7.E: Orbital Angular Momentum (Exercises)

1. A system is in the state $\psi = Y_{l,m}(\theta,\phi)$. Calculate $\langle L_x\rangle$ and $\langle L_x^2\rangle$.
2. Find the eigenvalues and eigenfunctions (in terms of the angles θ and ϕ) of L_x.
3. Consider a beam of particles with $l = 1$. A measurement of L_x yields the result $\hbar$. What values will be obtained by a subsequent measurement of L_z, and with what probabilities? Repeat the calculation for the cases in which the measurement of L_x yields the results 0 and $-\hbar$.
4. The Hamiltonian for an axially symmetric rotator is given by

$$H = \frac{L_x^2 + L_y^2}{2\,I_1} + \frac{L_z^2}{2\,I_2}. \tag{7.E.1}$$

   What are the eigenvalues of H?

8.E: Central Potentials (Exercises)

1. A particle of mass m is placed in a finite spherical well:

$$V(r) = \begin{cases} -V_0 & \text{for } r \leq a \\ 0 & \text{for } r > a \end{cases} \tag{8.E.1}$$

   with $V_0 > 0$ and $a > 0$. Find the ground-state by solving the radial equation with $l = 0$. Show that there is no ground-state if $V_0\,a^2 < \pi^2\,\hbar^2/(8\,m)$.
2. Consider a particle of mass m in the three-dimensional harmonic oscillator potential $V(r) = (1/2)\,m\,\omega^2\,r^2$. Solve the problem by separation of variables in spherical coordinates, and, hence, determine the energy eigenvalues of the system.
3. The normalized wavefunction for the ground-state of a hydrogen-like atom (neutral hydrogen, He^+, Li^{++}, et cetera.) with nuclear charge $Z\,e$ has the form

$$\psi = A\,\exp(-\beta\,r), \tag{8.E.2}$$

   where A and β are constants, and r is the distance between the nucleus and the electron. Show the following:
   1. $A^2 = \beta^3/\pi$.
   2. $\beta = Z/a_0$, where $a_0 = (\hbar^2/m_e)\,(4\pi\,\epsilon_0/e^2)$.
   3. The energy is $E = -Z^2\,E_0$ where $E_0 = (m_e/2\,\hbar^2)\,(e^2/4\pi\,\epsilon_0)^2$.
   4. The expectation values of the potential and kinetic energies are $2\,E$ and $-E$, respectively.
   5. The expectation value of r is $(3/2)\,(a_0/Z)$.
   6. The most probable value of r is a_0/Z.
4. An atom of tritium is in its ground-state. Suddenly the nucleus decays into a helium nucleus, via the emission of a fast electron that leaves the atom without perturbing the extranuclear electron, Find the probability that the resulting He^+ ion will be left in an $n = 1, l = 0$ state. Find the probability that it will be left in a $n = 2, l = 0$ state. What is the probability that the ion will be left in an $l > 0$ state?
5. Calculate the wavelengths of the photons emitted from the $n = 2, l = 1$ to $n = 1, l = 0$ transition in hydrogen, deuterium, and positronium.
6. To conserve linear momentum, an atom emitting a photon must recoil, which means that not all of the energy made available in the downward jump goes to the photon. Find a hydrogen atom's recoil energy when it emits a photon in an $n = 2$ to $n = 1$ transition. What fraction of the transition energy is the recoil energy?