Chapter 7
Introduction to Nuclear Physics

7.1 Introduction

Atomic nuclei are extended objects (Fig. 7.1), like the neutrons and protons of which they are made: we can measure their size, while fundamental particles, like electrons and quarks, behave as point-like objects.

The force which binds together the nucleons within a nucleus is the strong nuclear interaction, the same that binds together quarks to form baryons and mesons. However, this force is now acting on a longer range and it can be pictured as being a residual force that is left after forming the bound states of quarks, which are the nucleons. Using an analogy, the nuclear strong force is similar to the electromagnetic attraction which binds together two neutral atoms to form a molecule. In our case, two colourless nucleons are bound together to form a nucleus. Of course, the interaction is completely different in the two cases.

The simplest stable bound state of two nucleons is the *deuteron*, which is made of a proton and a neutron, as shown in Fig. 7.2. The presence of the proton stabilises the neutron, making it energetically unfavourable to undergo a beta decay. The next simplest nuclide is the *triton*, or *tritium nucleus*, with two neutrons and one proton: ^{3}H. It is unstable and has a lifetime of 12.3 years, decaying β to ^{3}He, which is stable. The next simple nuclide is ^{4}He, which is extremely stable and is also known as α particle. At the opposite end, we have nuclides which are made of a very large number of nucleons: an example is Uranium, which contains 238 nucleons. Different models are used to describe, to various levels of approximation, these systems, which are made of a number of nucleons ranging between 2 and 240. Unlike what occurs for atoms, where the energy levels can be precisely calculated, each of the nuclear models is successful to explain some features, but not all.

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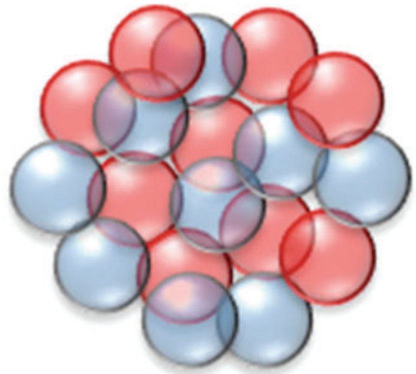


Fig. 7.1 Pictorial representation of a nucleus, made of neutrons and protons. Clearly, it is only a way to represent it and we don't have to take it literally

Fig. 7.2 Pictorial representation of a deuteron, bound state of (pn). It is a stable nuclide, an isotope of Hydrogen ^{2}H. Heavy water has one deuterium atom replacing hydrogen. Deuterium is not an element but is occasionally indicated with D, and heavy water with HDO

7.2 The Q-Value of a Reaction

The *Q-value* is defined as the difference between the kinetic energy of the final system and the kinetic energy of the initial system:

$$Q = E_k(\text{final}) - E_k(\text{initial}) \tag{7.1}$$

In a generic reaction $A + B \rightarrow C + D$, we can write the energy conservation as:

$$m_A c^2 + K_A + m_B c^2 + K_B = m_C c^2 + K_C + m_D c^2 + K_D \tag{7.2}$$

where K_X indicates the kinetic energy of the particle X. Rearranging:

$$K_C + K_D - K_A - K_B = Q \tag{7.3}$$

$$Q = m_A + m_B - (m_C + m_D) \tag{7.4}$$

We therefore derive that the Q-value is the mass difference between the initial and final states (note the inverted order with respect to the kinetic energy). As a consequence, an elastic scattering $A + B \rightarrow A + B$ has $Q = 0$. An example of elastic scattering is Compton scattering. A decay can occur spontaneously only if $Q > 0$. Of course, this is not the only condition: also electric charge, total angular momentum, and other quantum numbers have to be conserved. Reactions with $Q > 0$ transform mass into kinetic energy; reactions with $Q < 0$ transform kinetic energy into mass. An example of the latter is the creation of pairs of top quarks in proton–proton collisions. The proton mass is ≈ 1 GeV/c^2, and the *top* mass is $m_t \approx 174$ GeV/$c^2 \gg m_p$. These reactions are "endothermic" or "endo-energetic", and they require a large kinetic energy to create a pair of particles with large mass.

We define the *threshold energy for a reaction to occur* as the minimum kinetic energy required to the initial particles, when the product particles have zero kinetic energy. If the particles in the initial state have a lower value of kinetic energy, the reaction cannot take place.

In this case:

$$K_A + K_B = (m_C + m_D) - (m_A + m_B) \tag{7.5}$$

As an example, photons can interact with matter by creating an electron–positron pair, in the presence of a nucleus (to conserve momentum):

$$\gamma + (A, Z) \rightarrow e^+ + e^- + (A, Z) \tag{7.6}$$

The photon has no mass, so all its energy is kinetic. The threshold of the reaction above is

$$K_\gamma = 2m_e = 2 \times 511 \text{ keV} \tag{7.7}$$

In decays where $A \rightarrow C + D$ and $K_A = 0$, the requirement of $Q \geq 0$ translates into

$$m_A \geq m_C + m_D \ , \tag{7.8}$$

that is, the mass of the parent must be larger than the combined mass of the daughters. The Q-value is connected mathematically to the factor ρ_f the phase space density of the final states, Eq. (6.9).

7.3 Atomic Nuclei Phenomenology

One of the properties to look for at first is the nuclear size and the nuclear density. We'll address some stability issues and review one model of the atomic nucleus, before showing some nuclear reaction of practical interest. The size of the nuclei is experimentally measured by means of scattering experiments, which probe the nucleus with particles. These can be either α or electrons accelerated to medium or high energies up to a few MeV. The nuclear size, as it appears from the electromagnetic interaction, is measured experimentally by electron–nucleus scattering, which is sensitive to the electrical charge density. The nuclear radius R turns out to be

$$R = 1.21 A^{1/3}; \quad R \text{ in fm} \tag{7.9}$$

Assuming that nuclei have a spherical shape, their volume is $V = 4/3\pi R^3 = 7.42A$ (fm^3), and their density is 0.13 nucleons per fm^3. It is quite constant for $55 \leq A \leq 209$. To probe the nuclear density, as seen by the nuclear force, other particles can be used as "projectiles": $\pi^{\pm}, \pi^0$ or neutrons. Various models of atomic nuclei can be used, depending on the experiment and on the quantity being measured, like the *black disk* in a model and the *optical model*. The mass of nuclei is expressed in terms of the *atomic mass unit*, which is defined to be 1/12 of the mass of an atom of ^{12}C. We should note explicitly here that the mass of the electrons is included in this unit. The a.m.u. is

$$u = 931.4940\,\text{MeV/c}^2 = 1.6605389 \times 10^{-27}\ \text{kg}. \tag{7.10}$$

The mass of an atom, as determined experimentally, is less than the sum of the masses of its components:

$$M(Z, N) < Z(m_p + m_e) + N m_n \tag{7.11}$$

$$-B \equiv \Delta M(Z, N) \equiv M(Z, N) - Z(m_p + m_e) - N m_n \tag{7.12}$$

We call ΔM the *mass deficit*; when multiplied by c^2, we obtain the total *binding energy B*: if we want to separate completely all nucleons, we need to provide an energy equal to the binding energy. The interesting quantity is the binding energy per nucleon B/A. Its experimental value is plotted for stable nuclides in Fig. 7.3 as a function of A. On average, the value of B/A is between 7 and 9 MeV per nucleon. It increases for small nuclides, has a maximum for ^{56}Fe, then decreases. The nuclides in the raising part of the curve can, under appropriate conditions, *fuse* with other small nuclides and emit energy; in this case, we have a *nuclear fusion*. The nuclides at high A can gain energy if they split into smaller nuclides; this is why, we have *nuclear fission*.

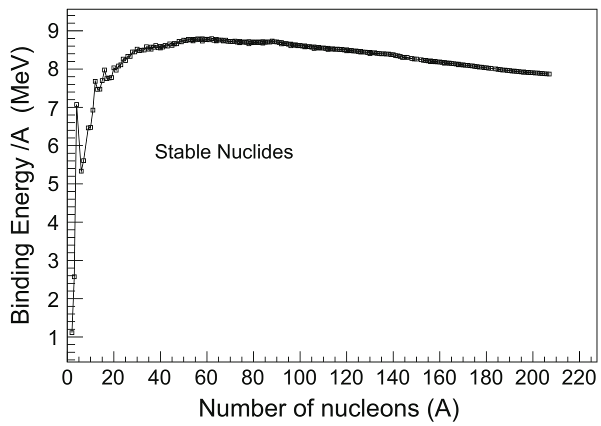


Fig. 7.3 Binding energy per nucleon as a function of the mass number $A = Z + N$ for stable or long-lived nuclides. Nuclides with low A and low binding energy per nucleon can undergo a *fusion* reaction, under appropriate conditions, to increase A and move towards higher binding energies. Conversely, nuclides with high A, when splitting into two nuclides with lower A, can increase the binding energy per nucleon in each of the daughter nuclides. Fission and fusion processes have $Q > 0$ for high and low A, respectively. It is worth noting that ^{4}He, or α particle, has a binding energy of 7 MeV per nucleon, making it a very stable nuclide (data from IAEA Nuclear Data Section)

7.4 The Liquid Drop Model

A spherical object with uniform density can be thought as resembling a liquid drop in the absence of gravity. We can use this model to describe nuclei and calculate their mass with the *semi-empirical* mass formula of Bethe–Weizsäcker[1] as the sum of six terms: the first is just the sum of the mass of components, while the other five represent the binding energy:

$$M(A, Z) = \sum m_i - a_1 f_1 + a_2 f_2 + a_3 f_3 + a_4 f_4 + f_5 \tag{7.13}$$

$$= \big(Z(m_p + m_e) + N m_n\big) - a_V A + a_S A^{\frac{2}{3}} + \tag{7.14}$$

$$+ a_C Z^2 A^{-\frac{1}{3}} + a_A (A - 2Z)^2 A^{-1} + f_5 \tag{7.15}$$

where the numerical values in MeV are

$$a_V = 15.76;\ a_S = 17.81;\ a_C = 0.711;\ a_A = 23.7 \tag{7.16}$$

[1] After Hans Bethe (1906–2005) and Carl Friedrich von Weizsäcker, (1912–2007). Both were from Germany, Bethe moved to Manchester, Bristol (UK) and later to Cornell, USA, because of racial laws.

