Cosmology 15

How big is the universe, how old is it, how did it come into being, what does its future development look like? *Cosmology* is concerned with such questions. The first philosophical reflections on the structure of the universe can be found already in the ancient peoples.

Distance determinations play a major role in understanding the structure of the universe, and here we mention three milestones:

Copernicus, Kepler: were concerned with distances and dynamics of the bodies of the solar system;

Bessel: First measurement of a stellar parallax. Bessel used the star 61 Cygni, which showed the largest known proper motion at that time: 4.1″/year in right ascension and 3.2″/year in declination. In 1838 he determined the parallax of this star: 0.3″. This measurement showed definitively: Our solar system is only a tiny part of the universe.

Hubble: From the Period-Luminosity Relationship of the Cepheids, he was able for the first time to determine the distance of the "Andromeda Nebula" and classify it as an extragalactic system.[1]

> Only since about 100 years we know that there are many other galaxies beside our galaxy.

[1] He reported at the annual meeting of the American Astronomical Society, 1925, "Cepheids in Spiral Nebula."

A. Hanslmeier, *Introduction to Astronomy and Astrophysics*,
https://doi.org/10.1007/978-3-662-64637-3_15

Modern cosmology has contributed significantly to the development of general relativity and modern particle physics.

15.1 Expansion of the Universe

In this section, observational results relevant to cosmology are discussed. These can then be used to build a model of the formation and evolution of the universe.

15.1.1 View into the Past

In general: The objects (galaxies) are very far away from us; recently there has been a great increase in knowledge due to the development of new detectors (CCD, CMOS) and large telescopes (> 8 m). The objects are at a distance D by us. The light propagates with finite speed c, and therefore we see a source at distance D today in a state in which light propagated by a time interval Δt earlier than today:

$$\Delta t = \frac{D}{c} \tag{15.1}$$

The present state of the universe is observable only in the near vicinity. We are, so to speak, looking into the past

If we observe galaxies at a distance of 10^{10} light years, we see light from a time when the age of the universe (about 13.7 billion years) was only about 1/3 of today's!

Suppose we are in a Euclidean space at the position $\mathbf{r} = 0$ at the time $t = t_0$. Since light propagates at finite speed, we only see space-time points for which:

$$|\mathbf{r}| = c(t_0 - t) \tag{15.2}$$

i.e. only this part of the universe is observable for us.

15.1.2 Olbers Paradox

Olbers (1758–1840) arose the question: Why is it dark at night? Let's assume, n_* is the number density of stars and n_* is constant in space and time. The number of stars within in a spherical shell of radius r and thickness dr is $n_* 4\pi r^2 dr$. Each star occupies a solid angle $\pi R_*^2/r^2$ so the stars in the spherical shell occupy the solid angle

$$d\omega = 4\pi r^2 dr n_* R_*^2 \pi / r^2 \tag{15.3}$$

i.e. r truncates out. So the sky would be completely filled with star discs and would be bright (shine like a body with a temperature of approx. 10^4 K).

Therefore, the universe (i) cannot be infinitely extended, (ii) cannot have existed for an infinitely long time.

15.1.3 Galaxy Counts

If one counts galaxies with a luminosity $n > L$ then the number of galaxies within a spherical shell of radius r and thickness dr is:

$$n(> L)4\pi r^2 dr \tag{15.4}$$

galaxies. We further assume that the luminosity function is constant in space and time.

The relation between flux S and luminosity L is

$$L = 4\pi r^2 S \tag{15.5}$$

and one finds for the number:

$$N(> L) = \int_0^\infty dr 4\pi r^2 n(> L) 4\pi r^2) \tag{15.6}$$

Because of $L = S4\pi r^2$, $r = \sqrt{L/(4\pi S)}$ and $dr = dL/(2\sqrt{4\pi LS})$ we find

$$\begin{aligned} N(> L) &= \int_0^\infty \frac{dL}{2\sqrt{4\pi LS}} \frac{L}{4\pi S} n(> L) \\ &= \frac{1}{16\pi^{3/2}} S^{-3/2} \int_0^\infty dL\sqrt{L} n(> L) \end{aligned} \tag{15.7}$$

We see: Independently of the luminosity function, the following holds for the count of galaxies in the universe:

$$N(> L) \propto S^{-3/2} \tag{15.8}$$

However, this contradicts observations.

The universe cannot be Euclidean, infinite and static.

15.1.4 The Redshift of the Galaxies

Let v be the measured Radial velocity of a galaxy, d their distance. *Hubble* found around 1926 that galaxies are moving away from us, the velocity being proportional to the distance of the galaxy.

The *Hubble relation*

$$v = dH_0 \tag{15.9}$$

states that (a) galaxies are moving away from us (except for those in our galaxy cluster) and (b) the escape velocity of galaxies is greater the farther they are from us (distance d). If one introduces the redshift z, then holds:

$$v = cz \qquad d = cz/H_0 \tag{15.10}$$

If all galaxies are moving away from us, you might think we are at the center of the universe. But the explanation is quite simple. Galaxy movements reflect the expansion of the universe. Imagine a balloon with galaxies marked as black dots. When the balloon is inflated, i.e. the universe expands, then you observe from any given point that the other points move away. Hubble's law applies to all galaxies in the universe.

The redshift of the galaxies is explained by the *Expansion* of the universe.

Einstein published his Field Equations in 1915. The solution of the field equations led to non static universe. At that time nothing was known about an expanding universe. Therefore he introduced the *Cosmological constant* introduced to keep the solutions static. This constant was also necessary because a universe consisting of mass would automatically contract if it were static.

Another problem is the exact determination of the Hubble constant H_0. In 1929 Hubble gave a value of 530 $\mathrm{km\,s^{-1}\,Mpc^{-1}}$. 1958 one was with a value of 75, later there were two variants with (a) 50 and (b) 100. Measurements with the HST and WMAP come to a value accepted today as valid from:

$$H_0 = (69.7 \pm 4.9)\ \mathrm{km\,s^{-1}Mpc^{-1}} \tag{15.11}$$

A galaxy at a distance of 1 Mpc is therefore moving away from us at about 70 km/s.

One also writes:

$$H_0 = 50h \qquad h = 1.4 \tag{15.12}$$

15.1.5 The Age of the Universe

From the expansion, one can estimate an age of the world; let us consider the units of the quantities of the Hubble relation:

- Hubble law: $V = RH$
- Unit of velocity: [v] = km/s
- Unit of distance: [R] = Mpc
- Unit of Hubble constant: [H] = $[\frac{\text{km/s}}{\text{Mpc}}]$
- $\rightarrow [1/H] = s$

The reciprocal of the Hubble constant provides an estimate of the age of the universe.

$$\tau_0 = r/v = 1/H_0 = \frac{1}{h50\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}} = 20 \times 10^9 h^{-1}\text{a} \tag{15.13}$$

So let's put in $h = 1.4$ (modern value of the Hubble constant), then the universe would be approx. 13.65×10^9 years old. This is consistent with the observed oldest globular clusters, whose age is 12 Ga.[2]

Of course, this assumes a uniform expansion of the universe. To a first approximation, this is given. However, there are two important exceptions:

- *Inflationary phase:* roughly between 10^{-35} and 10^{-33} s after the big bang the universe expanded extremely strong by a factor of 10^{26}.
- If one compares the present expansion of the universe with that of earlier epochs, it turns out: the universe is expanding at an accelerated rate. This is explained by the presence of *Dark energy*.

[2] 1 Ga = 1 Gigajahr so 10^9 a.

15.1.6 Homogeneity and Isotropy

The Hubble radius of the universe is:

$$R_H = \frac{c}{H_0} = 2997h^{-1}\text{Mpc} \tag{15.14}$$

There are structures in the universe:

- Galaxies,
- galaxy clusters,
- Superclusters.

