident ray, the reflected ray, the refracted ray, and the normal to the surface all lie in the same plane.

For light waves, **Snell's law of refraction** states that

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \tag{35.8}$$

where n_1 and n_2 are the indices of refraction in the two media.

Objective Questions

1. denotes answer available in *Student Solutions Manual/Study Guide*

1. In each of the following situations, a wave passes through an opening in an absorbing wall. Rank the situations in order from the one in which the wave is best described by the ray approximation to the one in which the wave coming through the opening spreads out most nearly equally in all directions in the hemisphere beyond the wall. (a) The sound of a low whistle at 1 kHz passes through a doorway 1 m wide. (b) Red light passes through the pupil of your eye. (c) Blue light passes through the pupil of your eye. (d) The wave broadcast by an AM radio station passes through a doorway 1 m wide. (e) An x-ray passes through the space between bones in your elbow joint.

2. A source emits monochromatic light of wavelength 495 nm in air. When the light passes through a liquid, its wavelength reduces to 434 nm. What is the liquid's index of refraction? (a) 1.26 (b) 1.49 (c) 1.14 (d) 1.33 (e) 2.03

3. Carbon disulfide ($n = 1.63$) is poured into a container made of crown glass ($n = 1.52$). What is the critical angle for total internal reflection of a light ray in the liquid when it is incident on the liquid-to-glass surface? (a) 89.2° (b) 68.8° (c) 21.2° (d) 1.07° (e) 43.0°

4. A light wave moves between medium 1 and medium 2. Which of the following are correct statements relating its speed, frequency, and wavelength in the two media, the indices of refraction of the media, and the angles of incidence and refraction? More than one statement may be correct. (a) $v_1/\sin\theta_1 = v_2/\sin\theta_2$ (b) $\csc\theta_1/n_1 = \csc\theta_2/n_2$ (c) $\lambda_1/\sin\theta_1 = \lambda_2/\sin\theta_2$ (d) $f_1/\sin\theta_1 = f_2/\sin\theta_2$ (e) $n_1/\cos\theta_1 = n_2/\cos\theta_2$

5. What happens to a light wave when it travels from air into glass? (a) Its speed remains the same. (b) Its speed increases. (c) Its wavelength increases. (d) Its wavelength remains the same. (e) Its frequency remains the same.

6. The index of refraction for water is about $\frac{4}{3}$. What happens as a beam of light travels from air into water? (a) Its speed increases to $\frac{4}{3}c$, and its frequency decreases. (b) Its speed decreases to $\frac{3}{4}c$, and its wavelength decreases by a factor of $\frac{3}{4}$. (c) Its speed decreases to $\frac{3}{4}c$, and its wavelength increases by a factor of $\frac{4}{3}$. (d) Its speed and frequency remain the same. (e) Its speed decreases to $\frac{3}{4}c$, and its frequency increases.

7. Light can travel from air into water. Some possible paths for the light ray in the water are shown in Figure

OQ35.7. Which path will the light most likely follow? (a) A (b) B (c) C (d) D (e) E

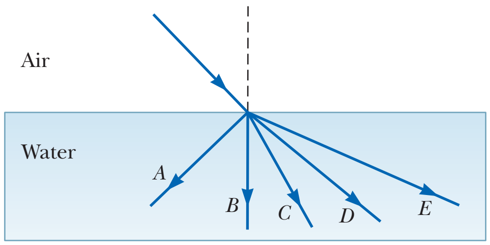


Figure OQ35.7

8. What is the order of magnitude of the time interval required for light to travel 10 km as in Galileo's attempt to measure the speed of light? (a) several seconds (b) several milliseconds (c) several microseconds (d) several nanoseconds

9. A light ray containing both blue and red wavelengths is incident at an angle on a slab of glass. Which of the sketches in Figure OQ35.9 represents the most likely outcome? (a) A (b) B (c) C (d) D (e) none of them

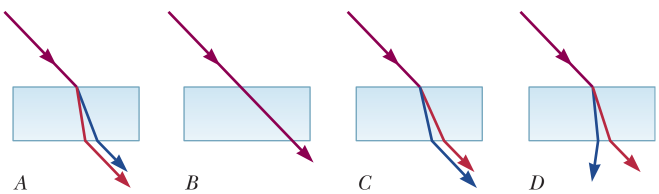


Figure OQ35.9

10. For the following questions, choose from the following possibilities: (a) yes; water (b) no; water (c) yes; air (d) no; air. **(i)** Can light undergo total internal reflection at a smooth interface between air and water? If so, in which medium must it be traveling originally? **(ii)** Can sound undergo total internal reflection at a smooth interface between air and water? If so, in which medium must it be traveling originally?

11. A light ray travels from vacuum into a slab of material with index of refraction n_1 at incident angle θ with respect to the surface. It subsequently passes into a second slab of material with index of refraction n_2 before passing back into vacuum again. The surfaces of the different materials are all parallel to one another. As the light exits the second slab, what can be said of the final angle ϕ that the outgoing light makes with the normal? (a) $\phi > \theta$ (b) $\phi < \theta$ (c) $\phi = \theta$ (d) The angle depends on the magnitudes of n_1 and n_2. (e) The angle depends on the wavelength of the light.

12. Suppose you find experimentally that two colors of light, A and B, originally traveling in the same direction in air, are sent through a glass prism, and A changes direction more than B. Which travels more slowly in the prism, A or B? Alternatively, is there insufficient information to determine which moves more slowly?

13. The core of an optical fiber transmits light with minimal loss if it is surrounded by what? (a) water (b) diamond (c) air (d) glass (e) fused quartz

14. Which color light refracts the most when entering crown glass from air at some incident angle θ with respect to the normal? (a) violet (b) blue (c) green (d) yellow (e) red

15. Light traveling in a medium of index of refraction n_1 is incident on another medium having an index of refraction n_2. Under which of the following conditions can total internal reflection occur at the interface of the two media? (a) The indices of refraction have the relation $n_2 > n_1$. (b) The indices of refraction have the relation $n_1 > n_2$. (c) Light travels slower in the second medium than in the first. (d) The angle of incidence is less than the critical angle. (e) The angle of incidence must equal the angle of refraction.

Conceptual Questions

1. denotes answer available in *Student Solutions Manual/Study Guide*

1. The level of water in a clear, colorless glass can easily be observed with the naked eye. The level of liquid helium in a clear glass vessel is extremely difficult to see with the naked eye. Explain.

2. A complete circle of a rainbow can sometimes be seen from an airplane. With a stepladder, a lawn sprinkler, and a sunny day, how can you show the complete circle to children?

3. You take a child for walks around the neighborhood. She loves to listen to echoes from houses when she shouts or when you clap loudly. A house with a large, flat front wall can produce an echo if you stand straight in front of it and reasonably far away. (a) Draw a bird's-eye view of the situation to explain the production of the echo. Shade the area where you can stand to hear the echo. For parts (b) through (e), explain your answers with diagrams. (b) **What If?** The child helps you discover that a house with an L-shaped floor plan can produce echoes if you are standing in a wider range of locations. You can be standing at any reasonably distant location from which you can see the inside corner. Explain the echo in this case and compare with your diagram in part (a). (c) **What If?** What if the two wings of the house are not perpendicular? Will you and the child, standing close together, hear echoes? (d) **What If?** What if a rectangular house and its garage have perpendicular walls that would form an inside corner but have a breezeway between them so that the walls do not meet? Will the structure produce strong echoes for people in a wide range of locations?

