Questions for Module 30

Q.1 A compressional wave of frequency 300 Hz is set up in an iron rod and passes from the iron rod into air. The speed of the wave is 4800 m/s in iron and 330 m/s in air. Find the wavelength in each material.

Q.2 A uniform inextensible string of length L and total mass M is fixed at one end and hangs freely under its own weight. It is tapped at the top end so that a transverse wave runs down it. At the same moment a small stone is released from rest and falls freely from the top of the string. How far from the top does the stone pass the wave?
Answer: x=8L/9

Q.3 A transverse wave is described by the equation

$$Y = Y_0 \sin 2\pi (ft - x/\lambda).$$

The maximum particle velocity is equal to four times the wave velocity if
(*a*) $\lambda = \pi Y_0/4$
(*b*) $\lambda = \pi Y_0/2$
(*c*) $\lambda = \pi Y_0$
(*d*) $\lambda = 2 \pi Y_0$
Tick the correct answer(s).

Q.4 A uniform flexible cable is 20 m long. It hangs vertically under its own weight and is vibrated from its upper end. Find the speed of transverse wave on the cable at its midpoint.

Q.5 *The air column in a pipe closed at one end is made to vibrate in its* ***second overtone*** *by a tuning fork of frequency 440 Hz. The speed of sound in air is 330 ms^{-1}. End corrections may be neglected. Let P_0 denote the mean pressure at any point in the pipe, and ΔP_0 the maximum amplitude of pressure variation.*

(a) Find the length L of the air column.
(b) What is the amplitude of pressure variation at the middle of the column?
(c) What are the maximum and minimum pressures at the open end of the pipe?
(d) What are the maximum and minimum pressures at the closed end of the pipe?

Q.6 Two identical travelling waves moving in the same direction are out of phase by 90°. What is the amplitude of the combined wave in terms of the common amplitude *A* of the two combining waves?

Q.7 A source S and a detector D of radio waves are a distance d apart on the ground. The direct wave from S is found to be in phase at D with the wave from S that is reflected from a horizontal layer at an altitude H (Fig. 6.10). The incident and reflected rays make the same angle with the reflecting layer.

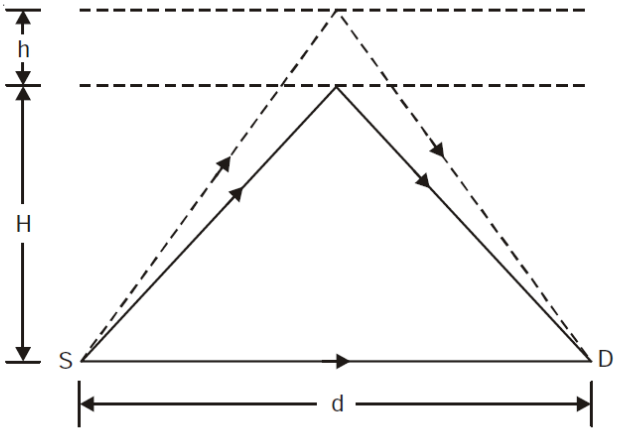

Fig. 6.10

When the layer rises a distance h, no signal is detected at D. Neglect absorption in the atmosphere and find the relation between d, h, H and the wavelength λ of the waves.

Q.8 *Let* $f(x) = \begin{cases} x, & for \quad 0 < x < \pi \\ -x, & for \quad -\pi < x < 0 \end{cases}$

where f (x) has period 2π. Draw the graph of f (x) and obtain the Fourier series for f(x). Considering the point x = 0 show that

$$\frac{1}{1^2} + \frac{1}{3^3} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}.$$

Q.9 *Find the Fourier series corresponding to the function*

$$f(x) = \begin{cases} 1 & for \quad 0 < x < l \\ 0 & for \quad -l < x < 0 \end{cases}$$

where the period is 2l.

Q.10 *(a) A displacement curve is given by*

$$f(t) = A\frac{t}{T}, \text{ for } 0 < t < T$$

and

$$f(t + T) = f(t),$$

where A is a constant. Draw the graph of f (t) and obtain the Fourier series expansion for f (t).

(b) Considering the point t = T/4, show that

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots = \frac{\pi}{4}.$$