The largest structures are the super clusters, with dimensions up to about $100h^{-1}$ Mpc. This is still small compared to the Hubble radius, so the universe can be considered homogeneous and isotropic.

> Although there are structures in the 13.65 billion year old universe, it can be simplistically described as homogeneous and isotropic.

This assumption greatly simplifies the complex calculations.

15.1.7 Methods of Distance Determination

In order to make statements about the structure and distribution of objects in the universe, it is necessary to know the distances. Most of the methods for distance determination have already been discussed in detail and shall only be listed here:

- Cepheids, RR Lyrae stars (from the period of brightness change follows the distance),
- globular clusters (standard candles),
- Novae (standard candles),
- supernovae (standard candles),
- Tully-Fisher relationship (rotational behavior of galaxies indicates their luminosity),
- luminosity of *planetary nebulae:* They emit 15 % of the light in the 500.7-nm-line ([OIII]). This part can be filtered out with a narrow band filter and then you have practically only the planetary nebulae of a galaxy. The absolute brightness then follows from the empirical Relationship:

$$M = -2.5\log(F_{500.7} - 13.74) \tag{15.15}$$

Table 15.1 Methods of distance determination applied to members of the Virgo cluster. The uncertainty of the method is given in absolute magnitudes ^{M}

Method	Uncertainty [^{M}]	Del. [Mpc]	Range [Mpc]
Cepheids	0.16	14.9 ± 1.2	20
Novae	0.40	21.1 ± 3.9	20
Planet. Nebula	0.16	15.4 ± 1.1	30
Globular cluster	0.40	18.8 ± 3.8	50
Tulley-Fisher	0.28	15.8 ± 1.5	> 100
Supernovae Ia	0.53	19.4 ± 5.0	> 1000

In this equation $F_{500.7}$ denotes the Radiation flux at 500.7 nm. Cepheids and RR Lyrae stars were identified in the closer galaxies (distance below 100 million light-years). With the HST, the horizon within which such objects can be seen directly has been substantially extended (Table 15.1).

15.2 Newtonian cosmology

In the universe, only gravitational force and electromagnetic force act on large length scales. Electric forces neutralize, leaving only gravity, and Einstein developed General Relativity in 1915, relating gravity to space-time curvature. Newton's theory is valid on small length scales, and one can consider simple world models.

15.2.1 Expansion

Although we have the impression that all galaxies are moving away from us—due to expansion of space—this does not mean that we occupy an excellent position in the universe. Because of

$$\mathbf{v} = H_0\mathbf{r} \tag{15.16}$$

is valid for any other galaxy, which is at a distance $\mathbf{r_1}$ relative to us and has the velocity $\mathbf{v}_1$ relative to us:

$$\mathbf{v} - \mathbf{v}_1 = H_0(\mathbf{r} - \mathbf{r}_1) \tag{15.17}$$

Therefore, a Hubble relation exists there as well: $\mathbf{v}_1 = H_0\mathbf{r}_1$.

We assume a homogeneous sphere expanding radially. At time $t = t_0$ the coordinate is $\mathbf{x}$ and it changes over time to:

$$\mathbf{r}(t) = a(t)\mathbf{x} \tag{15.18}$$

$a(t)$ ... cosmic scale factor. It holds $\mathbf{r}(t_0) = \mathbf{x}$ and thus $a(t_0) = 1$. The quantity t_0 can be chosen arbitrarily, and we set $t_0 =$ today. The expansion rate is given by

$$\mathbf{v}(\mathbf{r}, t) = \frac{d}{dt}\mathbf{r}(t) = \frac{da}{dt}\mathbf{x} = \dot{a}\mathbf{x} = \frac{\dot{a}}{a}\mathbf{r} = H(t)\mathbf{r} \tag{15.19}$$

So the expansion rate is (cf. Hubble constant!):

$$H(t) = \frac{\dot{a}}{a} \tag{15.20}$$

and to the time $t = t_0$ one has $H_0 = H(t_0)$.

15.2.2 Equation of Motion

If one considers a spherical shell with radius x at time t_0. For any t the radius is $r(t) = a(t)x$, and the mass is:

$$M(x) = \frac{4\pi}{3}\rho_0 r^3 = \frac{4\pi}{3}\rho(t)a^3(t)x^3 \tag{15.21}$$

The mass density of the present universe is ρ_0, and it holds:

$$\rho(t) = \rho_0 a^{-3}(t) \tag{15.22}$$

If we consider a particle on the surface of a sphere, we get an inward gravitational acceleration:

$$\ddot{r}(t) = -\frac{GM(x)}{r^2} = -\frac{4\pi G}{3}\frac{\rho_0 x^3}{r^2} \tag{15.23}$$

and if we substitute: $r(t) = xa(t)$ it follows that

$$\ddot{a}(t) = \frac{\ddot{r}(t)}{x} = -\frac{4\pi G}{3}\frac{\rho_0}{a^2(t)} = -\frac{4\pi G}{3}\rho(t)a(t) \tag{15.24}$$

15.2.3 Conservation of Energy

We multiply Eq. 15.24 by $2\dot{a}$, and because of $d(\dot{a}^2)/dt = 2\dot{a}\ddot{a}$ and $d(-1/a)/dt = \dot{a}/a^2$ is:

$$\dot{a}^2 = \frac{8\pi G}{3}\rho_0\frac{1}{a} - Kc^2 = \frac{8\pi G}{3}\rho(t)a^2(t) - Kc^2 \tag{15.25}$$

(Kc^2 is a constant of integration). Multiplying this equation by $x^2/2$ and find:

$$\frac{v^2(t)}{2} - \frac{GM}{r(t)} = -Kc^2\frac{x^2}{2} \tag{15.26}$$

The constant K is proportional to the total energy of the system. One can see immediately:

- $K < 0$ The right side of (15.25) positive, $da/dt > 0$ today, and remains positive for all times, universe expands eternally.
- $K = 0$: right side always positive, but for $t \to \infty$ goes $da/dt \to 0$.
- $K > 0$: right side of (15.25) becomes zero if $a = a_{\max} = (8\pi G\rho_0)/(3Kc^2)$, expansion stops, and contraction occurs.

The special case $K = 0$ at which the universe expands forever corresponds to a critical density ρ_{cr}:

$$\rho_{\text{cr}} = \frac{3H_0^2}{8\pi G} = 1.88 \times 10^{-29}h^2 \text{ g/cm}^3 \tag{15.27}$$

The density parameter Ω_0 is

$$\Omega_0 = \frac{\rho_0}{\rho_{\text{cr}}} \tag{15.28}$$

and one has: $K > 0 \to \Omega_0 > 1$.

Observations reveal: The density $\Omega_* \approx 0.01$ present due to stars and luminous matter is thus much too low (Fig. 15.1).

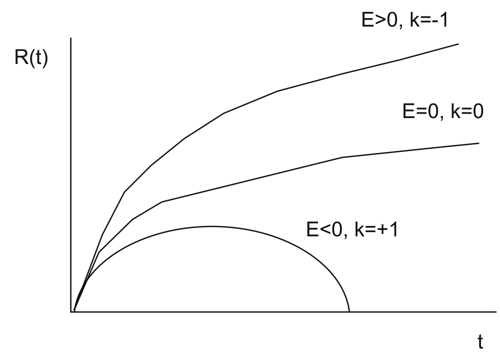

Fig. 15.1 Expansion of the universe for three values of k

There is a critical matter density. This determines whether the universe expands forever, or collapses again after a certain expansion phase.