The single terms are

- a mass term from the constituents: $Z(m_p + m_e) + Nm_n$;
- a volume term, which is negative and proportional to A; it accounts for the average binding energy between a nucleon and its immediate neighbours.
- a surface term, which is positive and proportional to $A^{2/3}$, accounts for the nucleons at the surface, which have fewer neighbours and therefore are less bound. If the radius is proportional to $A^{1/3}$, the surface of the sphere is proportional to $A^{2/3}$. Together with the next Coulomb interaction term, the surface term limits the maximum value of A for stable nuclei; this surface term gives the name to the model, it is like the surface tension of a liquid drop.
- the Coulomb term accounts for the mutual repulsion of protons; it is positive, as it decreases the binding energy, and is proportional to $Z^2A^{-1/3}$. This term is the potential energy of a sphere of radius $r_0A^{1/3}$ with a uniform charge $+Ze$. The theoretical value for $a_c = 0.714$ MeV matches remarkably well the experimental value in Eq. (7.16).
- the asymmetry term: nuclei tend to be stable when $Z \approx N$ (Fig. 7.4) so the term is

$$f_4 = a_A \frac{(Z - A/2)^2}{A}$$

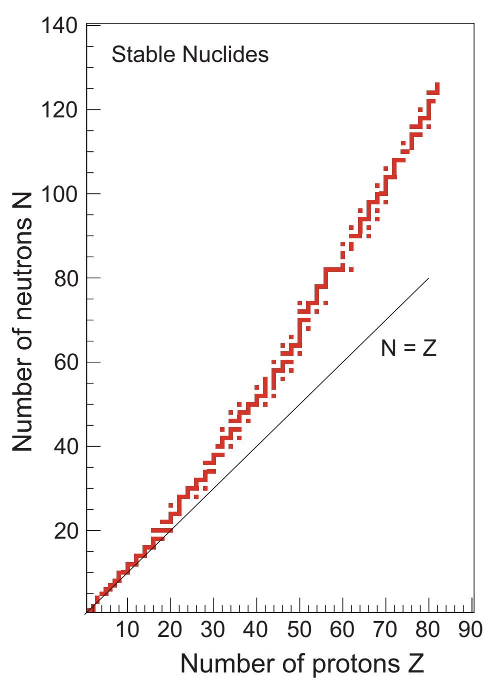


Fig. 7.4 Segrè chart of the stable nuclides: they tend to have $N \approx Z$ for low $Z \leq 20$, while heavier elements tend to have a slight neutron excess. This plot, together with Figs. 7.5 and 7.6, is the experimental basis of the asymmetry term in the semi-empirical mass formula (data from IAEA Nuclear Data Section)

- the *pairing term*: inside a nucleus, nucleons tend to form pairs (nn) and (pp) with opposite spin. Thus, nuclides with Z even and N even have more binding energy than nuclides where there is an odd number of either or both nucleons. Only four nuclides with both Z and N odd are stable, while 167 stable nuclides have both Z and N even. The term in the formula will be positive for A, N odd, zero for even–odd or odd–even, and negative for both A, N odd. This term has a purely empirical value of

$$f_5 = \pm 12 A^{-1/2} \mathrm{MeV/c^2} \text{ (+for odd-odd)}; \quad f_5(\text{even,odd}) = 0 \tag{7.17}$$

7.5 Beta Decays

The pairing term in Eq. (7.17) (Figs. 7.5 and 7.6) explains why some nuclides undergo β decays: the total energy is lower, or the binding energy is higher, if a neutron is replaced with a proton: in this case, we have a β^--emitting transition; when a lower-energy state is reached by replacing a proton with a neutron, the nuclide decays emitting a β^+. The decay

$$p \to n + e^+ + \nu_e \tag{7.18}$$

in which

$$u \to d + e^+ + \nu_e \tag{7.19}$$

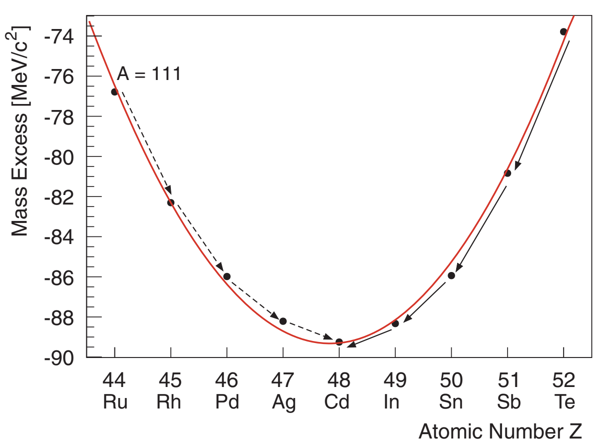


Fig. 7.5 Mass excess, which is minus the mass deficit, for the isobars with $A = 111$. These are all even–odd or odd–even nuclides, for which the pairing term is zero. The circles are the experimental values, the parabola is the calculation of the semi-empirical mass formula (SEMF). The arrows indicate beta decays: arrows towards the left hand side represent β^+ decays ($Z' \to Z - 1$), arrows towards the right-hand side represent β^- decays ($Z' \to Z + 1$). Data from IAEA Nuclear Data Section, plot modified after B.R. Martin, *Nuclear and Particle Physics*

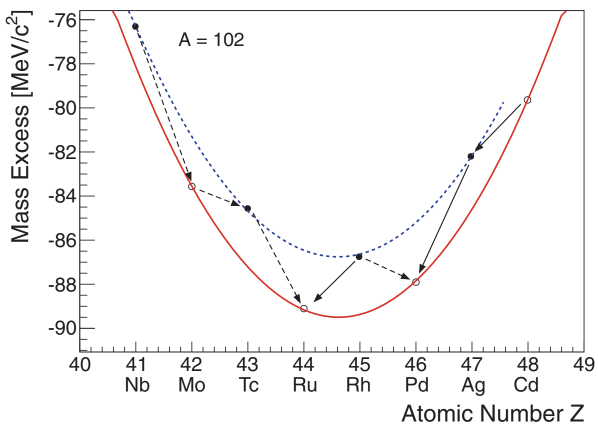


Fig. 7.6 Mass excess (or minus mass deficit) for the isobars $A = 102$. The experimental values now lay on two parabolas, and therefore the *pairing term* was introduced. Open circles represent (Z, N) even–even nuclides, filled circles represent (Z, N) odd–odd nuclides. Both ^{102}Ru and ^{102}Pd are stable nuclides. β^- transitions are indicated with a dashed arrow, and β^+ transitions with a full line. ^{102}Rh can undergo both transitions. Out of all the stable nuclides, only four have odd numbers of both protons and neutrons

is perfectly "legal" from the point of view of conservation laws. It has the same matrix element $|\mathcal{M}_{if}|^2$ as the "standard" beta decay:

$$d \to u + e^- + \overline{\nu}_e \tag{7.20}$$

but it is not energetically allowed, or in other terms the phase space density of the final state is zero. It occurs sometimes in nuclei, when the nucleus, as a whole, would reach a lower-energy state by replacing a proton with a neutron (Fig. 7.7).

Another possible process is *electron capture (Fig. 7.8)*: it may happen that one of the electrons which are closer to the nucleus is captured by it and the following reaction occurs:

$$p^+ + e^- \to n + \nu_e \ . \tag{7.21}$$

In this case, the atom is left in an excited state: an atomic energy level has become available, because one electron has been captured by the nucleus. When other electrons occupy this level they emit energy in the form of X-rays. The reaction at quark level is

$$u + e^- \to d + \nu_e \tag{7.22}$$

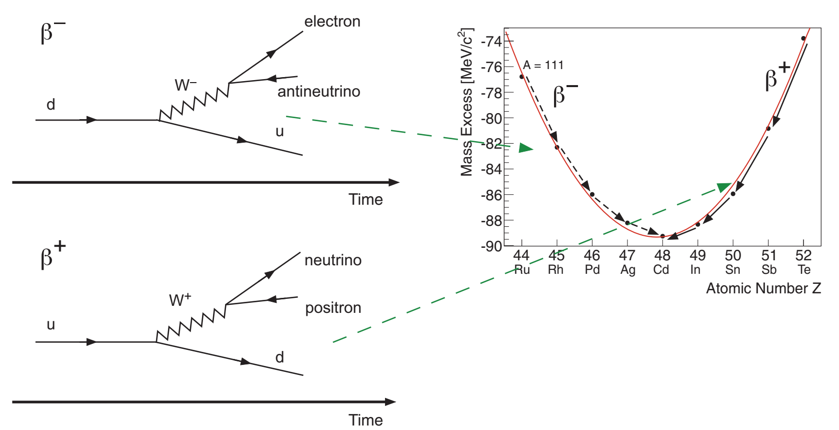


Fig. 7.7 Feynman diagrams at the quark level for the β^- (upper left) and β^+ decays (lower left). The two decays have almost identical Feynman diagrams, and therefore they have the same value for the matrix element $\mathcal{M}_{if}$. To determine whether the nuclide is stable or decays β^- or β^+, we have to calculate the energy variation related to this transition which is shown in the plot on the right-hand side. Isolated neutrons undergo β^- decays, because they have a mass which is slightly larger than the sum of the proton, electron and neutrino mass, i.e. the Q-value of the decay reaction is > 0. Protons are stable, because the β^+ decay is not energetically allowed, having a Q-value < 0. Inside a nucleus, the transition rate depends on the Q-value of the reaction at nuclear level, which depends on the pairing factor in the SEMF

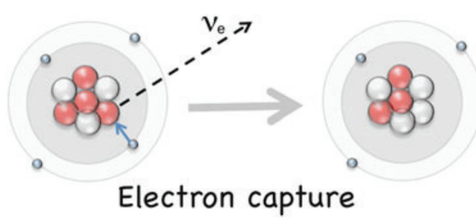


Fig. 7.8 Schematic representation of electron capture in the reaction $^7\text{Be} + e^- \rightarrow ^7\text{Li} + \nu_e$