4. The F-117A stealth fighter (Fig. CQ35.4) is specifically designed to be a *non*retroreflector of radar. What aspects of its design help accomplish this purpose?

Courtesy U.S. Air Force

Figure CQ35.4

5. Retroreflection by transparent spheres, mentioned in Section 35.4, can be observed with dewdrops. To do so, look at your head's shadow where it falls on dewy grass. The optical display around the shadow of your head is called *heiligenschein,* which is German for *holy light.* Renaissance artist Benvenuto Cellini described the phenomenon and his reaction in his *Autobiography,* at the end of Part One, and American philosopher Henry David Thoreau did the same in *Walden,* "Baker Farm," second paragraph. Do some Internet research to find out more about the heiligenschein.

6. Sound waves have much in common with light waves, including the properties of reflection and refraction. Give an example of each of these phenomena for sound waves.

7. Total internal reflection is applied in the periscope of a submerged submarine to let the user observe events above the water surface. In this device, two prisms are arranged as shown in Figure CQ35.7 so that an incident beam of light follows the path shown. Parallel tilted, silvered mirrors could be used, but glass prisms with no silvered surfaces give higher light throughput. Propose a reason for the higher efficiency.

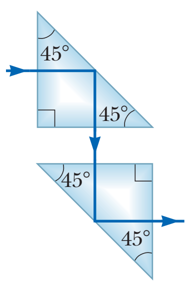


Figure CQ35.7

8. Explain why a diamond sparkles more than a glass crystal of the same shape and size.

9. A laser beam passing through a nonhomogeneous sugar solution follows a curved path. Explain.

10. The display windows of some department stores are slanted slightly inward at the bottom. This tilt is to decrease the glare from streetlights and the Sun, which would make it difficult for shoppers to see the display inside. Sketch a light ray reflecting from such a window to show how this design works.

11. At one restaurant, a worker uses colored chalk to write the daily specials on a blackboard illuminated with a spotlight. At another restaurant, a worker writes with colored grease pencils on a flat, smooth sheet of transparent acrylic plastic with an index of refraction 1.55. The panel hangs in front of a piece of black felt. Small, bright fluorescent tube lights are installed all along the edges of the sheet, inside an opaque channel. Figure CQ35.11 shows a cutaway view of the sign. (a) Explain why viewers at both restaurants see the letters shining against a black background. (b) Explain why the sign at the second restaurant may use less energy from the electric company than the illuminated blackboard at the first restaurant. (c) What would be a good choice for the index of refraction of the material in the grease pencils?

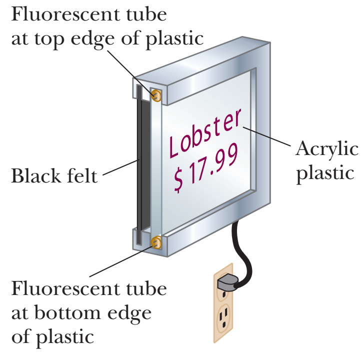


Figure CQ35.11

12. (a) Under what conditions is a mirage formed? While driving on a hot day, sometimes you see what appears to be water on the road far ahead. When you arrive at the location of the water, however, the road is perfectly dry. Explain this phenomenon. (b) The mirage called *fata morgana* often occurs over water or in cold regions covered with snow or ice. It can cause islands to sometimes become visible, even though they are not normally visible because they are below the horizon due to the curvature of the Earth. Explain this phenomenon.

13. Figure CQ35.13 shows a pencil partially immersed in a cup of water. Why does the pencil appear to be bent?

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Figure CQ35.13

14. A scientific supply catalog advertises a material having an index of refraction of 0.85. Is that a good product to buy? Why or why not?

15. Why do astronomers looking at distant galaxies talk about looking backward in time?

16. Try this simple experiment on your own. Take two opaque cups, place a coin at the bottom of each cup near the edge, and fill one cup with water. Next, view the cups at some angle from the side so that the coin in water is just visible as shown on the left in Figure CQ35.16. Notice that the coin in air is not visible as shown on the right in Figure CQ35.16. Explain this observation.

Figure CQ35.16

17. Figure CQ35.17a shows a desk ornament globe containing a photograph. The flat photograph is in air, inside a vertical slot located behind a water-filled compartment having the shape of one half of a cylinder. Suppose you are looking at the center of the photograph and then rotate the globe about a vertical axis. You find that the center of the photograph disappears when you rotate the globe beyond a certain maximum angle (Fig. CQ35.17b). (a) Account for this phenomenon and (b) describe what you see when you turn the globe beyond this angle.

Figure CQ35.17

Problems

The problems found in this chapter may be assigned online in Enhanced WebAssign

1. straightforward; **2.** intermediate; **3.** challenging

1. full solution available in the *Student Solutions Manual/Study Guide*

AMT Analysis Model tutorial available in Enhanced WebAssign

GP Guided Problem

M Master It tutorial available in Enhanced WebAssign

W Watch It video solution available in Enhanced WebAssign

Section 35.1 The Nature of Light

Section 35.2 Measurements of the Speed of Light

1. M Find the energy of (a) a photon having a frequency of 5.00×10^{17} Hz and (b) a photon having a wavelength of 3.00×10^{2} nm. Express your answers in units of electron volts, noting that 1 eV = 1.60×10^{-19} J.

2. The *Apollo 11* astronauts set up a panel of efficient corner-cube retroreflectors on the Moon's surface (Fig. 35.8a). The speed of light can be found by measuring the time interval required for a laser beam to travel from the Earth, reflect from the panel, and return to the Earth. Assume this interval is measured to be 2.51 s at a station where the Moon is at the zenith and take the center-to-center distance from the Earth to the Moon to be equal to 3.84×10^{8} m. (a) What is the measured speed of light? (b) Explain whether it is necessary to consider the sizes of the Earth and the Moon in your calculation.

3. AMT M In an experiment to measure the speed of light using the apparatus of Armand H. L. Fizeau (see Fig. 35.2), the distance between light source and mirror was 11.45 km and the wheel had 720 notches. The experimentally determined value of c was 2.998×10^{8} m/s when the outgoing light passed through one notch and then returned through the next notch. Calculate the minimum angular speed of the wheel for this experiment.

4. As a result of his observations, Ole Roemer concluded that eclipses of Io by Jupiter were delayed by 22 min during a six-month period as the Earth moved from the point in its orbit where it is closest to Jupiter to the diametrically opposite point where it is farthest from Jupiter. Using the value 1.50×10^{8} km as the average radius of the Earth's orbit around the Sun, calculate the speed of light from these data.

Section 35.3 The Ray Approximation in Ray Optics

Section 35.4 Analysis Model: Wave Under Reflection

Section 35.5 Analysis Model: Wave Under Refraction

Notes: You may look up indices of refraction in Table 35.1. Unless indicated otherwise, assume the medium surrounding a piece of material is air with $n = 1.000\ 293$.