15.3 Theory of Relativity

The theory of relativity developed by *A. Einstein* is divided into the special theory of relativity (1905), which describes the transformation between two reference systems moving with uniform velocity, and the general theory of relativity (1915), which specifies the transformation between reference systems moving arbitrarily against each other (and also includes gravity, as a curvature of space and time).

15.3.1 Special Theory of Relativity

Maxwell's equations of electrodynamics showed that electromagnetic waves (light) propagate. Since waves need a medium to propagate, it was assumed that light propagates in a medium called the *ether* and scientists tried to measure the motion relative to the ether. In 1887 there was the famous experiment of *Michelson,* to determine the motion of the Earth relative to the aether, but the result was negative. The result showed that all observers had the same speed of light c. Consider being in a train that moves with a velocity v. Then one would have expected:

– The person in the train measures for the propagation of light the speed c
– the speed of light an observer measures outside the moving train should be $c + v$ (in thel ight propagatipn direction of the train's motion).

However, this is not the case. Both observers measure the same speed of light c!

Let us consider two reference frames $f,\ f'$. One measures the following coordinates:

- in the system f: x, y, z, t
- in the system f' : x', y', $z,'$, t'.

The movement in x-direction is given by a classical *Galilean transformation:*

$$\Delta x' = \Delta x - v\Delta t \tag{15.29}$$

$$\Delta y' = \Delta y \tag{15.30}$$

$$\Delta z' = \Delta z \tag{15.31}$$

$$\Delta t' = \Delta t \tag{15.32}$$

However, it also follows that:

$$c' = c - v \tag{15.33}$$

That is, the speed of light would not be constant in all systems, and thus a contradiction to the experiment of *Michelson.* However, the *Lorentz transformation* solves this problem.

Let the Lorentz factor be:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{15.34}$$

The Lorentz transformation (again, we assume only motion in the x-direction) is then:

$$dx' = \gamma(dx - vdt) \tag{15.35}$$

$$dy' = dy \tag{15.36}$$

$$dz' = dz \tag{15.37}$$

$$dt' = \gamma\left[dt - \frac{v}{c^2}dx\right] \tag{15.38}$$

Thus we have the concept of *Space-time.* The space-time interval between two events is:

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2 \tag{15.39}$$

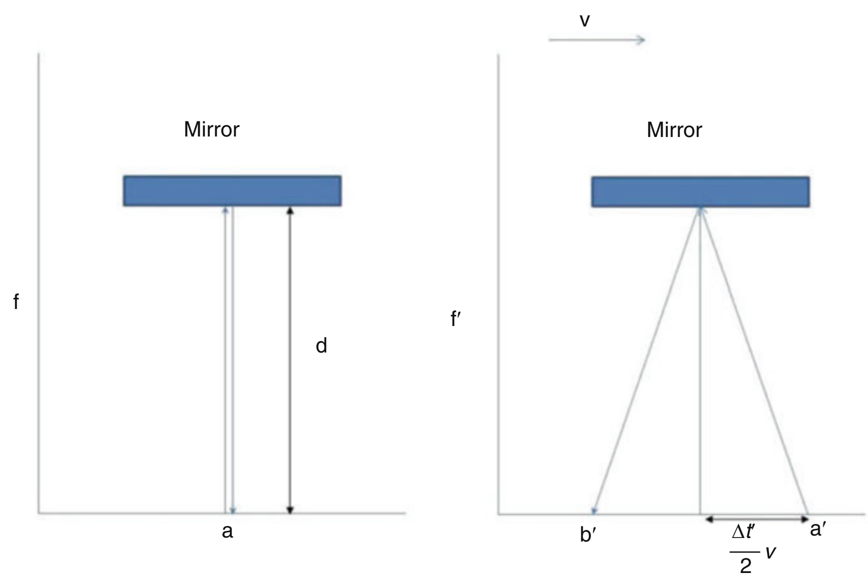

Fig. 15.2 To derive the time dilation. On the right, the light beam is emitted at a' and arrives, because of the motion of the system with the velocity v, at b'

Note: Between any two intervals is ds^2 independent of the motion of the observer, is a scalar invariant quantity. We denote the intervals as follows:

- $ds^2 < 0$ time-like,
- $ds^2 > 0$ space-like,
- $ds^2 = 0$ zero.

In relativity, one has the concept of a space-time continuum.

Wee derive from Fig. 15.2 the time dilation: On the left we have the system of rest, light is returned to a emits, reflects at b reflects and travels the distance $2d$ within the time Δt back. On the right, a ray of light is emitted at a' but arrives at b' because the system is moving with the speed v to the right – therefore light travels the distance $2d'$. We have therefore:

System at rest

$$\Delta t = \frac{2d}{c} \tag{15.40}$$

Moving system

$$\Delta t' = \frac{2d'}{c} = 2\frac{\sqrt{d^2 + \left(\frac{1}{2}v\Delta t'\right)^2}}{c} \tag{15.41}$$

Because of $\Delta t'^2 = 4d^2/c^2 + 4v^2\Delta t'^2/(4c^2)$ and $2d/c = \Delta t$ we get $\Delta t = \Delta t'\sqrt{1 - v^2/c^2} = \Delta t'/\gamma$.

A moving clock (system $'$) goes slower than a stationary → Time dilation.

$$\Delta t' = \gamma \Delta t \tag{15.42}$$

Analogous to that the *Length contraction* is obatined:

$$\Delta x' = \gamma \Delta x \tag{15.43}$$

For the addition of velocities we find: Let there be an observer B′ in a system which moves with respect to the observer B with the velocity v in the x-direction. The observer B′ measures the velocity of a body in its system to be u' the following velocity results for the observer B:

Velocity of the object measured in the moving system, is u'_x , and u_x is the velocity of the object measured in the system at rest. Then holds:

$$u_x = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}} \tag{15.44}$$

For energy and momentum holds:

$$E = \gamma mc^2 \qquad p = \gamma mv \tag{15.45}$$

Energy and mass are equivalent:

$$E = mc^2 \tag{15.46}$$

No object, no wave, i.e. no information can be transmitted faster than the speed of light. In order to even reach this speed, an infinite amount of energy must be expended. An object, which moves with superluminal speed from A to B, violates the *Causality Principle* (cause would then occur after effect).

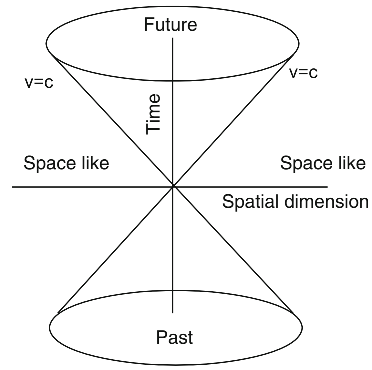

Fig. 15.3 The space-time diagram. Because of the principle of causality, the world lines can only go from the past cone to the future cone. The two 45°-lines mean $v = c$, photons with rest mass = 0 move along these lines. The space-like area is forbidden, because here $v > c$ would be

An event is represented by three space and one time coordinate.

In the space-time diagram, a space coordinate is plotted against the time-space-time cone (Fig. 15.3).

15.3.2 Four Vectors, Transformations

Figure 15.4 shows the Minkowski diagram: Two coordinate systems $K(x, ct)$ and $K'(x', ct')$ move against each other with the speed v. The events in the systems have the coordinates (x_1, y_1, z_1, t_1) as well as (x'_1, y'_1, z'_1, t'_1). Let us now apply the transformation equations:

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}, \qquad y = y', \qquad z = z', \qquad t = \frac{t' + v(x'/c^2)}{\sqrt{1 - v^2/c^2}} \tag{15.47}$$

and calculate

$$L^2 = x^2 + y^2 + z^2 - c^2t^2 = \ldots . \rightarrow \qquad L = L' \tag{15.48}$$

The length of the four-vector is independent of the coordinate system.