7.6 Double Beta Decay

Double beta decay can occur in even–even nuclei when the decay $(A, Z) \rightarrow (A, Z + 1)$ is energetically forbidden, but $(A, Z) \rightarrow (A, Z + 2)$ is energetically allowed, as shown in Fig. 7.9. It is an extremely rare decay because two almost independent transitions have to occur at the same time. It was observed for the first time in 1987, in the decay

$$^{82}_{34}\text{Se} \rightarrow ^{82}_{36}\text{Kr} + 2e^- + 2\,\overline{\nu}_e \qquad (7.23)$$

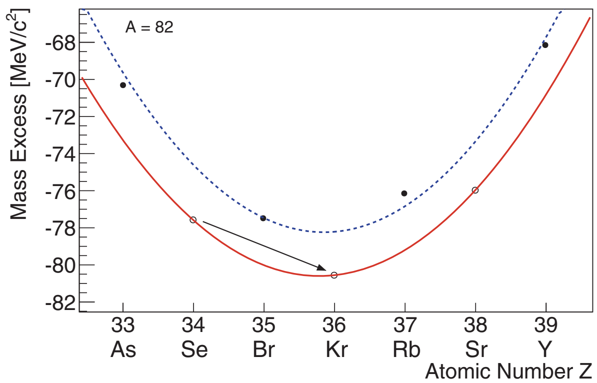


Fig. 7.9 Mass excess for the isobars with $A = 82$. The values for ${}^{82}_{34}$Se and ${}^{82}_{35}$Br are -77.594 and -77.496 MeV/c^2, respectively, so the decay ${}^{82}_{34}\text{Se} \rightarrow {}^{82}_{35}\text{Br} + \beta^- + \overline{\nu}_e$ is not allowed

and it has been observed in the decay of ten other nuclides since then. Double electron capture has not been observed directly, but there is indirect evidence of it in geochemical samples. It may be possible in

$$^{102}\text{Pd} + 2e^- \rightarrow {}^{102}\text{Ru} + 2\,\nu_e \tag{7.24}$$

This transition is energetically allowed, as shown in Fig. 7.6, while ^{102}Pd cannot decay to ^{102}Rh.

The category of fermions which are their own anti-particles is named after Ettore Majorana (b. 1906, ?), who first advanced the hypothesis of their existence. He went missing in March 1938, during or after his boat trip from Palermo to Naples, Italy.

Neutrinoless double beta decay has never been observed: it may be possible if neutrinos are *Majorana particles*. This category of particles refers to massive fermions which are their own anti-particles. In the case of neutrinoless double beta decay, the electron energy spectrum would be a characteristic single-energy line. The neutrinoless double beta decay, if exists, would violate the lepton number conservation (Eq. (6.46)) in the sector of charged leptons.

7.7 Other Models

Other models for the nucleus are the *Fermi gas model*, the *shell model* and the *collective model*.

In the Fermi gas model, nucleons are considered as non-interacting particles.

Their mutual interaction is replaced by the resultant of their attraction, which forms a potential well and keeps nucleons inside a sphere of radius R_0, as shown in Fig. 7.10. These non-interacting nucleons are subject to the Fermi statistics, which gives the name to the model, and, therefore, to the Pauli exclusion principle. Let's start considering neutrons only: any quantum state, or energy level, in the well can be occupied by two neutrons, with opposite spin. The same is valid for protons. Now, while the wells of the two types of nucleons are not required to have the same depth, it is assumed that they have the same radius. In addition, the proton potential has a different shape, to account for the electrostatic repulsion outside the well. Nucleons occupy all available energy states, up to a level E_F, which is called "Fermi energy level".

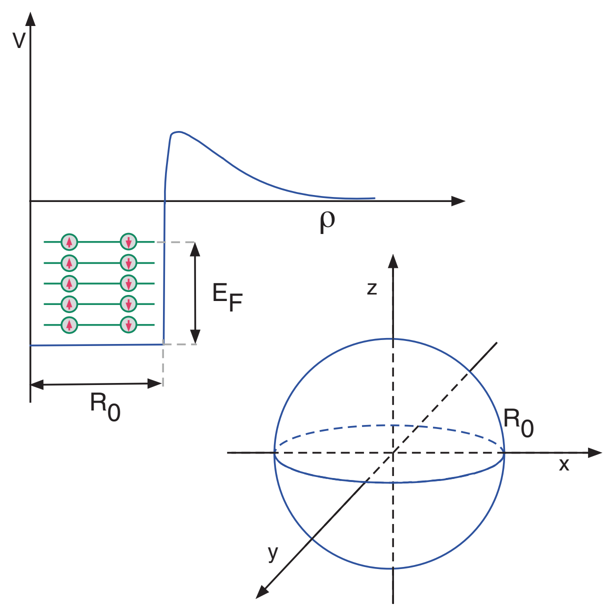


Fig. 7.10 A representation of the potential well for protons in the Fermi gas model. Protons are considered as non-interacting and are confined inside a sphere of radius R_0, the potential well is one-dimensional along the polar coordinate ρ; all available states are occupied up to the energy $E_F^{(p)}$. The Coulomb potential $1/\rho$ is sketched for $\rho > R_0$

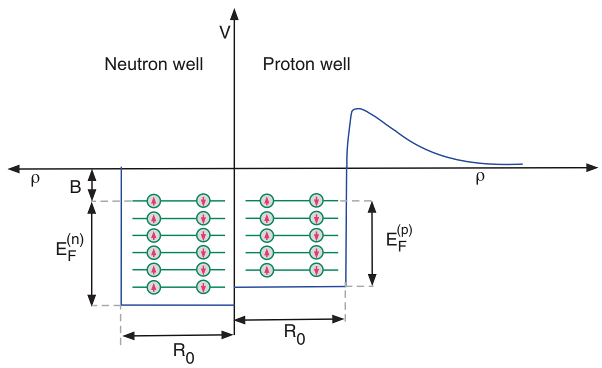


Fig. 7.11 The potential well for neutrons (left) and protons (right) in the Fermi gas model: all states are occupied up to the same energy level, but the depth of the two potential wells may be different

In a stable nucleus, this energy is required to be the same for protons and neutrons, as shown in Fig. 7.11, where the two wells are shown side by side, with the same vertical axis. When the two Fermi levels are not equal $E_F^{(n)} \neq E_F^{(p)}$, the nuclide will reach a stable configuration, with the same level, by undergoing a $\beta^{\pm}$ decay. The fact that at high A stable nuclides tend to have slightly more neutrons than protons is explained with a slightly deeper neutron well. To further advance in this model, we need to calculate how many quantum states are available, separately for protons and neutrons, in a potential well. In Quantum Mechanics, a state occupies a fundamental cell in the phase space. The volume of this cell is $h^3 = (2\pi\hbar)^3$. This is three orders of magnitudes larger than what could be naively expected based on the uncertainty relations $\Delta x\, \Delta p_x \geq \hbar/2$ (Eq. (6.2)) but it can be calculated using basic quantum mechanics. So, the number n_q of quantum states is simply

$$n_q = 2\frac{(4/3\pi R_0^2)(4/3\pi p_F^3)}{(2\pi\hbar)^3}\,, \tag{7.25}$$

where p_F is the momentum corresponding to the state of energy E_F and the factor of two accounts for the two possible spin orientations. Now, we assume that all states are occupied, so in the case of protons we know already that $n_q = Z$, and that $R_0 = r_0 A^{1/3} = 1.21\ \text{fm} A^{1/3}$, so we can calculate p_F:

$$Z = \frac{32}{9}\frac{\pi^2 r_0^3 A p_F^3}{8\pi^3\hbar^3} \; = \; \frac{4}{9}\frac{A}{\pi}\left(\frac{r_0 p_F}{\hbar}\right)^3$$

$$p_F = \frac{\hbar}{r_0}\sqrt[3]{\frac{9}{4}\pi\frac{Z}{A}} \approx \frac{\hbar}{r_o}\sqrt[3]{\frac{9}{8}\pi} = \frac{1.52 \times 0.658}{1.21} \text{ eV/(m/s)} \tag{7.26}$$

We have used the approximation: for stable nuclides $Z/A \approx 1/2$. Now, momenta are normally measured in MeV/c, where c is the speed of light, so we need to multiply the result above by an a-dimensional 3×10^8 to obtain

$$p_F \approx \frac{1.52 \times 0.658 \times 299{,}792{,}458}{1.21} \approx 250 \text{ MeV}/c \tag{7.27}$$

The proton mass is $m_p = 938.2720$ MeV/c^2 and therefore

$$\beta\gamma = \frac{p}{mc} = \frac{250}{928} = 0.26 < 1$$

protons inside nuclei can have quite a high momentum, but there is no need to use relativity. The Fermi energy can be calculated as:

$$E_F = \frac{p_F^2}{2m_p} = \frac{250}{2 \times 938} = 33 \text{ MeV} \tag{7.28}$$

The difference between E_F and the top of the potential well is the binding energy B, which for nuclides with $A > 25$ is about 8 MeV per nucleon, as shown in Fig. 7.3, therefore the depth of the potential well is

$$V_0 = E_F + B = 33 + 8 = 41 \text{ MeV}. \tag{7.29}$$

The shell model was proposed separately by Maria Göppert-Mayer (1906–1972), who was in Chicago, and Hans Jensen (1907–1973), who was in Heidelberg, in 1950. They were awarded the Nobel prize in 1963.