5. W The wavelength of red helium–neon laser light in air is 632.8 nm. (a) What is its frequency? (b) What is its wavelength in glass that has an index of refraction of 1.50? (c) What is its speed in the glass?

6. W An underwater scuba diver sees the Sun at an apparent angle of 45.0° above the horizontal. What is the actual elevation angle of the Sun above the horizontal?

7. A ray of light is incident on a flat surface of a block of crown glass that is surrounded by water. The angle of refraction is 19.6°. Find the angle of reflection.

8. W Figure P35.8 shows a refracted light beam in linseed oil making an angle of $\phi = 20.0°$ with the normal line NN'. The index of refraction of linseed oil is 1.48. Determine the angles (a) θ and (b) θ'.

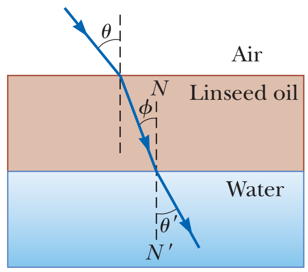

Figure P35.8

9. Find the speed of light in (a) flint glass, (b) water, and (c) cubic zirconia.

10. A dance hall is built without pillars and with a horizontal ceiling 7.20 m above the floor. A mirror is fastened flat against one section of the ceiling. Following an earthquake, the mirror is in place and unbroken. An engineer makes a quick check of whether the ceiling is sagging by directing a vertical beam of laser light up at the mirror and observing its reflection on the floor. (a) Show that if the mirror has rotated to make an angle ϕ with the horizontal, the normal to the mirror makes an angle ϕ with the vertical. (b) Show that the reflected laser light makes an angle 2ϕ with the vertical. (c) Assume the reflected laser light makes a spot on the floor 1.40 cm away from the point vertically below the laser. Find the angle ϕ.

11. A ray of light travels from air into another medium, making an angle of $\theta_1 = 45.0°$ with the normal as in Figure P35.11. Find the angle of refraction θ_2 if the second medium is (a) fused quartz, (b) carbon disulfide, and (c) water.

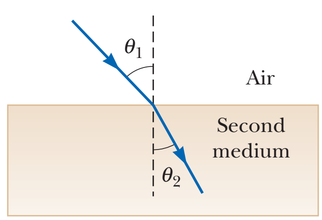

Figure P35.11

12. A ray of light strikes a flat block of glass ($n = 1.50$) of thickness 2.00 cm at an angle of 30.0° with the normal. Trace the light beam through the glass and find the angles of incidence and refraction at each surface.

13. M A prism that has an apex angle of 50.0° is made of cubic zirconia. What is its minimum angle of deviation?

14. W A plane sound wave in air at 20°C, with wavelength 589 mm, is incident on a smooth surface of water at 25°C at an angle of incidence of 13.0°. Determine (a) the angle of refraction for the sound wave and (b) the wavelength of the sound in water. A narrow beam of sodium yellow light, with wavelength 589 nm in vacuum, is incident from air onto a smooth water surface at an angle of incidence of 13.0°. Determine (c) the angle of refraction and (d) the wavelength of the light in water. (e) Compare and contrast the behavior of the sound and light waves in this problem.

15. A light ray initially in water enters a transparent substance at an angle of incidence of 37.0°, and the transmitted ray is refracted at an angle of 25.0°. Calculate the speed of light in the transparent substance.

16. A laser beam is incident at an angle of 30.0° from the vertical onto a solution of corn syrup in water. The beam is refracted to 19.24° from the vertical. (a) What is the index of refraction of the corn syrup solution? Assume that the light is red, with vacuum wavelength 632.8 nm. Find its (b) wavelength, (c) frequency, and (d) speed in the solution.

17. M A ray of light strikes the midpoint of one face of an equiangular (60°–60°–60°) glass prism ($n = 1.5$) at an angle of incidence of 30°. (a) Trace the path of the light ray through the glass and find the angles of incidence and refraction at each surface. (b) If a small fraction of light is also reflected at each surface, what are the angles of reflection at the surfaces?

18. The reflecting surfaces of two intersecting flat mirrors are at an angle θ ($0° < \theta < 90°$) as shown in Figure P35.18. For a light ray that strikes the horizontal mirror, show that the emerging ray will intersect the incident ray at an angle $\beta = 180° - 2\theta$.

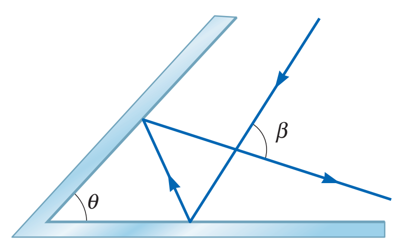

Figure P35.18

19. When you look through a window, by what time interval is the light you see delayed by having to go through glass instead of air? Make an order-of-magnitude estimate on the basis of data you specify. By how many wavelengths is it delayed?

20. Two flat, rectangular mirrors, both perpendicular to a horizontal sheet of paper, are set edge to edge with their reflecting surfaces perpendicular to each other. (a) A light ray in the plane of the paper strikes one of the mirrors at an arbitrary angle of incidence θ_1. Prove that the final direction of the ray, after reflection from both mirrors, is opposite its initial direction. (b) **What If?** Now assume the paper is replaced with a third flat mirror, touching edges with the other two and perpendicular to both, creating a corner-cube retroreflector (Fig. 35.8a). A ray of light is incident from any direction within the octant of space bounded by the reflecting surfaces. Argue that the ray will reflect once from each mirror and that its final direction will be opposite its original direction. The *Apollo 11* astronauts

placed a panel of corner-cube retroreflectors on the Moon. Analysis of timing data taken with it reveals that the radius of the Moon's orbit is increasing at the rate of 3.8 cm/yr as it loses kinetic energy because of tidal friction.

21. W The two mirrors illustrated in Figure P35.21 meet at a right angle. The beam of light in the vertical plane indicated by the dashed lines strikes mirror 1 as shown. (a) Determine the distance the reflected light beam travels before striking mirror 2. (b) In what direction does the light beam travel after being reflected from mirror 2?

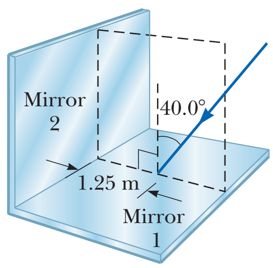

Figure P35.21

22. W When the light ray illustrated in Figure P35.22 passes through the glass block of index of refraction $n = 1.50$, it is shifted laterally by the distance d. (a) Find the value of d. (b) Find the time interval required for the light to pass through the glass block.

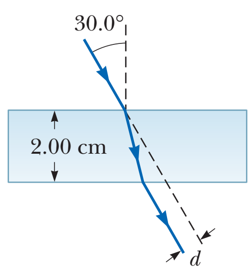

Figure P35.22

23. Two light pulses are emitted simultaneously from a source. Both pulses travel through the same total length of air to a detector, but mirrors shunt one pulse along a path that carries it through an extra length of 6.20 m of ice along the way. Determine the difference in the pulses' times of arrival at the detector.