The general transformation of a four-vector is ($\beta = v/c$):

$$A_1 = (A'_1 + \beta A'_4)\gamma, \qquad A_2 = A'_2, \qquad A_3 = A'_3 \tag{15.49}$$

$$A_4 = (A'_4 + \beta A'_1)\gamma \tag{15.50}$$

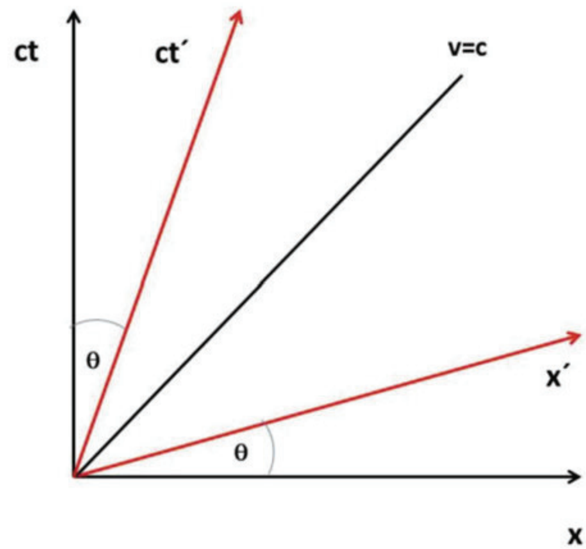

Fig. 15.4 The Minkowski diagram. Two coordinate systems K(x, ct) and K' (x', ct') move against each other. The following applies $tan\Theta = v/c$

Examples of four vectors:

- Momentum 4-vector

$$(p_x, p_y, p_z, E/c) \tag{15.51}$$

  with $E =$ Energy.
- The potential 4-vector in electrodynamics: consists of a vector potential (A_x, A_y, A_z) and as the 4th component the scalar potential Φs:

$$(A_x, A_y, A_z, \Phi) \tag{15.52}$$

From this one can deduce the *relativistic Doppler effect*. The energy is the fourth component of the four-vector ($\mathbf{p}$, E/c) and transforms as follows:

$$E = \gamma(E' + vp'_x) \tag{15.53}$$

and therefore

$$E = \frac{E' + (Ev/c)\cos\Theta'}{\sqrt{1 - v^2/c^2}} = E'(1 + \beta\cos\Theta')\gamma \tag{15.54}$$

and because of $E = h\nu$ becomes

$$\nu = \nu'(1 + \beta\cos\Theta')\gamma \tag{15.55}$$

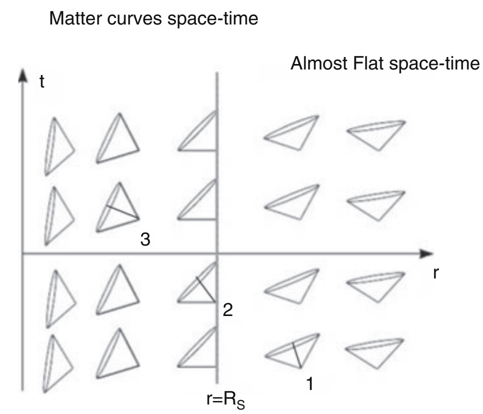

Fig. 15.5 Space-time diagram around a black hole. The event horizon is approached from the right. R_S. At this (at location 2) the escape velocity is $v = c$ and inside $r < R_s$ it is greater than c (e.g., at location 3). Space and time are practically reversed

Two special cases:

- Source moves radially to detector, it emits light waves of frequency ν_0; the frequency measured by the detector is then:

$$\nu = \nu_0 \sqrt{\frac{1-\beta}{1+\beta}} \tag{15.56}$$

- Transverse Doppler effect: the relative motion of the light source is perpendicular to the line that lies between the source and the observer:

$$\nu = \nu_0 \sqrt{1-\beta^2} \tag{15.57}$$

$\beta = v/c$. The *transverse Doppler effect* is a consequence of time dilation. The space-time diagram around a black hole is given in Fig. 15.5. At the event horizon given by the Schwarzschildraius, space and time coordinates get inversed.

15.3.3 General Theory of Relativity

In principle the structure of space depends on the line element ds^2. This gives the distance between two points in the four-dimensional space-time continuum. In the simplest case of Euclidean space the distance between two points is (x^1, x^2, x^3) and $(x^1 + dx^1, x^2 + dx^2, x^3 + dx^3)$ in *Cartesian coordinates* is:

$$ds^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2 \tag{15.58}$$

In special relativity, you have require a four-dimensional *Minkowski space,* and the line element is:

$$ds^2 = -(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2 \tag{15.59}$$

Note: Sometimes one writes the time coordinate as positive and the three space coordinates as negative, sometimes vice versa.

The most general form of the line element is:

$$ds^2 = g_{ik} dx^i dx^k \tag{15.60}$$

Here one assumes the *Einstein's sum convention*: indices occurring twice on one side of an equation are automatically summed, so here over i, k. g_{ik} means *metric tensor,* it determines the metric of the space.

> The metric tensor describes the metric of the space.

The metric given by g_{ik} and describing space-time is dynamically connected with matter. Matter curves space-time. Riemannian geometry describes this mathematically, namely the geometric properties of space-time are intrinsic, i.e. one does not need a super space to explain the curvature.

15.3.4 Matter and Space-Time Curvature

Einstein's field equations describe the connection between the matter distribution (described by the tensor T_{ik}) and the curvature properties of space (described by the Ricci tensor R_{ik}):

$$R_{ik} - \frac{R}{2} g_{ik} + \Lambda g_{ik} = -\kappa T_{ik} = -\frac{8\pi G}{c^4} T_{ik} \tag{15.61}$$

The Ricci tensor is obtained by summation over n obtained from the Riemann tensor:

$$R_{ik} = R^n_{ink} \tag{15.62}$$

The curvature scalar is found from:

$$R^n_m = g^{in} R_{km} \qquad R = R^n_n \tag{15.63}$$

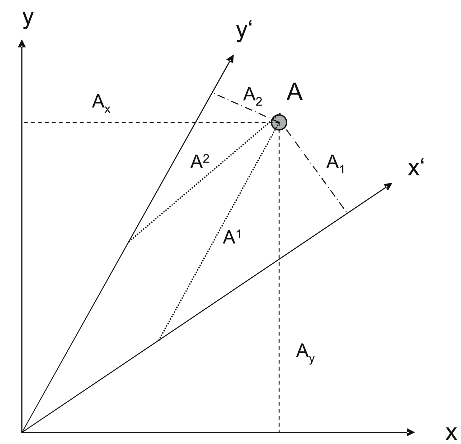

Fig. 15.6 A point is represented in the x–y-system by coordinates (A_x, A_y) and in the oblique x′-y′ system by its contra variant (A^1, A^2) and covariant components (A_1, A_2)

In an orthogonal coordinate system, there are only two components of a vector (Fig. 15.6, x–y system), whereas in a non orthogonal coordinate system there are the contravariant components (written with index above) and the covariant Components (written with index below) (Fig. 15.6, x′-y′-system).

If one wishes to determine the derivative of a vector or tensor field, one must take the difference of two vectors sitting at different points in space time. However, two such vectors cannot be compared directly because they are elements of different tangent spaces. One vector must first be transported parallel along a curve to the location of the other vector.

In principle, the Riemann tensor can be derived as follows: At each space-time point, the gravitational field is made to vanish → flat spaces. Now we investigate how these are related according to special relativity. Thus, the gravitational field is made to disappear by coordinate transformations. There is a simple analogy for this: in a free-falling elevator on Earth, the gravitational field disappears. If the elevator has a very weak gravitational field, then one can neglect the inhomogeneity of the field.