This model can explain quite well also the asymmetry term of the mass formula. However, there are features of nuclides that require a more refined model. It was observed that the abundance of elements and their stability is maximum for some particular values of Z and $(A-Z)$: 2, 8, 20, 28, 50, 82, and 126. These numbers have been called *magic numbers* before theory could explain them. This effect is similar to what happens with atoms, where completely filled electronic shells correspond to chemically stable and inactive elements, the six noble gases. Without entering into details, qualitatively the shell model adds a *spin–orbit* interaction to the square potential well of the Fermi gas model. The strong interaction between nucleons becomes now spin-dependent, along the same lines as the electromagnetic spin–orbit interaction, which slightly modifies the atomic energy levels. By adding a spin–orbit term to the potential,

$$V = V_{\text{well}}(r) + V_{\text{so}}(r)\, \vec{j} \cdot \vec{L}\ , \tag{7.30}$$

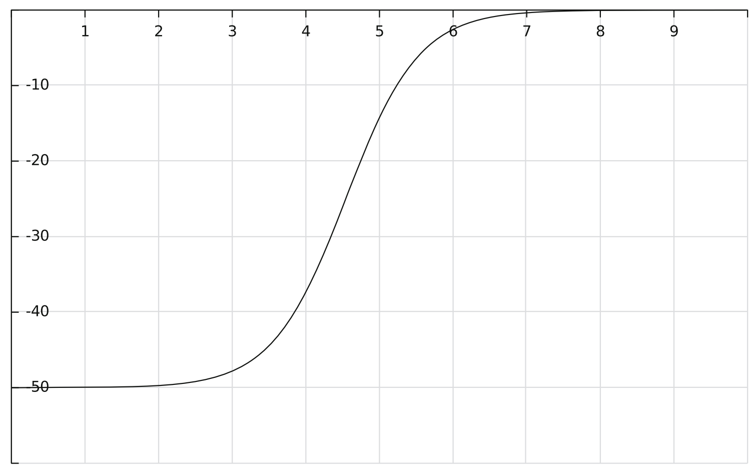


Fig. 7.12 The Woods–Saxon potential well for neutrons: $V(r) = -\frac{V_0}{(1+e^{\frac{r-R}{a}})}$

where $\vec{j}$ indicates the spin and $\vec{L}$ the orbital angular momentum, the model can explain all seven magic numbers.

Further improvements modify the shape of the potential well, as shown in Fig. 7.12. Some nuclides are extremely stable as they have both proton and neutron shell completed, like noble gases. Te α particle is one example, at the other end of the spectrum is the most abundant isotope of lead $^{208}_{82}\mathrm{Pb}_{126}$.

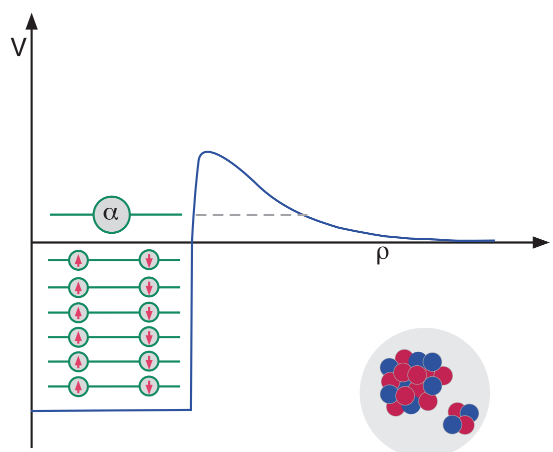
7.8 Alpha Decays

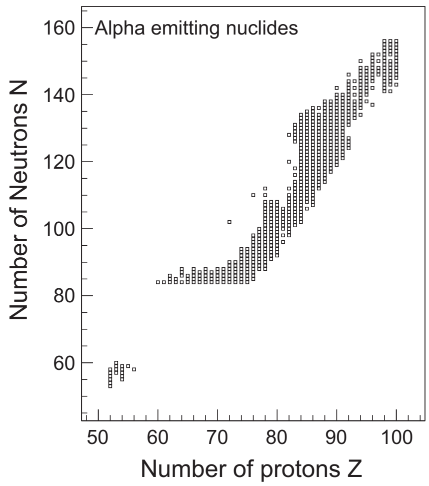
At this point, we can try to understand the mechanism of alpha decays. Its quantitative explanation requires quantum mechanics. The *tunnelling* effect has no equivalent in classical physics: a potential barrier of finite width can be crossed by a particle even if its energy is lower than the potential barrier. The probability for this process to occur depends on the width of the barrier at the energy level of the particle. It is outside the scope of this introductory book to calculate the decay rate with Gamov's tunnelling, see, e.g. Martin (2009). Remaining on the qualitative description, we first notice from Fig. 7.3 that emitting a nucleon is energetically allowed ($Q > 0$) only for heavy nuclides, with $Z \geq 105$. The α-emitting nuclide with lowest A is $^{105}_{52}$Te. The next question is why we observe α emission and not single neutron or single proton emission. Neutron emission by *evaporation* from a nucleus, many contributions fromes from a further analogy to the liquid drop, is a process that occurs normally, e.g. when fission fragments are produced. However, it is a very rapid process, which immediately follows the formation of new nuclei. Apart from this, there is an easier way for the nucleus to reach a lower-energy state, and this is β decay, which transforms neutrons into protons and vice versa. Very often, α and β emissions are competing processes, or different decay branches of the same nuclide. Alpha decay lifetimes range from 8×10^{-7} s to 6×10^{26} s. An alpha particle is a "doubly magic" nuclide ($Z = 2$, $N = 2$), with a binding energy of 7 MeV per nucleon, which is close to the maximum. In the *collective model* of heavy nuclei, we can consider them as formed by a tightly bound core and an outer layer, which may be modeled as a liquid drop, not necessarily spherical. For large nuclei, a part of this outer shell can be modeled as containing an α particle. In some very recent models also, some nuclear states of nuclides like ^{8}Be and ^{12}C are considered as α particle *condensates* (see, e.g. Yamada (2012)) of, respectively, two and three alpha particles, each α being a spin zero boson.

The alpha particle inside an unstable nucleus can be modeled as being in a potential well (Fig. 7.13); as it is electrically charged, the potential well is like the one of protons (Fig. 7.10), with a Coulomb barrier outside the well. We suppose that the α is formed among the most energetic nucleons with a certain probability. If the energy level of this alpha particle inside the nucleus is below the zero level, the α decay will not occur. If, however, the energy level is above zero, there is possibility of tunnelling through the Coulomb barrier. Alpha particles from a given nuclide are emitted with a single energy, which is often a marker to identify the nuclide that has emitted it. From the experimental point of view, the lifetime of pure alpha emitters is linked to the energy of the alpha particle by the Geiger–Nuttall relation:

$$\log \tau = a + b \frac{Z}{\sqrt{E_\alpha}} \ . \tag{7.31}$$

Fig. 7.13 Some nuclides can be modeled as having α particles as one of the components. If the energy level of this alpha particle is larger than zero, it can *tunnel* through the Coulomb barrier and be emitted





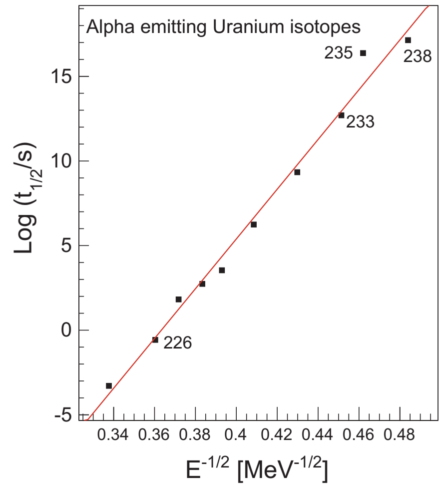


Fig. 7.14 Left: Segrè chart of the α-emitting nuclides: they all have $A \geq 105$. Right: the Geiger–Nuttall plot for isotopes of uranium, showing a linear relation between Log_{10} of the lifetime in seconds and the inverse of the square root of the kinetic energy. Nuclides with short lifetime emit more energetic α particles. The lifetime spans 20 orders of magnitude, while the α kinetic energy varies within a few MeV (data from IAEA Nuclear Data Section)

This relation can be derived with Gamow's tunnelling model (see, e.g. Martin (2009)) and is in good agreement with data, as shown in Fig. 7.14.

George Gamow (Russia 1904, USA 1968) taught in Leningrad (now St. Petersburgh), then moved to Europe end then to the USA, where he taught in various universities. He is known for many contributions from α decays to cosmology, DNA combinations and for authoring many popular science books.

7.9 Gamma Emission

Gamma rays are high-energy quanta of light. We can use the shell model to explain such an emission, when a transition occurs between nuclear energy levels. Unlike atomic physics, the shell model is presently unable to predict the exact energy levels, or rather their difference, and even less so the transition amplitudes. However, a detailed "mapping" of the experimentally measured energy levels for each nuclide is available in public databases. Both α and $\beta^{\pm}$ decays in most cases leave the daughter nuclide in an excited state.

Subsequent transitions, or decays, to intermediate states and to the ground state are marked by emission of one or more gamma rays. This is very similar to what

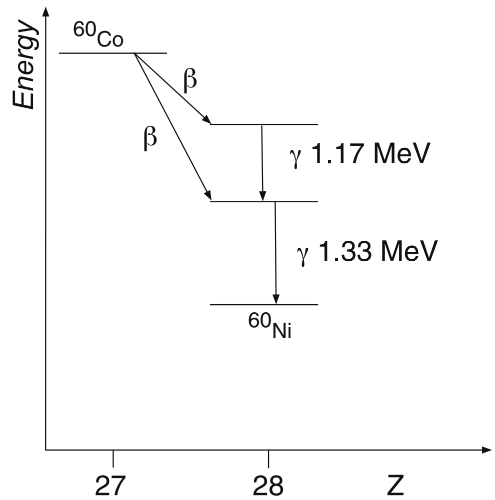


Fig. 7.15 Decay scheme of ^{60}Co, where both β and γ transitions are indicated, in the Energy vs Z plane

happens in atomic physics, where light emission occurs upon transition from excited electron states. Similarly to atomic physics, nuclear energy levels are indicated as horizontal segments in plots where the vertical axis represents the energy and the horizontal axis represents A or Z, as shown in Fig. 7.15. State transitions which emit γ's are indicated as vertical arrows, they do not change A or Z, while α and β emissions are indicated as arrows with a slope in the $(E - Z)$ plane. Also, the spin associated with the level can be indicated, together with the energy level and the lifetime or branching fraction of the transition.

7.10 Passage of Radiation Through Matter: Neutrons

Charged particles lose their kinetic energy by ionising the atoms: they interact with the electrons. Heavily ionising particles are either highly electrically charged, like alphas and fission fragments, or slow, or both. They lose all their energy and are stopped by a thin amount of solid material. High-energy, penetrating particles like *muons* ($\mu^{\pm}$) lose part of their energy with the same ionisation mechanism and are deviated in their trajectory by the electric field of the nuclei. The Bethe–Bloch equation (5.38) allows us to calculate the energy loss for charged particles. So far, we have not considered neutral particles. The only two types of neutral particles which live long enough to interact with matter are neutrinos and neutrons. Neutrons are particularly important, because they can provoke nuclear fission. Being neutral, they do not interact with the electrons of the atoms. They are baryons, so they have a residual strong force and they interact with the nuclei (Fig. 7.16).