24. Light passes from air into flint glass at a nonzero angle of incidence. (a) Is it possible for the component of its velocity perpendicular to the interface to remain constant? Explain your answer. (b) **What If?** Can the component of velocity parallel to the interface remain constant during refraction? Explain your answer.

25. A laser beam with vacuum wavelength 632.8 nm is incident from air onto a block of Lucite as shown in Figure 35.10b. The line of sight of the photograph is perpendicular to the plane in which the light moves. Find (a) the speed, (b) the frequency, and (c) the wavelength of the light in the Lucite. *Suggestion:* Use a protractor.

26. A narrow beam of ultrasonic waves reflects off the liver tumor illustrated in Figure P35.26. The speed of the wave is 10.0% less in the liver than in the surrounding medium. Determine the depth of the tumor.

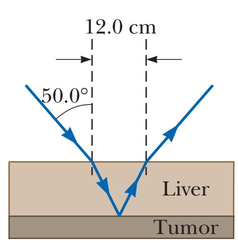

Figure P35.26

27. AMT M W An opaque cylindrical tank with an open top has a diameter of 3.00 m and is completely filled with water. When the afternoon Sun reaches an angle of 28.0° above the horizon, sunlight ceases to illuminate any part of the bottom of the tank. How deep is the tank?

28. A triangular glass prism with apex angle 60.0° has an index of refraction of 1.50. (a) Show that if its angle of incidence on the first surface is $\theta_1 = 48.6°$, light will pass symmetrically through the prism as shown in Figure 35.17. (b) Find the angle of deviation δ_{min} for $\theta_1 = 48.6°$. (c) **What If?** Find the angle of deviation if the angle of incidence on the first surface is 45.6°. (d) Find the angle of deviation if $\theta_1 = 51.6°$.

29. Light of wavelength 700 nm is incident on the face of a fused quartz prism ($n = 1.458$ at 700 nm) at an incidence angle of 75.0°. The apex angle of the prism is 60.0°. Calculate the angle (a) of refraction at the first surface, (b) of incidence at the second surface, (c) of refraction at the second surface, and (d) between the incident and emerging rays.

30. Figure P35.30 shows a light ray incident on a series of slabs having different refractive indices, where $n_1 < n_2 < n_3 < n_4$. Notice that the path of the ray steadily bends toward the normal. If the variation in n were continuous, the path would form a smooth curve. Use this idea and a ray diagram to explain why you can see the Sun at sunset after it has fallen below the horizon.

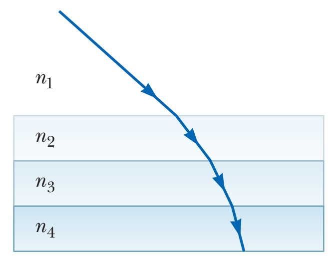

Figure P35.30

31. Three sheets of plastic have unknown indices of refraction. Sheet 1 is placed on top of sheet 2, and a laser beam is directed onto the sheets from above. The laser beam enters sheet 1 and then strikes the interface between sheet 1 and sheet 2 at an angle of 26.5° with the normal. The refracted beam in sheet 2 makes an angle of 31.7° with the normal. The experiment is repeated with sheet 3 on top of sheet 2, and, with the same angle of incidence on the sheet 3–sheet 2 interface, the refracted beam makes an angle of 36.7° with the normal. If the experiment is repeated again with sheet 1 on top of sheet 3, with that same angle of incidence on the sheet 1–sheet 3 interface, what is the expected angle of refraction in sheet 3?

32. A person looking into an empty container is able to see the far edge of the container's bottom as shown in Figure P35.32a. The height of the container is h, and its width is d. When the container is completely filled with a fluid of index of refraction n and viewed from the same angle, the person can see the center of a coin at

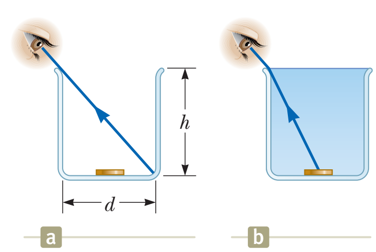

Figure P35.32

the middle of the container's bottom as shown in Figure P35.32b. (a) Show that the ratio h/d is given by

$$\frac{h}{d} = \sqrt{\frac{n^2 - 1}{4 - n^2}}$$

(b) Assuming the container has a width of 8.00 cm and is filled with water, use the expression above to find the height of the container. (c) For what range of values of n will the center of the coin not be visible for any values of h and d?

33. A laser beam is incident on a 45°–45°–90° prism perpendicular to one of its faces as shown in Figure P35.33. The transmitted beam that exits the hypotenuse of the prism makes an angle of $\theta = 15.0°$ with the direction of the incident beam. Find the index of refraction of the prism.

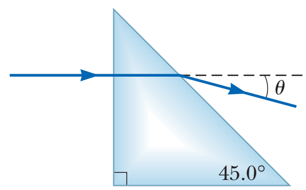

Figure P35.33

34. GP A submarine is 300 m horizontally from the shore of a freshwater lake and 100 m beneath the surface of the water. A laser beam is sent from the submarine so that the beam strikes the surface of the water 210 m from the shore. A building stands on the shore, and the laser beam hits a target at the top of the building. The goal is to find the height of the target above sea level. (a) Draw a diagram of the situation, identifying the two triangles that are important in finding the solution. (b) Find the angle of incidence of the beam striking the water–air interface. (c) Find the angle of refraction. (d) What angle does the refracted beam make with the horizontal? (e) Find the height of the target above sea level.

35. A beam of light both reflects and refracts at the surface between air and glass as shown in Figure P35.35. If the refractive index of the glass is n_g, find the angle of incidence θ_1 in the air that would result in the reflected ray and the refracted ray being perpendicular to each other.

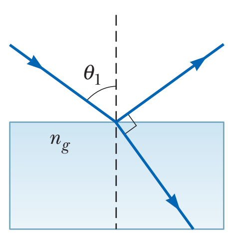

Figure P35.35

Section 35.6 Huygens's Principle

Section 35.7 Dispersion

36. The index of refraction for red light in water is 1.331 and that for blue light is 1.340. If a ray of white light enters the water at an angle of incidence of 83.0°, what are the underwater angles of refraction for the (a) red and (b) blue components of the light?

37. A light beam containing red and violet wavelengths is incident on a slab of quartz at an angle of incidence of 50.0°. The index of refraction of quartz is 1.455 at 600 nm (red light), and its index of refraction is 1.468 at 410 nm (violet light). Find the dispersion of the slab, which is defined as the difference in the angles of refraction for the two wavelengths.

38. The speed of a water wave is described by $v = \sqrt{gd}$, where d is the water depth, assumed to be small compared to the wavelength. Because their speed changes, water waves refract when moving into a region of different depth. (a) Sketch a map of an ocean beach on the eastern side of a landmass. Show contour lines of constant depth under water, assuming a reasonably uniform slope. (b) Suppose waves approach the coast from a storm far away to the north–northeast. Demonstrate that the waves move nearly perpendicular to the shoreline when they reach the beach. (c) Sketch a map of a coastline with alternating bays and headlands as suggested in Figure P35.38. Again make a reasonable guess about the shape of contour lines of constant depth. (d) Suppose waves approach the coast, carrying energy with uniform density along originally straight wave fronts. Show that the energy reaching the coast is concentrated at the headlands and has lower intensity in the bays.