If one now transfers this to a curved space, a tangential surface can be defined on a curved spherical surface to every point.

- Curvature of the spherical surface ⇔ Space curvature;
- Tangent surface ⇔ flat space.

The problem is how to describe the relationship between different tangent surfaces in space → Covariant derivatives of a vector field.

The energy-momentum tensor is:

$$(T^{ik}) = \begin{pmatrix} w & \frac{S_x}{c} & \frac{S_y}{c}\ddot{a} & \frac{S_z}{c} \\ \frac{S_x}{c} & G_{xx} & G_{xy} & G_{xz} \\ \frac{S_y}{c} & G_{yx} & G_{yy} & G_{yz} \\ \frac{S_z}{c} & G_{zx} & G_{zy} & G_{zz} \end{pmatrix} \tag{15.64}$$

w—energy density, (S_x, S_y, S_z)—energy-current density, G_{ij}—stress tensor—whose diagonal components correspond to the pressure exerted by a radiation field.

In cosmology, one usually uses the following for the energy-momentum tensor:

$$T^{ik} = \left(\rho + \frac{P}{c^2}\right) u^i u^k - P g^{ik} \tag{15.65}$$

where $u^i u^k$ is the quadratic velocity, P the pressure (radiation field), ρ is the mass density. This corresponds to the tensor for a fluid field.

Galaxies are therefore taken to be elements of an ideal cosmic fluid, The pressure is assumed as isotropic.

15.3.5 Metric of the Space

If one knows the metric, then one can derive the curvature tensor from it.

For this one needs the *Christoffel symbols, affine connections*

$$\Gamma^i{}_{k\ell} = \frac{1}{2} g^{im} \left(\frac{\partial g_{mk}}{\partial x^\ell} + \frac{\partial g_{m\ell}}{\partial x^k} - \frac{\partial g_{k\ell}}{\partial x^m} \right) = \frac{1}{2} g^{im} (g_{mk,\ell} + g_{m\ell,k} - g_{k\ell,m})$$

Here $g_{kl,m}$ denotes a derivative of the component g_{kl} with respect to component m. From this one finds:

$$R^\rho{}_{\sigma\mu\nu} = \partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda} \Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda} \Gamma^\lambda_{\mu\sigma} \tag{15.66}$$

with $\partial_\mu = \partial / \partial x^\mu$.

So we outline the whole procedure:

- Give some metric of space-time $ds^2 = \ldots$
- $\rightarrow$ Calculate Christoffel symbols

- → Calculate the Riemann tensor
- → Insert this into Einstein's field equations . . .

Motion in the Gravitational Field

Motion occurs along a geodesic line in Riemannian space; the equation of motion, according to special relativity, is given by ($d\tau = ds$):

$$\frac{d^2x^\mu}{ds^2} = -\Gamma^\kappa_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} \tag{15.67}$$

Basic statement of Einstein's field equations: Matter distribution determines space-time curvature.

Schwarzschild Metric

The Schwarzschild metric applies to a very simple situation: just one mass M in space-time, outside $T_{ik} = 0$

$$ds^2 = \left(1 - \frac{R_s}{r}\right)c^2dt^2 - \frac{dr^2}{1-(R_s/r)} - r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{15.68}$$

with

$$R_s = \frac{2GM}{c^2} \tag{15.69}$$

as *Schwarzschild radius.* Application: non-rotating black hole.

For practice, let us consider the components of the metric tensor in the Schwarzschild metric. These are obtained from the factors that precede the squares of the coordinate differentials, so for example:

$$g_{00} = \left(1 - \frac{R_s}{r}\right) \tag{15.70}$$

$$g_{11} = \frac{1}{1 - R_s/r} \tag{15.71}$$

Kerr Metric

If one also takes the angular momentum (rotation) into account, the result is the Kerr metric.

This metric can be used to understand various special processes in the vicinity of black holes:

- the generation of jets,
- the quasi-periodic oscillations in stellar black holes,
- the relativistic broadening of emission lines,
- the *Lense-Thirring effect* in accretion disks, *frame-dragging.*

Any rotating mass virtually drags the local reference frame along with it *(frame dragging).* The more mass rotating in small space, the larger this effect. It was found by *Lense* and *Thirring*.[3] The area within which a particle is forced to co-rotate is called *Ergosphere.*

Here again an analogue to electrodynamics becomes visible. One can deduce the magnetic field **B** from a vector potential **A** :

$\mathbf{B} = \mathrm{curl}\mathbf{A}$.

Let us conceive of the space-time continuum as three-dimensional subspaces with constant time. The lapse function mediates from one of these hyper surfaces to the next later time, the shift function from one location to another on the hyper surface $\rightarrow$ gravitomagnetic field. A mass flow generates a gravitomagnetic field, just as an electric current generates a magnetic field in its vicinity.

In April 2004, the satellite Gravity Probe B was launched to demonstrate the Earth's space-time curvature and the Lense-Thirring effect using gyroscopes. Due to the rotating space-time of the Earth, the gyroscopes start a precession motion. The effect is additive and increases per revolution around the Earth. The Lense-Thirring effect after one year of revolutions around the earth amounts to $42 \times 10^{-3}{}''$. It is orders of magnitude smaller than the geodetic precession caused by space-time curvature.

The gravitomagnetic dynamo caused by the Lense-Thirring effect is of great importance for astrophysics: in the vicinity of rotating black holes, tube-shaped magnetic fields are generated. By accretion then the jets are formed as well as the also observed quasiperiodic oscillations, which coincide with the Lense-Thirring frequency (produced by gyroscopic motion). They lie in the Microquasars in the kHz range.

Robertson-Walker Metric

This line element is used to describe the expanding universe. $a(t)$ is the cosmic scale factor. We assume a homogeneous, isotropic universe $\rightarrow$ Universe with constant curvature. The curvature is given by the curvature parameter k.

- $k = 0$ Euclidean, circumference of the circle $2\pi r$, area of the circle $r^2\pi$.
- $k > 0$ spherical or elliptic; circumference of the circle $< 2\pi r$, area $< r^2\pi$.
- $k < 0$ hyperbolic; circumference of the circle $> 2r\pi$, area $> r^2\pi$.

[3] In 1918.

The line element here is

$$ds^2 = c^2dt^2 - a(t)^2\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \tag{15.72}$$

We roughly sketch the derivation of this metric. What is the equation of a hypersurface of constant curvature in a higher dimensional space? Solution:

$$x_1^2 + x_2^2 + x_3^2 = \frac{1}{k}R^2 \tag{15.73}$$

In Euclidean space, the metric is:

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 \tag{15.74}$$

If one substitutes from (15.73) the expression for x_3 then follows:

$$ds^2 = dx_1^2 + dx_2^2 + \frac{(x_1dx_1 + x_2dx_2)^2}{\frac{R^2}{k} - x_1^2 - x_2^2} \tag{15.75}$$

and with $x_1 = r\cos\Theta,\ x_2 = r\sin\Theta$:

$$ds^2 = \frac{R^2dr^2}{R^2 - kr^2} + R^2r^2d\Theta^2 \tag{15.76}$$

Robertson-Walker metric: One substitutes R by $a(t)$, calculate in 3 coordinates, then comes the term $R^2\sin^2\theta d\Phi^2$ to it.

15.3.6 Friedmann-Lemaître Equations

If one takes the Robertson-Walker-metric as a basis, then from the field equations follow the Friedmann-Lemaître equations[4]:

$$\left(\frac{\dot{a}}{a}\right)^2 = -k\frac{c^2}{a^2} + \frac{8\pi G}{3}\rho + \frac{c^2}{3}\Lambda \tag{15.77}$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = -k\frac{c^2}{a^2} - \frac{8\pi G}{c^2}P + c^2\Lambda \tag{15.78}$$

[4] *Friedmann* 1922, *Lemaître* 1927.