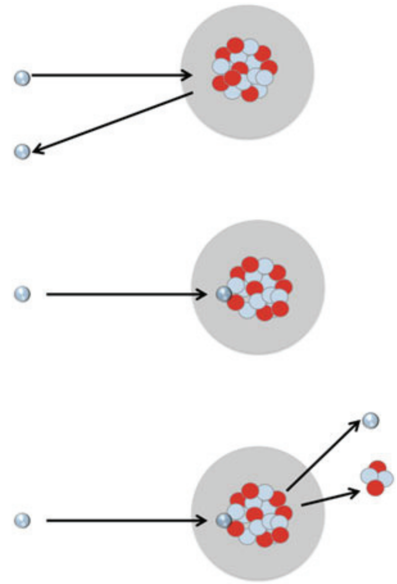


Fig. 7.16 Schematic visualisation of the interactions of neutrons with the matter. All interactions occur with nuclei: elastic scattering (top), neutron absorption (middle) and generic nuclear reactions (bottom)

There are three possible reactions:

- elastic scattering $n + A \rightarrow n + A$
- capture $n + A \rightarrow (A + 1)$
- other nuclear reactions: $n + A \rightarrow B + C$

Elastic Scattering

In the elastic scattering process, neutrons lose their energy by transmitting part of it to the nuclei. Both parts remain unchanged by the scattering. For kinetic energies lower than about 1 MeV, this is the only possible scattering process, and it can be treated mathematically just as a classical billiard balls scattering, with balls of different mass. Neglecting the mass difference between protons and neutrons, and the binding energy, A represents the atomic mass. We have maximum energy transfer when the collision occurs in one dimension: the neutron bounces back, and the nucleus recoils along the initial direction of the neutron. The heavier the nucleus, the lower fraction of neutron energy is transmitted to it. The minimum energy transfer is zero for extremely peripheral collisions. After one scattering event, the neutron has a kinetic energy E which is a fraction of the initial energy E_0

$$\left(\frac{A-1}{A+1}\right)^2 \leq \frac{E}{E_0} \leq 1 \; . \tag{7.32}$$

All energy losses within this range are equally probable. In case of scattering on hydrogen, $A = 1$ after n scattering processes the neutron average kinetic energy is:

$$< E_n >= \frac{E_0}{2^n} \ . \tag{7.33}$$

Neutron Capture

In the capture process, the neutron remains bound to the nucleus, which now has increased by one its mass number $A+1$. The atomic number is unchanged, unless, of course, there is a subsequent beta decay. A very important neutron capture process is

$$n +^{113} \mathrm{Cd} \rightarrow^{114} \mathrm{Cd}^* \rightarrow^{114} \mathrm{Cd} + \gamma \ . \tag{7.34}$$

This is used in nuclear reactors to control the flux of neutrons in the core. Another example of capture, which occurs in nuclear reactors, produces radiocarbon:

$$n \ + \ ^{13}\mathrm{C} \rightarrow^{14} \mathrm{C} \ + \ \gamma \ . \tag{7.35}$$

Other Reactions

Other reactions include the charge exchange scattering:

$$n \ +^{14} \mathrm{N} \rightarrow \ p \ +^{14} \mathrm{C} \tag{7.36}$$

which is the main reaction to produce radiocarbon in the atmosphere. In this case, neutrons originate from cosmic rays and have high energy. The same reaction can occur in a nuclear plant and in this case it has a cross section of 1.8 b, for thermal neutrons.

In a generic nuclear reaction, anything can happen, provided that all conservation laws are respected. Another important reaction is

$$n +^{10}\mathrm{B} \rightarrow \alpha +^{7} \mathrm{Li}^* + \ \gamma\,(0.48\ \mathrm{MeV}) \quad (\sigma \ = 3480\,\mathrm{barn}) \tag{7.37}$$

The cross section for the above process, for very low-energy neutrons ($E_k \approx 0.025$ eV), is 3480 barn. Also, this reaction is used to remove neutrons from the core of a nuclear plant, to control the reaction.

The neutron-induced fission was discovered by Lise Meitner and Otto Hahn, in 1938–1939. Meitner was the first woman to become a full professor in physics in Germany; she had to leave in 1938 to Sweden. Hahn was awarded the Nobel prize for chemistry in 1944; the artificial element *meitnerium* $_{109}$Mt is named after L. Meitner. The most stable isotope $^{278}_{109}$Mt has a lifetime of about 10 s.

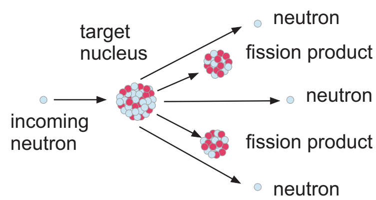


Fig. 7.17 A schematic representation of the neutron-induced nuclear fission. Notice that one or more neutrons are produced, which is essential for a self-sustained nuclear reaction

7.11 Spontaneous and Induced Fission

We have a spontaneous fission when a nucleus breaks into two daughter nuclei of approximately equal mass without any external action. As shown in Fig. 7.3, it occurs only for some nuclides with $A > 100$, which reach a higher binding energy per nucleon if they move to a lower A.

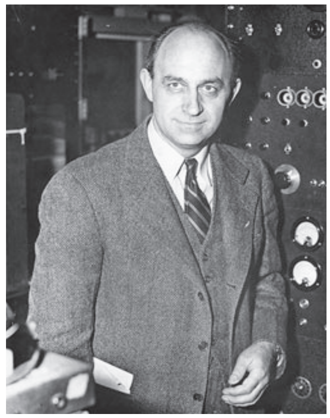
$$^{238}\mathrm{U} \rightarrow ^{145}\mathrm{La} + ^{90}\mathrm{Br} + 3n; \quad \mathcal{BR} \approx 10^{-7} \tag{7.38}$$

It is a fairly rare process, which "competes" in terms of probability, or decay branching ratio, with α-emission. The associated production of one or more neutrons is very important, because they can in turn start a neutron-induced fission, which is at the basis of nuclear reactors. The neutron-induced fission is a form of nuclear reaction, whose effect is breaking a nucleus into two or more daughter nuclei of about the same mass; an example is

$$^{235}\mathrm{U} + n \rightarrow ^{92}\mathrm{Kr} + ^{141}\mathrm{Ba} + 2n \tag{7.39}$$

Many odd-A nuclides are *fissile*, i.e. they have a large probability to undergo fission when a low-energy neutron interacts with the nucleus (Fig. 7.17). This is the case for ^{235}U, ^{239}Pu and ^{241}Pu.

Certain even-Z/even-N nuclides, like ^{232}Th, ^{238}U and ^{240}Pu, require energetic neutrons to undergo a fission process.



Enrico Fermi (1901–1954) from Italy, was where he received the prize in 1934 for his work on neutron-induced radioactivity. From Stockholm he moved to the USA, where he taught at Columbia University. He designed and operated the first nuclear reactor, which reached criticality in Chicago on December 2nd 1942. He also taught at the Universities of Rome and Chicago. More than 25 physics effects and models are named after him (photo: courtesy of Argonne National Laboratory)

7.12 Applications: Fission-Based Nuclear Reactors

The principle of fission reactors is based upon neutron-induced fission and an accurate neutron balance. The most stable uranium isotopes have $A = 238$ and $A = 235$. The former, ^{238}U, is more abundant in nature, but the probability that upon meeting a slow neutron it breaks into fission fragments is very small. On the contrary, ^{235}U only makes 0.7% of natural uranium, but the *cross section* for the reaction:

$$^{235}\mathrm{U} + n \rightarrow \text{ fission fragments} + N_n\, n \tag{7.40}$$

is a factor 10^8 larger. The cross section depends on the neutron kinetic energy, and it is larger for low-energy neutrons. When the energy of the neutrons is of the same order of magnitude of the average thermal energy kT, where k is the Boltzmann's constant and T the temperature in K, these neutrons are called *thermal* ($kT \approx 0.025$ eV). The fission fragments are not unique, as many different fission reactions can take place. They are often radioactive and most of them decay with medium-slow lifetimes. On average, the fission fragments carry 180 MeV per fission, the neutrons carry about 2.5% of the energy, and an additional 13% is obtained by a later-stage decay of radioactive nuclides. This is a significant fraction of the heat created with the fission. The kinetic energy is transformed into heat:

The energy loss for charged particles is calculated with the Bethe–Bloch formula, Eq. (5.38), while neutrons lose their kinetic energy by elastically scattering, as in Eq. (7.32), or by reacting with nuclei and producing heavy charged particles. A cooling system, typically water-operated, generates steam for a turbine.

> Boltzmann's constant links the average energy of a gas to its temperature: $k = 8.61733 \times 10^{-5}$eV K^{-1}. It is named after Ludwig Boltzmann 1844–1906, from Vienna.

The key point to the nuclear *pile* is that the fission reactions generate neutrons, which in turn can induce fission to other ^{235}U nuclei. The number N_n of neutrons generated in each fission process, Eq. (7.40), is very important to establish a *chain reaction*. These neutrons are not thermal, and they carry quite a substantial amount of energy. The cross section for a fission reaction is much larger for thermal neutron. In order to thermalise them, a *moderator* has to be used. A moderator is some inert material which absorbs energy from fast neutrons, without absorbing them. The main process is the elastic scattering and therefore hydrogen, with $A = 1$, is the best possible moderator, from Eq. (7.32). Water and graphite are used as moderators as well as heavy water, which contains deuterium. The chemical symbol D is used for deuterium, and heavy water is also indicated with HDO. With reference to Fig. 7.18, we define the *criticality of a chain reaction* as:

$$k = \frac{\text{number of useful neutrons produced at stage}(n)}{\text{number of useful neutrons produced at stage}(n-1)} \tag{7.41}$$

Neutrons are *useful* in a reactor when they don't escape, they are not absorbed by other material which may be present, but, after some multiple elastic scattering, they induce a fission process. We have a self-sustained and steady reaction when $k = 1$. At this point, the reactor has reached criticality. The reaction continues as long as there is nuclear fuel (^{235}U). We have an explosive reaction if $k > 1$: this is the principle of the nuclear bomb. The reaction will not continue if $k < 1$; in this case, the reactor is said to be *sub-critical*. In order to operate a reactor, we need to have a way to regulate and control the number of useful neutrons. This is done thanks to the properties of cadmium and boron, which we have seen in the reaction (7.34). There are three stable isotopes of Cd, and ^{114}Cd has a neutron capture cross section of about 2500 barn for thermal neutrons. For this reason, when cadmium rods are inserted in the nuclear fuel, they can remove neutrons and control the criticality of the nuclear reaction. Also, boron is used for the same purpose.