Andy Ryan/Stone/Getty Images

Figure P35.38

39. M The index of refraction for violet light in silica flint glass is 1.66, and that for red light is 1.62. What is the angular spread of visible light passing through a prism of apex angle 60.0° if the angle of incidence is 50.0°? See Figure P35.39.

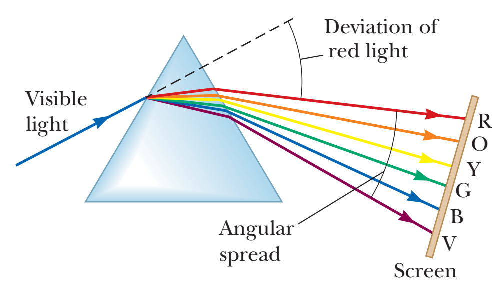

Figure P35.39 Problems 39 and 40.

40. The index of refraction for violet light in silica flint glass is n_V, and that for red light is n_R. What is the angular spread of visible light passing through a prism of apex angle Φ if the angle of incidence is θ? See Figure P35.39.

Section 35.8 Total Internal Reflection

41. A glass optical fiber ($n = 1.50$) is submerged in water ($n = 1.33$). What is the critical angle for light to stay inside the fiber?

42. For 589-nm light, calculate the critical angle for the following materials surrounded by air: (a) cubic zirconia, (b) flint glass, and (c) ice.

43. A triangular glass prism with apex angle $\Phi = 60.0°$ has an index of refraction $n = 1.50$ (Fig. P35.43). What is the smallest angle of incidence θ_1 for which a light ray can emerge from the other side?

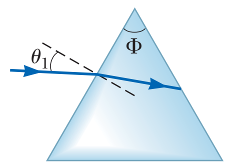

Figure P35.43
Problems 43 and 44.

44. A triangular glass prism with apex angle Φ has an index of refraction n (Fig. P35.43). What is the smallest angle of incidence θ_1 for which a light ray can emerge from the other side?

45. Assume a transparent rod of diameter $d = 2.00\ \mu$m has an index of refraction of 1.36. Determine the maximum angle θ for which the light rays incident on the end of the rod in Figure P35.45 are subject to total internal reflection along the walls of the rod. Your answer defines the size of the *cone of acceptance* for the rod.

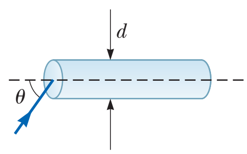

Figure P35.45

46. Consider a light ray traveling between air and a diamond cut in the shape shown in Figure P35.46. (a) Find the critical angle for total internal reflection for light in the diamond incident on the interface between the diamond and the outside air. (b) Consider the light ray incident normally on the top surface of the diamond as shown in Figure P35.46. Show that the light traveling toward point P in the diamond is totally reflected. **What If?** Suppose the diamond is immersed in water. (c) What is the critical angle at the diamond–water interface? (d) When the diamond is immersed in water, does the light ray entering the top surface in Figure P35.46 undergo total internal reflection at P? Explain. (e) If the light ray entering the diamond remains vertical as shown in Figure P35.46, which way should the diamond in the water be rotated about an axis perpendicular to the page through O so that light will exit the diamond at P? (f) At what angle of rotation in part (e) will light first exit the diamond at point P?

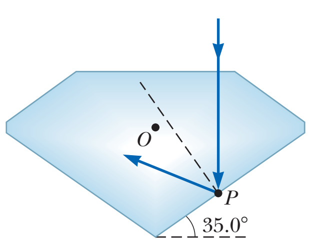

Figure P35.46

47. Consider a common mirage formed by superheated air immediately above a roadway. A truck driver whose eyes are 2.00 m above the road, where $n = 1.000\ 293$, looks forward. She perceives the illusion of a patch of water ahead on the road. The road appears wet only beyond a point on the road at which her line of sight makes an angle of 1.20° below the horizontal. Find the index of refraction of the air immediately above the road surface.

48. A room contains air in which the speed of sound is 343 m/s. The walls of the room are made of concrete in which the speed of sound is 1 850 m/s. (a) Find the critical angle for total internal reflection of sound at the concrete–air boundary. (b) In which medium must the sound be initially traveling if it is to undergo total internal reflection? (c) "A bare concrete wall is a highly efficient mirror for sound." Give evidence for or against this statement.

49. An optical fiber has an index of refraction n and diameter d. It is surrounded by vacuum. Light is sent into the fiber along its axis as shown in Figure P35.49. (a) Find the smallest outside radius R_{min} permitted for a bend in the fiber if no light is to escape. (b) **What If?** What result does part (a) predict as d approaches zero? Is this behavior reasonable? Explain. (c) As n increases? (d) As n approaches 1? (e) Evaluate R_{min} assuming the fiber diameter is 100 μm and its index of refraction is 1.40.

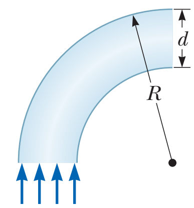

Figure P35.49

50. Around 1968, Richard A. Thorud, an engineer at The Toro Company, invented a gasoline gauge for small engines diagrammed in Figure P35.50. The gauge has no moving parts. It consists of a flat slab of transparent plastic fitting vertically into a slot in the cap on the gas tank. None of the plastic has a reflective coating. The plastic projects from the horizontal top down nearly to the bottom of the opaque tank. Its lower edge is cut with facets making angles of 45° with the horizontal. A lawn mower operator looks down from above and sees a boundary between bright and dark on the gauge. The location of the boundary, across the width of the plastic, indicates the quantity of gasoline in the tank. (a) Explain how the gauge works. (b) Explain the design requirements, if any, for the index of refraction of the plastic.

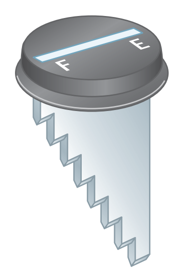

Figure P35.50

Additional Problems

51. A beam of light is incident from air on the surface of a liquid. If the angle of incidence is 30.0° and the angle of refraction is 22.0°, find the critical angle for total internal reflection for the liquid when surrounded by air.

52. Consider a horizontal interface between air above and glass of index of refraction 1.55 below. (a) Draw a light ray incident from the air at angle of incidence 30.0°. Determine the angles of the reflected and refracted rays and show them on the diagram. (b) **What If?** Now suppose the light ray is incident from the glass at an angle of 30.0°. Determine the angles of the reflected and refracted rays and show all three rays on a new diagram. (c) For rays incident from the air onto the air–glass surface, determine and tabulate the angles of reflection and refraction for all the angles of incidence at 10.0° intervals from 0° to 90.0°. (d) Do the same for light rays coming up to the interface through the glass.

53. M A small light fixture on the bottom of a swimming pool is 1.00 m below the surface. The light emerging from the still water forms a circle on the water surface. What is the diameter of this circle?