ρ contains the density of matter as well as radiation and also dark matter. From the two equations, by subtraction, we get:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) + \frac{c^2}{3}\Lambda \tag{15.79}$$

A cosmological constant $\Lambda > 0 \rightarrow$ means repulsive acceleration, counteracts gravity. The static world model results from the Friedmann-Lemaître equations by $\dot{a} = 0, k = \Lambda a^2$, $\Lambda = 4\pi G\rho/c^2$.

One can attribute to Λ an equivalent mass density ρ_Λ

$$\Lambda = \frac{8\pi G}{c^2}\rho_\Lambda \tag{15.80}$$

and then the Friedmann-Lemaître equations read:

$$\left(\frac{\dot{a}}{a}\right)^2 = -k\frac{c^2}{a^2} + \frac{8\pi G}{3}(\rho_M + \rho_\Lambda) \tag{15.81}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left[(\rho_M + \rho_\Lambda) + 3\left(\frac{P}{c^2} - \rho_\Lambda\right)\right] \tag{15.82}$$

15.3.7 The Cosmological Constant and Vacuum Energy

Quantum field theories are concerned with a unified description of fields and particles. As we know from quantum physics, properties like energy and momentum of particles are quantized. In the so-called second quantization, the fields are also quantized.

- First quantization: properties of particles quantized, e.g. energy, momentum.
- Second quantization: fields describe interactions between particles; these fields are also quantized.

According to the *Quantum Field Theory* the vacuum consists of fluctuating matter fields and virtual particles, and it can have a non-zero energy density. This provides an explanation for the cosmological constant:

$$\rho_\Lambda = \rho_{\text{vac}}$$

The cosmological constant corresponds to the energy density of the vacuum. The associated pressure is negative $P_{\text{vac}} = -\rho_{\text{vac}}c^2$.

The negative pressure of the vacuum energy can be made plausible: Consider the first law of thermodynamics. A change of the internal energy dU is equal to (i) sum of the heat supplied or dissipated and (ii) the work done pdV. An increase in volume means an output of work, hence the negative sign. The internal energy is therefore $U \propto$ volume V. Thus, if as a result of an increase in volume the internal energy is increased, because of

$$dU = -PdV \tag{15.83}$$

P to be negative.[5]

Three components can be given for the density:

$$\rho = \rho_m + \rho_r + \rho_{\text{vac}} = \rho_{m+r} + \rho_{\text{vac}}, \qquad P = P_r + P_{\text{vac}} \tag{15.84}$$

density of radiation (ρ_r) and density of matter (ρ_m) were combined and the dust was considered to be free of pressure. The pressure of a gas is determined by the thermal movement of the particles. Air molecules move at about $v \approx 300$ m/s, the pressure $P \ll \rho v^2$ i.e. the pressure is gravitationally insignificant. Such is the case in the universe today, *pressureless matter*. If the thermal velocity is equal to the speed of light, then you have the limiting case of radiation. An example of this is cosmic microwave background (CMB) radiation. Also particles with vanishing rest mass belong to it as well as particles whose thermal energy is much larger than the rest energy:

$$kT \gg mc^2 \tag{15.85}$$

The radiation pressure is:

$$P_r = \frac{1}{3}\rho_r c^2 \tag{15.86}$$

If ρc^2 is the energy density, then

$$\frac{d}{dt}(c^2 \rho a^3) = -P\frac{da^3}{dt} \tag{15.87}$$

Let us now consider the evolution of density:

$$\rho_m(t) = \rho_{m,0} a^{-3}(t) \tag{15.88}$$

$$\rho_r(t) = \rho_{r,0} a^{-4}(t) \tag{15.89}$$

$$\rho_{\text{vac}}(t) = \rho_{\text{vac}} = \text{const} \tag{15.90}$$

[5] For an adiabatic volume change dV the work is $dU = -PdV$.

The index “0” stands for today, $t = t_0$. Note the a^{-4} dependence of the radiation flux:

- As with matter, the number density of photons changes proportionally to a^{-3}.
- Photons are furthermore red shifted by cosmic expansion, $\lambda \propto a$, $E = hc/\lambda$; $E \propto a^{-1}$.

One can write the dimensionless density parameters for matter, radiation and vacuum as:

$$\Omega_m = \frac{\rho_{m,0}}{\rho_{\text{crit}}} \qquad \Omega_r = \frac{\rho_{r,0}}{\rho_{\text{crit}}} \qquad \Omega_\Lambda = \frac{\rho_{\text{vac}}}{\rho_{\text{crit}}} \tag{15.91}$$

and

$$\Omega_0 = \Omega_m + \Omega_r + \Omega_\Lambda \tag{15.92}$$

$\Omega_m > 0.3$ if one considers the galaxies and their halos. The energy density of radiation today is very small (photons from the CMB as well as neutrinos from the early universe).

$$\frac{\rho_r(t)}{\rho_m(t)} = \frac{\rho_{r,0}}{\rho_{m,0}} \frac{1}{a(t)} = \frac{\Omega_r}{\Omega_m} \frac{1}{a(t)} \tag{15.93}$$

Radiation and dust had the same energy density when

$$a_{\text{eq}} = 4{,}2 \times 10^{-5} (\Omega_m h^2)^{-1} \tag{15.94}$$

amounted to.

Because of $\rho = \rho_{m+r} = \rho_{m,0} a^{-3} + \rho_{r,0} a^{-4}$ becomes:

$$H^2(t) = H_0^2 \left[a^{-4}(t)\Omega_r + a^{-3}(t)\Omega_m - a^{-2}(t)\frac{Kc^2}{H_0^2} + \Omega_\Lambda \right] \tag{15.95}$$

Now we put in the values for today:

$$H(t_0) = H_0 \qquad a(t_0) = 1 \tag{15.96}$$

$\rightarrow$ Integration constant:

$$K = \left(\frac{H_0}{c}\right)^2 (\Omega_0 - 1) \approx \left(\frac{H_0}{c}\right)^2 (\Omega_m + \Omega_\Lambda - 1) \tag{15.97}$$

Table 15.2 Characteristic parameters of the universe

Age	$13.7 \pm 0.5 \times 10^9$ Years
Diameter	96×10^9 Light years
Mass	10^{53} kg
Number of galaxies	10^{11}
Number of particles	$\approx 10^{79}$
Number of photons	10^{88}
Present temperature	2.75 K
Average matter density	2.3×10^{-26} kg/m^3

$\rightarrow$ K stands for the curvature of the Space:

- $K = 0$ Space Euclidean, flat.
- $K > 0$: two-dimensional analogue is spherical surface. Radius of curvature is $1/\sqrt{K}$, sum of angles in a triangle is greater than 180 deg.
- $K < 0$: hyperbolic space, sum of angles in a triangle is less than 180 degrees.

Note: General relativity states that matter is associated with the space time continuum, but nothing about the topology. If the universe possesses a simple topology, it is in the case of $K > 0$ finite, in the case $K \leq 0$ infinite. However, it is always unbounded, even a spherical surface is a finite but unbounded space.

One still defines the *Deceleration parameter*:

$$q_0 = -\frac{\ddot{a}a}{\dot{a}^2} = -\frac{1}{2}\Omega_m - \Omega_\Lambda \tag{15.98}$$

From this we see: If $\Omega_\Lambda = 0$ then the expansion is slowed down (due to gravity). If Ω_Λ is large enough, then q_0 can become positive, i.e. accelerated expansion (Table 15.2).