An example of fission reaction is

$$^{235}\mathrm{U} + n \rightarrow ^{144}_{56}\mathrm{Ba} + ^{90}_{36}\mathrm{Kr} + 2\,n \tag{7.42}$$

In this case, the Q-value is 180 MeV. This is a small amount of energy from a macroscopic point of view, but we need to consider the huge number of atoms in a

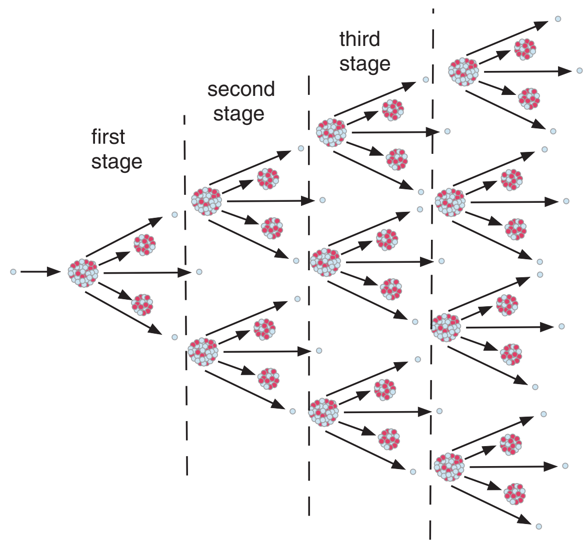


Fig. 7.18 Representation of a chain of fission reactions, initiated by a neutron. Different stages represent consecutive time intervals

mole. In case of natural Uranium, a mole is 238 g; it contains $N_A = 6.02 \times 10^{23}$ atoms. Of these, 0.7% are of the fissile isotope ^{235}U. The total amount of energy we can obtain from them is

$$W_T = 180 \times 10^6 \text{eV} \times 0.007 \times 6.0 \times 10^{23} \times 1.6 \times 10^{-19} \text{ J/eV} = 1.2 \times 10^{11} \text{ J}$$

Considering that 1 J = 2.78×10^{-7} kWh, the energy above can be expressed in a more practical unit as 3.36×10^4 kWh. This energy is mostly in the form of heat, although some of it, about 2%, escapes as neutrinos. The efficiency to convert heat to electric energy in nuclear plants is about 30%, obtaining 11,200 kWh. A household in Europe uses, on average, 3600 kWh of electricity a year. This makes a mole of natural uranium (238 g) just about sufficient for the electrical needs of three households in Europe for a year; it is equivalent to the energy of 2200 kg of liquefied natural gas. The total yearly production of uranium is about 6×10^4 kg at present. It is produced as "yellow cake" oxide, or U_3O_8.

Present nuclear reactors have somehow a "standard" size to produce a maximum of about 1 GW of electrical power. More than one reactor unit can be clustered in the same nuclear power plant, to share services, cooling and optimise the transport of fuel and spent fuel. At present, 450 reactor units are operating in the world (source: IAEA, 2016), producing a maximum electrical power of 390 GW. Different technologies are used for power plant reactors. They can be characterised by the nature of the moderator, by the coolant, by the type of fuel and by the type of nuclear reaction that occurs.

- *Pressurised Water Reactors (PWR)* They are moderated and cooled by normal water, a schematic is shown in Fig. 7.19. The pressurised vessel avoids that

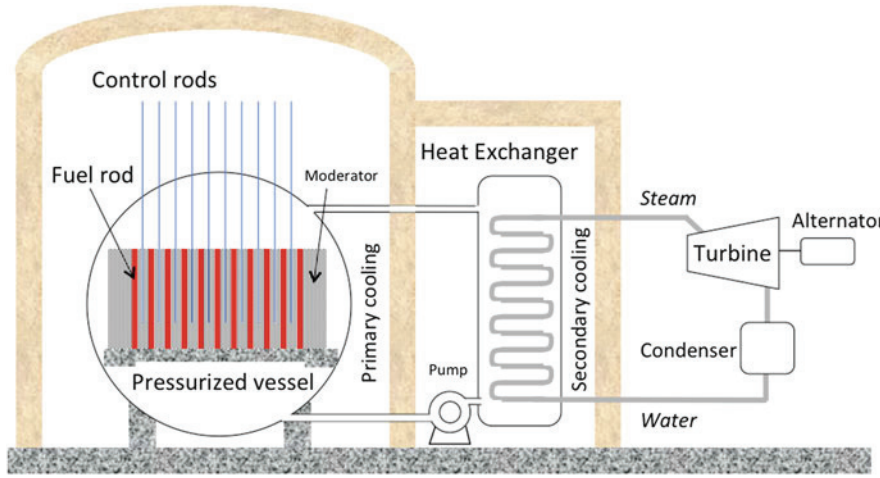


Fig. 7.19 A simplified diagram of a Uranium-based nuclear fission reactor, of the type PWR. (Modified after Lilley (2001).) For safety reasons, the coolant which circulates in the reactor core is not the same fluid which powers the turbine. The moderator is indicated as a separate element, but in Pressurised Water reactors the same pressurised water for primary cooling also acts as a moderator. CANDU reactors have a similar scheme, but the coolant in the primary circuit is heavy water (HDO)

coolant water boils inside the reactor core. For this reason, there are two cooling circuits, a primary and secondary.

- *Boiling Water Reactors (BWR)* only have one cooling circuit, otherwise they are very similar. They both use enriched uranium as fuel. This means that in the fuel rods the concentration of fissile isotope ^{235}U is increased with respect to the natural 0.7%. The fuel chemical compound is UO_2.
- *Heavy Water Reactors (HWR)* they are moderated and cooled by heavy water (HDO), where one atom of hydrogen is replaced by an atom of deuterium (^{2}H). The CANDU (CANadian Deuterium Uranium) reactors use natural uranium UO_2, and they can operate with a percentage of thorium. The schematics is very similar to the BWR.
- *Graphite Moderated Reactors* use the same moderator as the first reactor in Chicago. They can use either gas as a coolant (CO_2, He or N_2) or water, as in the case of the Chernobyl reactor.
- *Breeder Reactors* don't require a moderator and produce more fissile material than they consume. They are cooled with a molten salt, or with liquid sodium, as in case of the Superphenix reactor, which was operated in France. Their fuel is a mixture of ^{239}Pu which is fissile, and ^{235}U, which is *fertile*. The main reaction in a Pu-U breeder reactor are

$$^{239}\text{Pu} \rightarrow 3n + F_1 + F_2, \tag{7.43}$$

$$n + {}^{238}_{92}\mathrm{U} \rightarrow {}^{239}_{92}\mathrm{U} \xrightarrow{\beta} {}^{239}_{93}\mathrm{Np} \xrightarrow{\beta} {}^{239}_{94}\mathrm{Pu} \tag{7.44}$$

where F_1 and F_2 are fission fragments.

New types of reactors are being developed as "Generation IV", where new cooling schemes and type of fuel are being tested with the aim of improving efficiency and safety.

Although nuclear fission reactors don't produce directly any greenhouse gas, they have many other inconveniences. The main problem is production of nuclear waste, which is radioactive and has a decay time of million years. The other non-negligible problem is proliferation of nuclear armament: fission nuclear reactors can be used to produce nuclides which are most suitable for bombs. Other aspects are more common to any other human activity which involves large quantities of energy: operation safety, and protection from extreme natural events like earthquakes and tsunamis, and human mistakes. An additional factor is the magnitude and duration of the consequences that a possible accident would produce, which is far beyond the worst accident which could occur in a non-nuclear plant with a similar energy content. The consequences of the major nuclear plant accidents extended beyond the border of single countries: the release of radioactivity reached locations which were thousands of kilometres away from the plant, and entire cities had to be evacuated for decades.

> The two most severe accidents to nuclear plants were the Chernobyl accident, which occurred on 26 Aprseventeen small naturalil 1986 near Pripyat, Ukraine, and the Fukushima Daiichi accident, occurred on 11 March 2011 in Japan. The first was due to a series of human mistakes, the second to a tsunami following an earthquake of magnitude 9.

Nuclear waste can be divided into three categories: fission products, *actinides* and activated material. Fission products contribute for a weight that is about the same, or slightly lower, than the amount of initial fissile material. If fuel is enriched to 3.5% of fissile uranium, from 0.7% of the natural uranium, we expect about 3% of fission products. Some of them have a relatively short lifetime, like ^{131}I, while others have a lifetime comparable to the human life. This is the case of ^{137}Cs and ^{90}Sr. However, some have a very large lifetime: ^{135}Cs and ^{129}I decay in 2.3 and 15 million years, respectively. The large flux of neutrons produced in a reactor is absorbed by the shielding material, which, in turn, becomes radioactive. Control rods are extremely radioactive; some tritium is produced in water, especially when heavy water is used as a moderator. In general, the lifetime of activated materials can be kept low by selecting low-A material and special steels for infrastructures. The third and most important type of waste is made of *actinides*, i.e. nuclides with $Z \geq 89$. They are produced by neutron capture by ^{238}U, which is the major component of fuel rods. About 0.5% of spent fuel is fissile ^{239}Pu, which is mostly used in nuclear warheads. An example of a nuclide which is present with a concentration of about 0.4% in spent fuel is ^{236}U. This is not present in nature; it has a lifetime of 2.3×10^7 years, and it is at the start of a chain of ten decays, before ending as stable lead:

$$^{236}\mathrm{U} \xrightarrow{\alpha} {}^{232}\mathrm{Th} \xrightarrow{\alpha} {}^{228}\mathrm{Ra} \xrightarrow{\beta} {}^{228}\mathrm{Ac} \xrightarrow{\beta} {}^{228}\mathrm{Th} \xrightarrow{\beta} {}^{224}\mathrm{Ra} \xrightarrow{\alpha}$$
$$\to {}^{220}\mathrm{Rn} \xrightarrow{\alpha} {}^{216}\mathrm{Po} \xrightarrow{\alpha} {}^{212}\mathrm{Bi} \xrightarrow{\alpha} {}^{208}\mathrm{Tl} \xrightarrow{\alpha} {}^{208}\mathrm{Pb} \qquad (7.45)$$

About 4×10^8 kg of radioactive spent fuel have been produced worldwide so far, with a production rate of 1×10^8 kg/y.