54. *Why is the following situation impossible?* While at the bottom of a calm freshwater lake, a scuba diver sees the Sun at an apparent angle of 38.0° above the horizontal.

55. A digital video disc (DVD) records information in a spiral track approximately 1 μm wide. The track consists of a series of pits in the information layer (Fig. P35.55a) that scatter light from a laser beam sharply focused on them. The laser shines in from below through transparent plastic of thickness t = 1.20 mm and index of refraction 1.55 (Fig. P35.55b). Assume the width of the laser beam at the information layer must be a = 1.00 μm to read from only one track and not from its neighbors. Assume the width of the beam as it enters the transparent plastic is w = 0.700 mm. A lens makes the beam converge into a cone with an apex angle $2\theta_1$ before it enters the DVD. Find the incidence angle θ_1 of the light at the edge of the conical beam. This design is relatively immune to small dust particles degrading the video quality.

Andrew Syred/Photo Researchers, Inc.

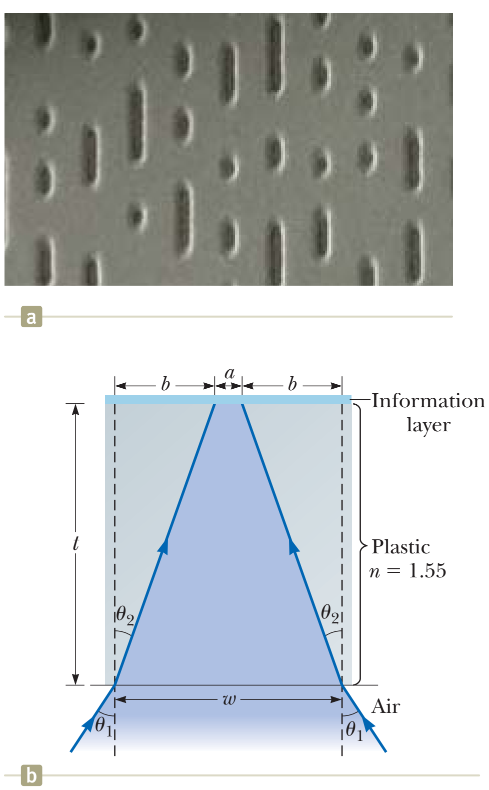

Figure P35.55

56. AMT How many times will the incident beam shown in Figure P35.56 be reflected by each of the parallel mirrors?

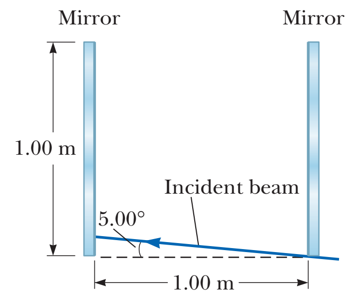

Figure P35.56

57. When light is incident normally on the interface between two transparent optical media, the intensity of the reflected light is given by the expression

$$S_1' = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2 S_1$$

In this equation, S_1 represents the average magnitude of the Poynting vector in the incident light (the incident intensity), S_1' is the reflected intensity, and n_1 and n_2 are the refractive indices of the two media. (a) What fraction of the incident intensity is reflected for 589-nm light normally incident on an interface between air and crown glass? (b) Does it matter in part (a) whether the light is in the air or in the glass as it strikes the interface?

58. Refer to Problem 57 for its description of the reflected intensity of light normally incident on an interface between two transparent media. (a) For light normally incident on an interface between vacuum and a transparent medium of index n, show that the intensity S_2 of the transmitted light is given by $S_2/S_1 = 4n/(n + 1)^2$. (b) Light travels perpendicularly through a diamond slab, surrounded by air, with parallel surfaces of entry and exit. Apply the transmission fraction in part (a) to find the approximate overall transmission through the slab of diamond, as a percentage. Ignore light reflected back and forth within the slab.

59. M A light ray enters the atmosphere of the Earth and descends vertically to the surface a distance h = 100 km below. The index of refraction where the light enters the atmosphere is 1.00, and it increases linearly with distance to have the value n = 1.000 293 at the Earth's surface. (a) Over what time interval does the light traverse this path? (b) By what percentage is the time interval larger than that required in the absence of the Earth's atmosphere?

60. A light ray enters the atmosphere of a planet and descends vertically to the surface a distance h below. The index of refraction where the light enters the atmosphere is 1.00, and it increases linearly with distance to have the value n at the planet surface. (a) Over what time interval does the light traverse this path? (b) By what fraction is the time interval larger than that required in the absence of an atmosphere?

61. A narrow beam of light is incident from air onto the surface of glass with index of refraction 1.56. Find the angle of incidence for which the corresponding angle of refraction is half the angle of incidence. *Suggestion:* You might want to use the trigonometric identity $\sin 2\theta = 2 \sin \theta \cos \theta$.

62. One technique for measuring the apex angle of a prism is shown in Figure P35.62. Two parallel rays of light are directed onto the apex of the prism so that the rays reflect from opposite faces of the prism. The angular separation γ of the two reflected rays can be measured. Show that $\phi = \frac{1}{2}\gamma$.

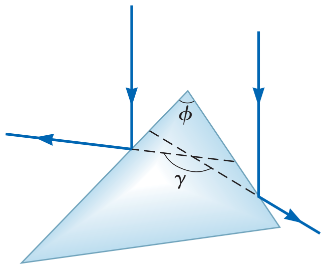

Figure P35.62

63. A thief hides a precious jewel by placing it on the bottom of a public swimming pool. He places a circular raft on the surface of the water directly above and centered over the jewel as shown in Figure P35.63. The surface of the water is calm. The raft, of diameter d = 4.54 m, prevents the jewel from being seen by any observer above the water, either on the raft or on the side of the pool. What is the maximum depth h of the pool for the jewel to remain unseen?

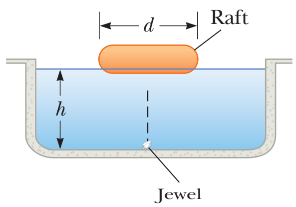

Figure P35.63

64. Review. A mirror is often "silvered" with aluminum. By adjusting the thickness of the metallic film, one can make a sheet of glass into a mirror that reflects anything between 3% and 98% of the incident light, transmitting the rest. Prove that it is impossible to construct a "one-way mirror" that would reflect 90% of the electromagnetic waves incident from one side and reflect 10% of those incident from the other side. *Suggestion:* Use Clausius's statement of the second law of thermodynamics.

65. M The light beam in Figure P35.65 strikes surface 2 at the critical angle. Determine the angle of incidence θ_1.

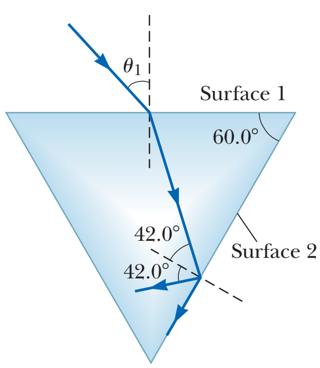

Figure P35.65

66. *Why is the following situation impossible?* A laser beam strikes one end of a slab of material of length L = 42.0 cm and thickness t = 3.10 mm as shown in Figure P35.66 (not to scale). It enters the material at the center of the left end, striking it at an angle of incidence of θ = 50.0°. The index of refraction of the slab is n = 1.48. The light makes 85 internal reflections from the top and bottom of the slab before exiting at the other end.