One takes the following values today:

$$\Omega_m = 0.3 \qquad \Omega_\Lambda = 0.7 \tag{15.99}$$

15.3.8 Gravitational Waves

The gravitational waves predicted by general relativity have already been detected in double pulsars. They only occur in when masse get accelerated (cf. electromagnetism: accelerated charges emit electromagnetic waves). They are most likely to be detected in:

- rapidly orbiting objects, e.g. double pulsars,
- non-rotationally symmetric, very rapidly rotating objects,
- asymmetric collapse or explosion of a massive object.
- Merging of massive objects such as neutrons stars, black holes.

Consider a binary star system with masses M_1, M_2 with period P, large semi-axis a and reduced mass $\mu = M_1 M_2/(M_1 + M_2)$; $M = M_1 + M_2$. Then, for the luminosity of the gravitational wave, we find (*Peters* and *Matthews*):

$$L = \frac{32}{5}\frac{G^5}{c^{10}}\frac{\mu^2 M^3}{a^5} L_0 f(e) \tag{15.100}$$

where is:

$$L_0 = c^5/G = 3.63 \times 10^{59}\,\mathrm{erg/s} \tag{15.101}$$

and

$$f(e) = \frac{1 + \frac{73}{24}e^2 + \frac{37}{96}e^4}{(1 - e^2)^{7/2}} \tag{15.102}$$

with e for the orbital eccentricity. In practical units this gives:

$$L \approx 3.0 \times 10^{33}\mathrm{erg/s}\left(\frac{\mu}{M_\odot}\right)^2\left(\frac{M}{M_\odot}\right)^{4/3}\left(\frac{P}{1h}\right)^{-10/3} f(e) \tag{15.103}$$

For a deformed neutron star Gravitational wave emission with mass M , radius R, rotation period P, moment of inertia $I = 2/5MR^2/5$ and ellipticity ϵ a luminosity due to gravitational wave emission can also be calculated from:

$$L = \frac{32}{5}\frac{G}{c^5} I^2 \epsilon^2 \left(\frac{2\pi}{P}\right)^6 \tag{15.104}$$

Gravitational waves are perturbations of space-time that propagate at the speed of light. A sensitive interferometer system (Fig.15.7) can measure these extremely small changes in position due to the space-time ripples. Thus, in September 2015, the first direct experimental evidence of gravitational waves was obtained, almost exactly 100 years after Einstein postulated them. The measured signal came from two black holes that merged. The distance of the objects to us was about 500 Mpc. A length change $h = 10^{-20}$ (strain) was detected by LIGO (Laser Interferometer Gravitational Observatory).

15.4 Dark Energy, Accelerated Expansion

15.4.1 Observations

Type Ia Supernovae , as already discussed, are very well suited as standard candles up to distances of 500 Mpc. Their true brightness is in a narrow range except for a few outliers. With the help of the HST, Cepheids have been found in nearby galaxies, calibrating the

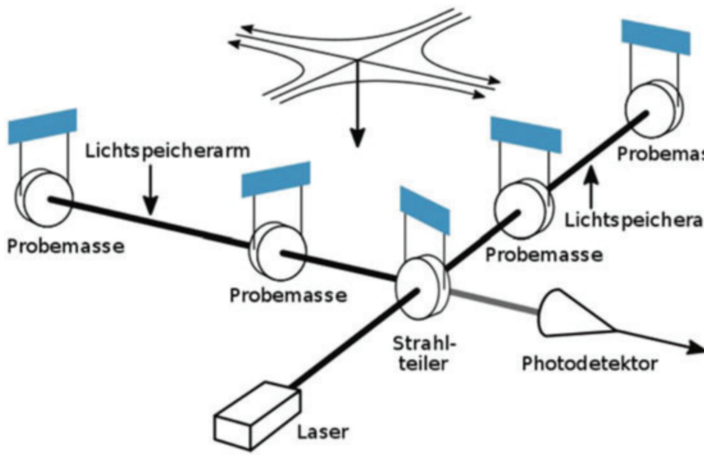

Fig. 15.7 Gravitational wave detector using two interferometer arms, LIGO concept, laser interferometer gravitational observatory. LIGO

magnitudes of SN Ia. There have been two major surveys, the Supernovae Cosmology Project (SCP) and the High Z Supernovae Search Team (HZT). These groups were able to find 70 type Ia supernovae.

The measurements are practically only possible with $\Omega_\Lambda \neq 0$ to explain it. So the cosmological constant, originally introduced to maintain a static universe, now describes the accelerated expansion of the universe.

Today's universe is expanding faster than it used to.

Another clear indication is the anisotropy of cosmic microwave radiation.

The early universe was extremely hot, the atoms were completely ionized, there were only:

- free electrons,
- atomic nuclei, mostly protons.

Because of the many free electrons, photons were extremely scattered, so the universe was opaque.

When, as a result of expansion, the universe cooled to about 3000 K, recombination occurred. The electrons recombined to form atoms, and the universe became transparent.

At the time of recombination, which was about 400 000 years after the Big Bang, the universe was anisotropic in some places. There were condensations, gravitational potential wells. Photons falling into such potential wells gain energy. If a photon falls into a potential well, it has to expend energy to get out, the same energy it gained falling in: δE_1. However, as the universe expands, the potential well, while the photon is in it, has become shallower, it therefore requires less energy, δE_2 to get out. Therefore $\delta E_2 < \delta E_1$, the photon therefore gains energy (which is lost to the potential well due to expansion). This is called *Sachs-Wolfe effect*[6] and through this the cosmic background radiation became anisotropic in some places.

15.4.2 Dark Energy

In 2001 the satellite WMAP[7] could find further evidence for the existence of dark energy. Dark energy opposes gravity in its effect, i.e. it is antigravitational (repulsive). Therefore, there is an accelerated expansion of the universe, in agreement with the type-SNIa observations. Dark energy accounts for about 70−74% of the energy of the universe.

This dark energy is to be distinguished from dark matter. Baryonic matter makes up only between 2–5% of the matter of the universe, and the dynamics of galaxy clusters show that the sum of baryonic matter and dark matter cannot exceed about 30% (Fig. 15.8). Dark matter consists of particles that do not radiate electromagnetically, or at least radiate extremely weakly, and interact only weakly. Candidates for this would be the *(weakly interacting particles)* WIMPs and super symmetric particles.

What does dark energy consist of?

- vacuum-energy (see previous section): the quantum vacuum can be explained by the *Casimir effect*. Between two metal plates certain modes are missing—outside the plates they exist. This creates a quantum pressure that pushes the plates together.
- Effect of a scalar field, quintessence. Associated with this field are extremely light elementary particles elementary ($10^{-82}m_e$).
- String theories; the universe consists of more than four dimensions. At large distances, gravitational interaction weakens, and space expands more.

[6] R.K. Sachs, A.M. Wolfe, 1967.

[7] Wilkinson Microwave Anisotropy Probe.

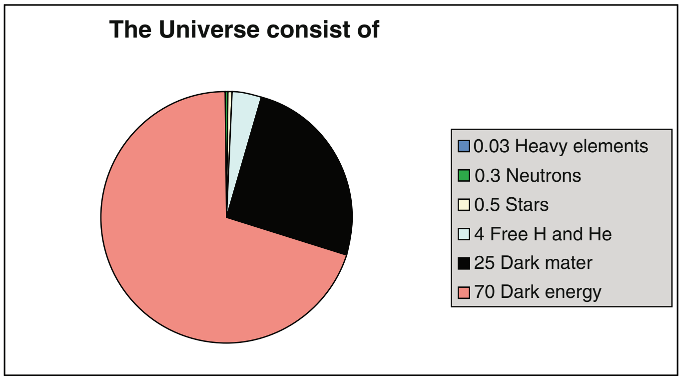

Fig. 15.8 Composition of the universe. The figures are in %

- Topological defects, imperfections as a result of spontaneous symmetry breaking in the early universe during the inflationary phase.
- Phantom energy: According to this theory, which was proposed in 2003, the universe would tear apart after 10^{50} years including all particles within *(big rip)*. However, this would already be noticeable by ultra-energy particles in cosmic rays, which have not been observed. Interestingly, this Big Rip could be observed: First, the most distant galaxies would disappear, and then successively closer and closer ones.