> A set of 17 small natural reactors has been discovered in Gabon, West Africa, near the equator, in a place called Oklo, near Franceville. These reactors operated about 2×10^9 years ago, for about one million years. They have been discovered because the uranium ore of these mines has a lower concentration of the fissile isotope.

7.13 The Thorium Cycle

The shortage of Uranium in some countries has revived interest in the thorium cycle, which was abandoned in the USA in 1973. ^{232}Th is the only naturally occurring isotope, for all practical purposes. It is not stable, but decays α with a half-life of 1.4×10^{10} years. It is also not fissile, but it is *fertile*: it can be used to "easily" generate an isotope of uranium, which is fissile, by neutron absorption:

$$n + {}^{232}\mathrm{Th} \to {}^{233}\mathrm{Th} \to {}^{233}\mathrm{Pa} \to {}^{233}\mathrm{U} \qquad (7.46)$$

All the decays in the chain above (Eq. (7.46)) are β^-. The nuclide ^{233}U is fissile. In order to take place, this reaction needs some initial nuclear chain reaction to take place and provide the neutrons to obtain the fissile nuclides. This is an example of a so-called *breeder reactor*, which produces both energy and nuclear fuel. The advantages of thorium are that it is more abundant than uranium, and it is not as diluted.

Also, the spent fuel has a shorter average lifetime, with respect to uranium-based fuel, although some long-lived nuclides are also produced. The only two active thorium-operated research reactors are in India. A recent project, the "energy amplifier", proposes to use a particle accelerator to sustain the criticality condition to an otherwise sub-critical thorium-based reactor, as sketched in Fig. 7.20. One of the advantages is the possibility to operate without initial fissile Pu, which is needed in all other breeder reactors (Eq. (7.43)), and the other is the possibility to fission the actinides.

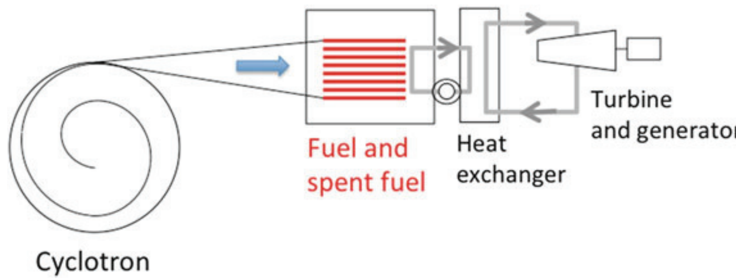


Fig. 7.20 Schematics of the principle of an Energy Amplifier. A proton beam is used as a spallation neutron source to sustain the fission reaction in thorium. This method can also be used to burn actinides from spent nuclear fuel. This generator is at the stage of feasibility study

7.14 Nuclear Fusion and Nucleosynthesis

Nuclear fusion powers the only "controlled" fusion nuclear power source so far: the sun, and other stars. Several fusion reactions occur in sequence in the sun and many have *branches*, which means that they can occur in more than one way. The predominant reaction is called the *proton–proton cycle*, or *pp*-I: protons fuse together to produce deuterons. Other branches of this cycle are also active (*pp*-II and *pp*-III). A star's temperature is much larger than the ionisation temperature of all elements: the sun is made by a neutral plasma of electrons and nuclei. The gravitational force holds them together, creating an enormous pressure. These two conditions make it possible that nuclei get in close contact, penetrating the electrostatic shield that surrounds them. The reactions in the proton–proton (I) cycle are (Fig. 7.21):

$$p + p \rightarrow {}^2\mathrm{H} + e^+ + \nu_e \quad Q = 0.42\ \mathrm{MeV}\ \text{proton–proton fusion} \tag{7.47}$$

$$p + {}^2\mathrm{H} \rightarrow {}^3\mathrm{He} + \gamma \quad Q = 5.49\ \mathrm{MeV}\ \text{proton–deuteron fusion} \tag{7.48}$$

$${}^3\mathrm{He} + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + 2p \quad Q = 12.86\ \mathrm{MeV}\ \text{Helium–Helium fusion} \tag{7.49}$$

Another fusion cycle is very important for stellar evolution: the *carbon-catalysed cycle* (CNO). Carbon acts as a catalyst, meaning that it facilitates the reaction, but carbon nuclei are not created or destroyed by the chain of reactions (Fig. 7.22):

$$p + {}^{12}\mathrm{C} \rightarrow {}^{13}\mathrm{N} + \gamma \quad Q = 1.95\ \mathrm{MeV}\ \text{proton–carbon fusion} \tag{7.50}$$

$${}^{13}\mathrm{N} \rightarrow {}^{13}\mathrm{C} + e^+ + \nu_e \quad Q = 1.20\ \mathrm{MeV}\ \beta\text{-decay} \tag{7.51}$$

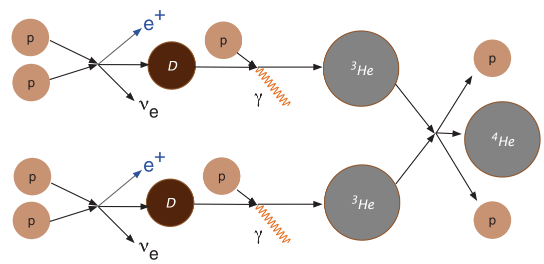


Fig. 7.21 Schematics of the pp-I fusion reaction, one of the main reactions occurring in the sun. The overall reaction is $4p \rightarrow^4 \text{He} + 2e^+ + 2\nu_e + 2\gamma$, and the total energy released is 24.67 MeV

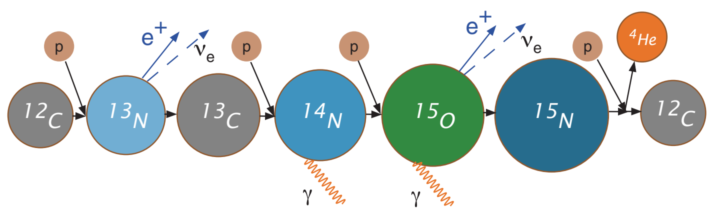


Fig. 7.22 The carbon-catalysed fusion reaction cycle, CNO. Carbon acts as a catalyst

$$p +^{13} \text{C} \rightarrow^{14} \text{N} + \gamma \quad Q = 7.55 \text{ MeV proton–carbon} \tag{7.52}$$

$$p +^{14} \text{N} \rightarrow^{15} \text{O} + \gamma \quad Q = 7.34 \text{ MeV proton–nitrogen} \tag{7.53}$$

$$^{15}\text{O} \rightarrow^{15} \text{N} + e^+ + \nu_e \quad Q = 1.68 \text{ MeV } \beta\text{-decay} \tag{7.54}$$

$$p +^{15} \text{N} \rightarrow^{12} \text{C} +^4 \text{He} \quad Q = 4.96 \text{ MeV proton–nitrogen} \tag{7.55}$$

Carbon takes part in the process; it is consumed, but it is also re-generated, so we find the same amount of carbon in the final state as in the initial state. The overall reaction is

$$4p \rightarrow^4 \text{He} + 2e^+ + 2\nu_e + 3\gamma \quad Q = 24.68 \text{ MeV} \tag{7.56}$$

From all the above reactions, we see that the sun is a large source of electron neutrinos, which are called *solar neutrinos*. Their energy has a range that extends up to 0.4 MeV for pp-I reactions and from 1 to 12 MeV, for the CNO reactions. Neutrinos of this energy have a cross section of the order of $\sigma_\nu \approx 10^{-44}$ cm^2. They escape from the sun, which is quite transparent to them, and relatively few of them, for purely geometrical reasons, reach the planet Earth and only very occasionally do they interact with matter, sometimes with neutrino detectors in underground laboratories. By *nucleosynthesis*, we mean the process of formation of elements in stars. When the proton content of stars, including the sun, is depleted, the fusion continues with other reactions, synthetising heavier elements. The sun is a *yellow dwarf star* and when its hydrogen will be consumed, in about 5×10^6 years, it will become a *red giant* star, with a much larger diameter and a core which will start burning helium, to form beryllium, carbon and oxygen; it will not go on further, because the temperature and pressure will not be enough to ignite other fusion reactions: it will become a cold *white dwarf* star. Stars which are more massive continue their fusion processes, which provides less and less energy as heavier nuclides are used for fusion, till producing nuclides with $A \approx 56$. This is the maximum of binding energy per nucleon, as shown in Fig. 7.3; above this nuclear size, adding nucleons would decrease the binding energy and would not be energetically favourable ($Q < 0$). Therefore, heavier elements cannot be produced with normal fusion processes. When a star of a large size, about 10 times the sun mass, has used all its fuel available to fusion, it ends up as an iron core and no radiation to balance the gravitational attraction, which heats up the remaining material and ignites a supernova explosion. Temperature increases above 10^{10} K, and photons have enough energy to break iron nuclei, for $E_\gamma > 2.5$ MeV

$$\gamma + {}^{56}\text{Fe} \rightarrow 13\,{}^{4}\text{He} + 4n \quad (Q = 145 \;\; \text{MeV}) \tag{7.57}$$

The process can continue further, to break alpha particles and create a hot plasma of protons and electrons, which have enough energy to produce an *inverse beta decay*

$$e^- + p \rightarrow n + \nu_e \tag{7.58}$$

What is left is similar to a giant nucleus, with $A \approx 10^{57}$, with a radius of a few kilometres and most of its nucleons in the form of neutrons (a *neutron star*). Under certain conditions, a *supernova* explosion (type-II) takes place, which produces a shock wave of neutrons and neutrinos. Models of supernova explosions predict short time intervals during which emission of a very large neutron flux takes place. This process forms heavier elements with two mechanisms: a sequence of neutron captures and beta decays, starting from initial iron nuclei, or a rapid aggregation of neutrons around an Fe nucleus, and subsequent beta decays. Emission of an even higher flux of neutrinos of all three kinds is also predicted by some theoretical models, and was detected in the laboratory in the last supernova explosion which occurred in proximity of our galaxy. The sun and the solar system have emerged from the remnants of a supernova explosion. The abundance of elements in the solar system is shown in Fig. 7.23.