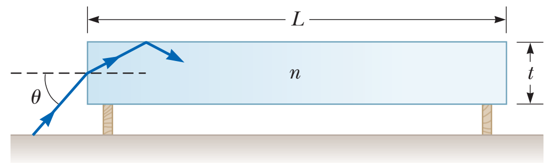

Figure P35.66

67. W A 4.00-m-long pole stands vertically in a freshwater lake having a depth of 2.00 m. The Sun is 40.0° above the horizontal. Determine the length of the pole's shadow on the bottom of the lake.

68. A light ray of wavelength 589 nm is incident at an angle θ on the top surface of a block of polystyrene as shown in Figure P35.68. (a) Find the maximum value of θ for which the refracted ray undergoes total internal reflection at the point P located at the left vertical face of the block. **What If?** Repeat the calculation for the case in which the polystyrene block is immersed in (b) water and (c) carbon disulfide. Explain your answers.

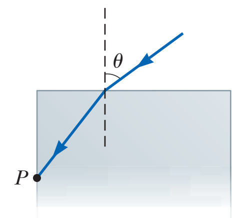

Figure P35.68

69. AMT A light ray traveling in air is incident on one face of a right-angle prism with index of refraction n = 1.50 as shown in Figure P35.69, and the ray follows the path shown in the figure. Assuming θ = 60.0° and the base of the prism is mirrored, determine the angle ϕ made by the outgoing ray with the normal to the right face of the prism.

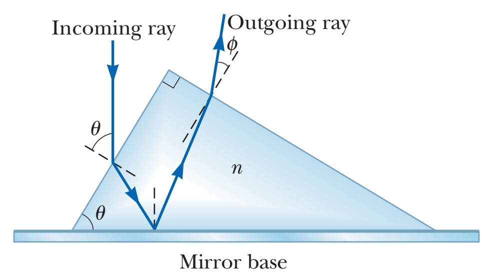

Figure P35.69

70. As sunlight enters the Earth's atmosphere, it changes direction due to the small difference between the speeds of light in vacuum and in air. The duration of an *optical* day is defined as the time interval between the instant when the top of the rising Sun is just visible above the horizon and the instant when the top of the Sun just disappears below the horizontal plane. The duration of the *geometric* day is defined as the time interval between the instant a mathematically straight line between an observer and the top of the Sun just clears the horizon and the instant this line just dips below the horizon. (a) Explain which is longer, an optical day or a geometric day. (b) Find the difference between these two time intervals. Model the Earth's atmosphere as uniform, with index of refraction 1.000 293, a sharply defined upper surface, and depth 8 614 m. Assume the observer is at the

Earth's equator so that the apparent path of the rising and setting Sun is perpendicular to the horizon.

71. A material having an index of refraction n is surrounded by vacuum and is in the shape of a quarter circle of radius R (Fig. P35.71). A light ray parallel to the base of the material is incident from the left at a distance L above the base and emerges from the material at the angle θ. Determine an expression for θ in terms of n, R, and L.

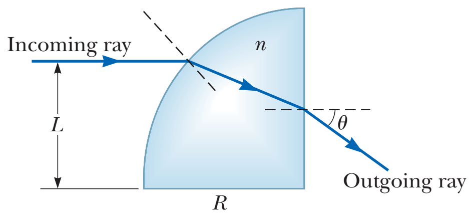

Figure P35.71

72. A ray of light passes from air into water. For its deviation angle $\delta = |\theta_1 - \theta_2|$ to be 10.0°, what must its angle of incidence be?

73. As shown in Figure P35.73, a light ray is incident normal to one face of a 30°–60°–90° block of flint glass (a prism) that is immersed in water. (a) Determine the exit angle θ_3 of the ray. (b) A substance is dissolved in the water to increase the index of refraction n_2. At what value of n_2 does total internal reflection cease at point P?

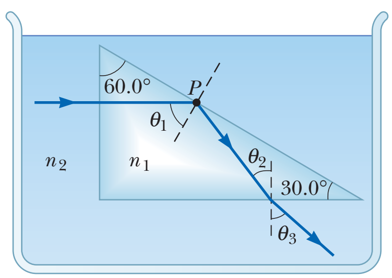

Figure P35.73

74. A transparent cylinder of radius $R = 2.00$ m has a mirrored surface on its right half as shown in Figure P35.74. A light ray traveling in air is incident on the left side of the cylinder. The incident light ray and exiting light ray are parallel, and $d = 2.00$ m. Determine the index of refraction of the material.

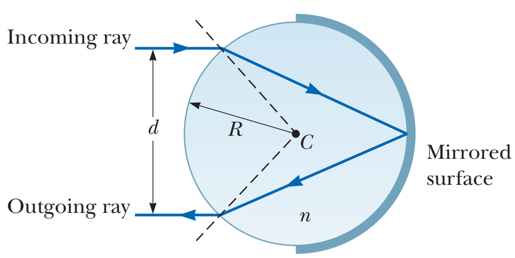

Figure P35.74

75. Figure P35.75 shows the path of a light beam through several slabs with different indices of refraction. (a) If $\theta_1 = 30.0°$, what is the angle θ_2 of the emerging beam? (b) What must the incident angle θ_1 be to have total internal reflection at the surface between the medium with $n = 1.20$ and the medium with $n = 1.00$?

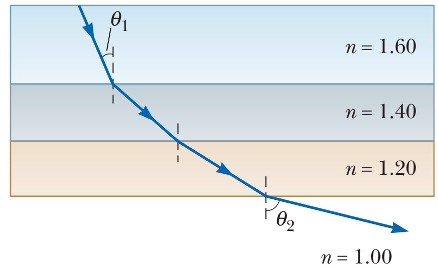

Figure P35.75

76. A. H. Pfund's method for measuring the index of refraction of glass is illustrated in Figure P35.76. One face of a slab of thickness t is painted white, and a small hole scraped clear at point P serves as a source of diverging rays when the slab is illuminated from below. Ray PBB' strikes the clear surface at the critical angle and is totally reflected, as are rays such as PCC'. Rays such as PAA' emerge from the clear surface. On the painted surface, there appears a dark circle of diameter d surrounded by an illuminated region, or halo. (a) Derive an equation for n in terms of the measured quantities d and t. (b) What is the diameter of the dark circle if $n = 1.52$ for a slab 0.600 cm thick? (c) If white light is used, dispersion causes the critical angle to depend on color. Is the inner edge of the white halo tinged with red light or with violet light? Explain.

