Questions for Module#34

5. If the atoms in a regular crystal are separated by 3×10^{-10} m demonstrate that an accelerating voltage of about 1.5 kV would be required to produce an electron diffraction pattern from the crystal.
6. The relationship between wavelength and frequency for electromagnetic waves in a waveguide is

$$\lambda = \frac{c}{\sqrt{\nu^2 - \nu_0^2}}, \tag{2.13.2}$$

where c is the velocity of light in vacuum. What are the group- and phase-velocities of such waves as functions of ν_0 and λ?
7. Nuclei, typically of size 10^{-14} m, frequently emit electrons with energies of 1–10 MeV. Use the uncertainty principle to show that electrons of energy 1 MeV could not be contained in the nucleus before the decay.
8. A particle of mass m has a wavefunction

$$\psi(x,t) = A \exp\left[-a\left(\frac{m\,x^2}{\hbar} + \mathrm{i}\,t\right)\right], \tag{2.13.3}$$

where A and a are positive real constants. For what potential function $V(x)$ does ψ satisfy the Schrödinger equation?

1. Monochromatic light with a wavelength of 6000Å passes through a fast shutter that opens for 10^{-9} sec. What is the subsequent spread in wavelengths of the no longer monochromatic light?
2. Calculate $\langle x\rangle$, $\langle x^2\rangle$, and σ_x, as well as $\langle p\rangle$, $\langle p^2\rangle$, and σ_p, for the normalized wavefunction

$$\psi(x) = \sqrt{\frac{2\,a^3}{\pi}}\,\frac{1}{x^2 + a^2}. \tag{3.11.1}$$

Use these to find $\sigma_x\,\sigma_p$. Note that $\int_{-\infty}^{\infty} dx/(x^2+a^2) = \pi/a$.
3. Classically, if a particle is not observed then the probability of finding it in a one-dimensional box of length L, which extends from $x=0$ to $x=L$, is a constant $1/L$ per unit length. Show that the classical expectation value of x is $L/2$, the expectation value of x^2 is $L^2/3$, and the standard deviation of x is $L/\sqrt{12}$.
4. Demonstrate that if a particle in a one-dimensional stationary state is bound then the expectation value of its momentum must be zero.
5. Suppose that $V(x)$ is complex. Obtain an expression for $\partial P(x,t)/\partial t$ and $d/dt \int P(x,t)\,dx$ from Schrödinger's equation. What does this tell us about a complex $V(x)$?
6. $\psi_1(x)$ and $\psi_2(x)$ are normalized eigenfunctions corresponding to the same eigenvalue. If

$$\int_{-\infty}^{\infty} \psi_1^*\,\psi_2\,dx = c, \tag{3.11.2}$$

where c is real, find normalized linear combinations of ψ_1 and ψ_2 that are orthogonal to (a) ψ_1, (b) $\psi_1 + \psi_2$.