Some of the solar neutrinos reach our planet, with a flux of about $\Phi_\nu \approx 6.5 \times 10^{10}$ cm^{-2} s^{-1}. A very small fraction of them are detected in underground laboratories. Some of the electron neutrinos transform into μ and τ neutrinos during their flight. The long standing problem of missing solar neutrino flux is now understood in terms of this phenomenon, called *neutrino oscillations*. The solar neutrino problem was first observed by Ray Davis (USA, 1914–2006) Nobel Prize 2002, using the Standard Solar Model calculations by John N. Bahcall (USA, 1934–2005).

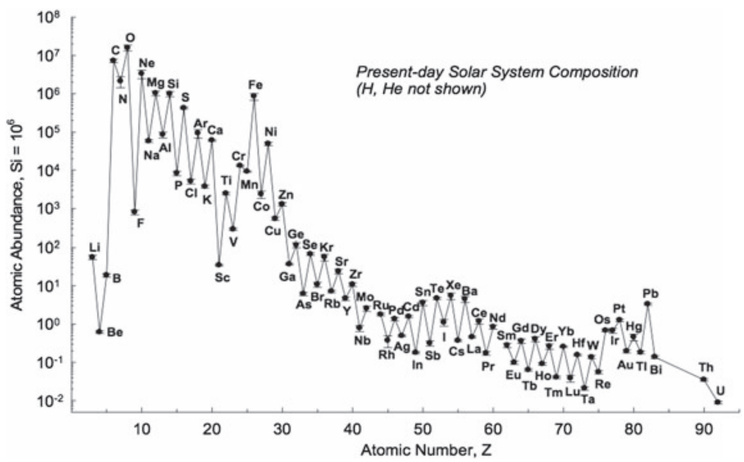


Fig. 7.23 Abundance of elements in the solar system. *From K. Lodders, in "Principles and perspectives in Cosmochemistry", Springer (2010)*

On 23 February 1987, a supernova appeared in the southern hemisphere. The originating star was a blue super giant with originally 10–100 solar masses located in the Large Magellanic Cloud, at "only" 167 kly (thousands of light years) from the Earth. A burst of neutrinos was detected in three neutrinos observatories, at 7 : 35 : 35 universal time, about 3 h before the visible light reached the Earth, which is consistent with a flux of 10^{58} neutrinos in two pulses a few seconds apart. The expected remnant neutron star has not been observed yet.

7.15 Fusion Reactors

The induced nuclear fusion has been achieved on the Earth, but not in a sustained way: in thermonuclear bombs the process is not under control, while in reactors fusion only occurs for a very short time and, so far, not in a self-sustained reaction. The main reactions are

$$^2\mathrm{H} + ^2\mathrm{H} \rightarrow ^3\mathrm{He} + n; \quad Q = 3.27\,\mathrm{MeV}\ \text{Helium production} \tag{7.59}$$

$$^2\mathrm{H} + ^2\mathrm{H} \rightarrow ^3\mathrm{H} + p; \quad Q = 4.03\,\mathrm{MeV}\ \text{Tritium production} \tag{7.60}$$

$$^2\mathrm{H} + ^3\mathrm{H} \rightarrow ^4\mathrm{He} + n; \quad Q = 17.62\,\mathrm{MeV}\ \text{Helium production} \tag{7.61}$$

The main problem is to beat the Coulomb repulsion between protons or tritium nuclei. Magnetic confinement or laser-induced pressure is being used (separately) to reach nuclear fusion in a laboratory. A laser-induced fusion lab in Livermore, CA, has recently reached a breakthrough point: more energy was produced by the fusion process than the energy absorbed by the fuel target. However, this quantity is far less than the energy needed to operate the lasers.

7.16 Problems

For these problems, we need to use more decimal digits than usual, because some of them are based on detailed calculations, involving differences between quantities which differ by less than 1%. Some of the data needed is available in the text.

7.1 The deuteron is a (pn) bound state. Its *atomic* mass is 2.014101 a.m.u. (or u). Calculate its binding energy. The mass of the proton is 1.0072764 u and the mass of the neutron is 1.008665 u.

7.2 Repeat the calculation above using MeV/c^2 for masses. The value of the masses are: deuteron: 1875.613 MeV/c^2; proton: 938.272 MeV/c^2; neutron: 939.565 MeV/c^2.

7.3 The atomic mass of $^{241}_{95}$Am is 241.056829 u. What is its mass deficit? It decays to $^{237}_{93}$Np by emitting an α particle. The atomic mass of Np is 237.048173 u. What is the kinetic energy of the α emitted?

7.4 In a fission nuclear reactor, a possible reaction is

$$n +^{235} \mathrm{U} \rightarrow^{92}_{37} \mathrm{Rb} +^{140}_{55} \mathrm{Cs} + N \text{ neutrons}$$

How many neutrons are produced? (the baryon number is conserved).

7.5 In the above reaction, the atomic masses are

$$M(^{235}_{92}\mathrm{U}) = 235.04393 \text{ u}$$

$$M(^{92}_{37}\mathrm{Rb}) = 91.919729 \text{ u}$$

$$M(^{140}_{55}\mathrm{Cs}) = 139.91728 \text{ u}$$

$$M(n) = 1.008665 \text{ u}$$

Neglecting the kinetic energy of the incoming neutron, is the reaction endo-energetic or exo-energetic? What is the Q-value? (in MeV).

7.6 ^{232}Th is the only naturally occurring isotope of thorium. Its half-life is 14.05×10^9 years. What is its mean lifetime in seconds? A mole of thorium (232 g) is made of 6.02×10^{23} atoms (Avogadro's number).

(a) What is the activity of a mole of thorium, in Bq ([s^{-1}])?

(b) The decay reaction is $^{232}\text{Th} \rightarrow^{228}$ Ra $+ \alpha$ with a kinetic energy of the α-particle of 4.083 MeV. Suppose that all the radiation is absorbed by the brick containing our thorium sample and transformed into heat. How much energy per second (power) is generated?

7.17 Solutions

Solution to 7.1 As for all nuclides, the mass of the deuteron is smaller than the sum of the mass of its components. The binding energy is equal to this mass difference. In this calculation, care must be taken to include or not the electron mass. This is normally included in the value of the *atomic mass*, but not in the proton mass, even if it is expressed in units of u. The electron mass is 510.9989 keV/c^2, $u = 931{,}494.0$ keV/c^2, so $m_e = 0.0005485\, u$.

$$\Delta M = M_D - m_e - m_p - m_n =$$
$$= 2.0141018 - 0.0005485 - 1.0072764 - 1.0086649 = -0.0023880\,\text{u}$$

Expressing it in MeV/c^2, we have

$$B = 931.4940 \times 0.0023880 = 2.2244 \text{ MeV, or } 1.1122 \text{ MeV per nucleon.}$$

Solution to 7.2 When masses are given in MeV, normally electrons are not included. In the data of this problem, it is specified that the mass of the *deuteron* is given. The calculation is straightforward:

$$\Delta M = m_D - m_p - m_n \;;$$

$$\Delta M = 1875.613 - 938.272 - 939.565 = -2.224\,\text{MeV/c}^2$$

Solution to 7.3 The Am isotope has 95 protons and 146 neutrons. Its mass deficit is

$$\Delta M = M_{\text{Am}} = 241.056827 - 95 \times (1.0072764 + 0.0005485) - 146 \times 1.008665$$

$$\Delta M = -1.9516285\text{ u}$$

$$B = -\Delta M c^2 = 1.9516285 \times 931.4940 = 1817.93 \text{ MeV} = 7.543 \text{ MeV per nucleon}$$

The Q-value of the reaction

$$^{241}_{95}\text{Am} \rightarrow \alpha +^{237}_{93}\text{Np} + 2\,e^- \qquad \text{is}$$

$$Q = M_{\text{Am}} - M_{\text{Am}} - M_\alpha - 2m_e$$

$$Q = 241.056827 - 237.048173 - 4.0015061 - 2 \times 0.0005485 = 0.006050\,\text{u}$$

which translates into 5.636 MeV. The largest part of this energy is converted into kinetic energy of the alpha particle, recoiling against a ^{237}Np nucleus: $E_k(\alpha) =$ 5.485 MeV.

Solution to 7.4

$$n +^{235}\text{U} \rightarrow^{92}_{37}\text{Rb} +^{140}_{55}\text{Cs} + N\ \text{neutrons}$$

Uranium has 92 protons. The number of protons and neutrons is conserved.
Protons: $92 = 37 + 55$
Neutrons: $143 + 1 = 55 + 85 + N \Rightarrow N = 4$

Solution to 7.5

$$Q = 1.008665 + 235.04393 - 91.919729 - 139.91728 - 4 \times 1.008665 = +0.180926$$

This value is positive, the reaction is exo-energetic; it releases energy.

Solution to 7.6

$$\tau = 14.05 \times 10^9 \times 3.1536 \times 10^7 = 44.3 \times 10^{16}\ s$$

$$\mathcal{A} = N/\tau = 6.02 \times 10^{23}/44.3 \times 10^{16} = 1.36 \times 10^6\ \text{Bq}$$

$$P = 1.36 \times 10^6 \times 4.083 \times 10^6 \times 1.602 \times 10^{-19}\ \text{W}$$

$$P = 8.89 \times 10^{-7} W$$

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