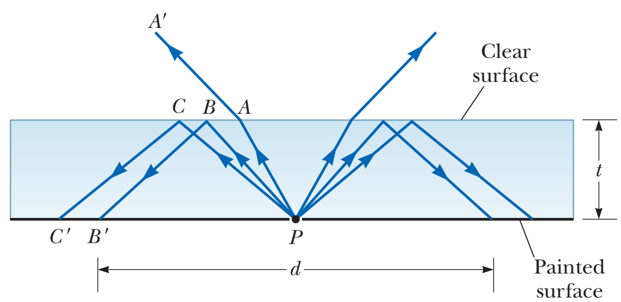

Figure P35.76

77. **W** A light ray enters a rectangular block of plastic at an angle $\theta_1 = 45.0°$ and emerges at an angle $\theta_2 = 76.0°$ as shown in Figure P35.77. (a) Determine the index of refraction of the plastic. (b) If the light ray enters the plastic at a point $L = 50.0$ cm from the bottom edge, what time interval is required for the light ray to travel through the plastic?

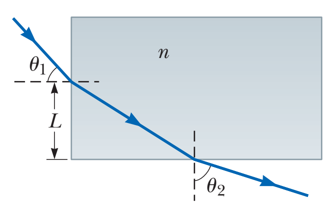

Figure P35.77

78. Students allow a narrow beam of laser light to strike a water surface. They measure the angle of refraction for selected angles of incidence and record the data shown in the accompanying table. (a) Use the data to verify Snell's law of refraction by plotting the sine of the angle of incidence versus the sine of the angle of refraction. (b) Explain what the shape of the graph demonstrates. (c) Use the resulting plot to deduce the index of refraction of water, explaining how you do so.

Angle of Incidence (degrees)	Angle of Refraction (degrees)
10.0	7.5
20.0	15.1
30.0	22.3
40.0	28.7
50.0	35.2
60.0	40.3
70.0	45.3
80.0	47.7

79. The walls of an ancient shrine are perpendicular to the four cardinal compass directions. On the first day of spring, light from the rising Sun enters a rectangular window in the eastern wall. The light traverses 2.37 m horizontally to shine perpendicularly on the wall opposite the window. A tourist observes the patch of light moving across this western wall. (a) With what speed does the illuminated rectangle move? (b) The tourist holds a small, square mirror flat against the western wall at one corner of the rectangle of light. The mirror reflects light back to a spot on the eastern wall close beside the window. With what speed does the smaller square of light move across that wall? (c) Seen from a latitude of 40.0° north, the rising Sun moves through the sky along a line making a 50.0° angle with the southeastern horizon. In what direction does the rectangular patch of light on the western wall of the shrine move? (d) In what direction does the smaller square of light on the eastern wall move?

80. Figure P35.80 shows a top view of a square enclosure. The inner surfaces are plane mirrors. A ray of light enters a small hole in the center of one mirror. (a) At what angle θ must the ray enter if it exits through the hole after being reflected once by each of the other three mirrors? (b) **What If?** Are there other values of θ for which the ray can exit after multiple reflections? If so, sketch one of the ray's paths.

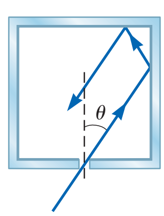

Figure P35.80

Challenge Problems

81. A hiker stands on an isolated mountain peak near sunset and observes a rainbow caused by water droplets in the air at a distance of 8.00 km along her line of sight to the most intense light from the rainbow. The valley is 2.00 km below the mountain peak and entirely flat. What fraction of the complete circular arc of the rainbow is visible to the hiker?

82. *Why is the following situation impossible?* The perpendicular distance of a lightbulb from a large plane mirror is twice the perpendicular distance of a person from the mirror. Light from the lightbulb reaches the person by two paths: (1) it travels to the mirror and reflects from the mirror to the person, and (2) it travels directly to the person without reflecting off the mirror. The total distance traveled by the light in the first case is 3.10 times the distance traveled by the light in the second case.

83. Figure P35.83 shows an overhead view of a room of square floor area and side L. At the center of the room is a mirror set in a vertical plane and rotating on a vertical shaft at angular speed ω about an axis coming out of the page. A bright red laser beam enters from the center point on one wall of the room and strikes the mirror. As the mirror rotates, the reflected laser beam creates a red spot sweeping across the walls of the room. (a) When the spot of light on the wall is at distance x from point O, what is its speed? (b) What value of x corresponds to the minimum value for the speed? (c) What is the minimum value for the speed? (d) What is the maximum speed of the spot on the wall? (e) In what time interval does the spot change from its minimum to its maximum speed?

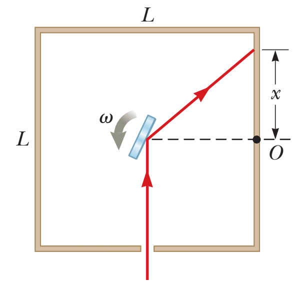

Figure P35.83

84. Pierre de Fermat (1601–1665) showed that whenever light travels from one point to another, its actual path is the path that requires the smallest time interval. This statement is known as *Fermat's principle.* The simplest example is for light propagating in a homogeneous medium. It moves in a straight line because a straight line is the shortest distance between two points. Derive Snell's law of refraction from Fermat's principle. Proceed as follows. In Figure P35.84, a light ray travels from point P in medium 1 to point Q in medium 2. The two points are, respectively, at perpendicular distances a and b from the interface. The displacement from P to Q has the component d parallel to the interface, and we let x represent the coordinate of the point where the ray enters the second medium. Let $t = 0$ be the instant the light starts from P. (a) Show that the time at which the light arrives at Q is

$$t = \frac{r_1}{v_1} + \frac{r_2}{v_2} = \frac{n_1\sqrt{a^2 + x^2}}{c} + \frac{n_2\sqrt{b^2 + (d - x)^2}}{c}$$

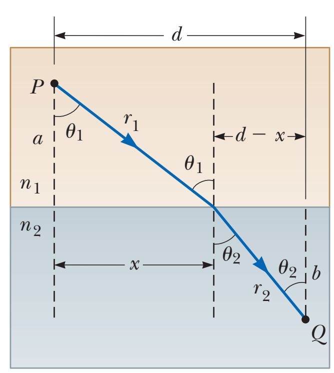

Figure P35.84 Problems 84 and 85.

(b) To obtain the value of x for which t has its minimum value, differentiate t with respect to x and set the derivative equal to zero. Show that the result implies

$$\frac{n_1 x}{\sqrt{a^2 + x^2}} = \frac{n_2(d - x)}{\sqrt{b^2 + (d - x)^2}}$$

(c) Show that this expression in turn gives Snell's law,

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

85. Refer to Problem 84 for the statement of Fermat's principle of least time. Derive the law of reflection (Eq. 35.2) from Fermat's principle.

86. Suppose a luminous sphere of radius R_1 (such as the Sun) is surrounded by a uniform atmosphere of radius $R_2 > R_1$ and index of refraction n. When the sphere is viewed from a location far away in vacuum, what is its apparent radius (a) when $R_2 > nR_1$ and (b) when $R_2 < nR_1$?

87. This problem builds upon the results of Problems 57 and 58. Light travels perpendicularly through a diamond slab, surrounded by air, with parallel surfaces of entry and exit. The intensity of the transmitted light is what fraction of the incident intensity? Include the effects of light reflected back and forth inside the slab.