7. Demonstrate that $p = -\mathrm{i}\,\hbar\,\partial/\partial x$ is an Hermitian operator. Find the Hermitian conjugate of $a = x + \mathrm{i}\,p$.
8. An operator A, corresponding to a physical quantity α, has two normalized eigenfunctions $\psi_1(x)$ and $\psi_2(x)$, with eigenvalues a_1 and a_2. An operator B, corresponding to another physical quantity β, has normalized eigenfunctions $\phi_1(x)$ and $\phi_2(x)$, with eigenvalues b_1 and b_2. The eigenfunctions are related via

$$\psi_1 = (2\,\phi_1 + 3\,\phi_2)\,/\sqrt{13},$$
$$\psi_2 = (3\,\phi_1 - 2\,\phi_2)\,/\sqrt{13}.$$

α is measured and the value a_1 is obtained. If β is then measured and then α again, show that the probability of obtaining a_1 a second time is $97/169$.
9. Demonstrate that an operator that commutes with the Hamiltonian, and contains no explicit time dependence, has an expectation value that is constant in time.
10. For a certain system, the operator corresponding to the physical quantity A does not commute with the Hamiltonian. It has eigenvalues a_1 and a_2, corresponding to properly normalized eigenfunctions

$$\phi_1 = (u_1 + u_2)\,/\sqrt{2},$$
$$\phi_2 = (u_1 - u_2)\,/\sqrt{2},$$

where u_1 and u_2 are properly normalized eigenfunctions of the Hamiltonian with eigenvalues E_1 and E_2. If the system is in the state $\psi = \phi_1$ at time $t = 0$, show that the expectation value of A at time t is

$$\langle A\rangle = \left(\frac{a_1 + a_2}{2}\right) + \left(\frac{a_1 - a_2}{2}\right)\cos\left(\frac{[E_1 - E_2]\,t}{\hbar}\right). \quad (3.11.3)$$