15.5 The Early Universe

15.5.1 Big Bang: Observational Hints

Expansion of Space

We described in the previous section the expansion of the universe, which had to start from a singular point. This is a first hint to the *Big Bang*. When this took place can be given by the world age, with the scale factor $a \to 0$. Theoretically, it could be that $\dot{a}$ was zero at some point in the past, i.e. a took a minimal value. However, this can be ruled out by observation: $\Omega_m > 0.1 \to a_{\min} > 0.3$. Since moreover a is related to the red shift z :

$$z_{\max} = \frac{1}{a_{\min}} - 1 \qquad (15.105)$$

there should be a maximum red shift. In the meantime, however, one has observed quasars with $z > 6$ observed!

Background Radiation

A further indication of a hot big bang was the detection of the *Cosmic Background Radiation, CBR* detected in 1965 by *Penzias* and *Wilson*[8] . It corresponds to a Planck radiation curve of a body with a temperature of about 3 K, so its maximum lies in the microwave range. The background radiation is isotropic and originates from the time when the redshift was $z \approx 1000$, a relic from the *Recombination Era of* the universe (whose age was then about 400,000 years). When the universe was younger than 400,000 years the temperature was high and there were only free particles, atomic nuclei and electrons. Because of expansion the universe cooled (adiabatic cooling), and when temperature dropped to 3000 K, electrons were captured by (mainly) hydrogen nuclei, and the universe became transparently → decoupling of radiation and matter.

> The decoupling of radiation and matter occurred at a time when the universe was about 400,000 years old. The Cosmic Background radiation also dates from this time.

As early as 1945 *Gamow*, when studying thermodynamics in the radiation-dominated early universe, suggested that there must have been primordial nucleosynthesis, and *Alpher* and *Herman* postulated the existence of a 5-K background radiation.

Primordial Nucleosynthesis

Another important clue is primordial nucleosynthesis. The simplest process is that of Deuterium *D* formation:

$$p + n \rightarrow \mathrm{D} + \gamma \tag{15.106}$$

Here, the temperature must be lower than the binding energy E_b of the deuterium so $kT \ll E_b$; at higher temperatures the formed D is destroyed again by photodissociation, only when $T_D \approx 8 \times 10^8$ K during the first $t \approx 3$ min the formation of D is predominant. D then forms into He^4, where the binding energy of 28 MeV is even higher, thus even more stable against photodissociation. Other fusion reactions will be discussed later.

> In total, in the first three minutes after the Big Bang, the following elements were formed: 75% hydrogen, 24% helium, and 1% lithium.

[8] They were awarded the Nobel Prize in Physics in 1978.

15.5.2 Sunyaev-Zel'dovich Effect

Compton Effect

We start with the photoelectric effect. *Hallwachs* discovered that the energy of the electrons triggered by the photoelectric effect is determined only by the frequency of the triggering light, their number only by the intensity of this light. Einstein assumed that only certain energy contributions $h\nu$ are available for electron triggering, which leads to the concept of photons and is of the same amount of energy that is exchanged between the oscillator and the radiation field according to Planck. Photons have the energy $h\nu$, move in vacuum with the speed of light c and have zero rest mass and thus momentum $p = h\nu/c = h/\lambda$. *Compton* found the effect named after him in 1922: Monochromatic X-rays are scattered by matter with an increase in wavelength. The wavelength of the scattered light is larger, the larger the scattering angle θ Is. For the backward scattering $\theta = \pi$ he found a wavelength increase of 0.0485 Å, independent of the irradiated wavelength. This effect can be interpreted as a collision process between the X-ray photon and the electron of the scattering matter; by conservation of energy and momentum this process is completely described. One finds after long calculation:

$$\lambda' - \lambda = \frac{2h}{mc} \sin^2 \frac{\theta}{2} \tag{15.107}$$

Inverse Compton Effect

Due to the free electrons in the universe there is a change in intensity ΔI_ν:

$$\frac{\Delta I_\nu}{I_\nu} = \frac{y x \exp^x}{\exp^x - 1} \left(x \frac{\exp^x + 1}{\exp^x - 1} - 4 \right) \tag{15.108}$$

with $x = \frac{h\nu}{kT}$ and the Compton parameter:

$$y = \int \frac{kT_e}{m_e c^2} \sigma_T n_e dl \tag{15.109}$$

n_e—density of free electrons, T_e—their temperature, m_e—their mass, σ_T—the Thomson cross section, l—the path length of the photons. The effect is also called *Sunyaev-Zel'dovich effect*. The measurements show $y \leq 1.5 \times 10^{-5}$. As photons pass through a cluster of galaxies, scattering of photons occurs, and some gain energy (a kind of inverse Compton effect). For example, if one observes at a wavelength of 1 cm, the gain is 0.05%. Thus, one observes a deficit of background cosmic photons, since some have been shifted to higher energies as a result of Compton scattering (Fig. 15.9).

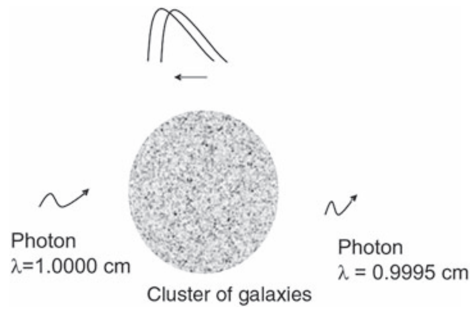

Fig. 15.9 Spectrum of the 3-K background radiation (indicated above by two Planck functions); due to the Sunyaev-Zel'dovich effect there is a shift to higher energies

R is the radius of a galaxy cluster and σ_T the Thomson effective cross section, then one finds for the resulting temperature deviations:

$$\frac{\Delta T}{T} = \frac{4Rn_e kT_e\sigma_T}{m_e c^2} \tag{15.110}$$

The 3-K background radiation is isotropic and exhibits deviations only on very small scales, which were measured with COBE (Cosmic Background Explorer) and WMAP very accurately(Fig. 15.10) :

- 10^{-3}: Fluctuations of this magnitude show an angular dependence of the temperature. This can be represented with the help of a dipole distribution:

$$T(T_0)\left(1 + \frac{v}{c}\cos\theta\right) \tag{15.111}$$

  This dipole asymmetry can be interpreted as a motion of the Earth relative to the co-moving reference frame, which follows from the expansion of the universe and in which the background radiation appears isotropic. One gets a velocity of 350 km/s in the direction of the galactic coordinates $\mathrm{l} = 264.25° \pm 0.33°,\ b = 48.22° \pm 0.14°$.
- With an accuracy of 10^{-5} the background radiation exhibits temperature fluctuations which no longer satisfy a simple dipole distribution. These are interpreted as fluctuations in the early universe and are therefore of great importance for the structure formation in the early Universe.

In mid-May 2009, the space probe PLANCK together with the probe HERSCHEL (observes in the IR region) was launched. It is located at a distance of 1.5 million km from Earth, at the Lagrangian point L2, and the entire sky has been mapped in the microwave range. Thus one looks back to the young universe, when it was about 400,000 years old (Fig. 15.11).

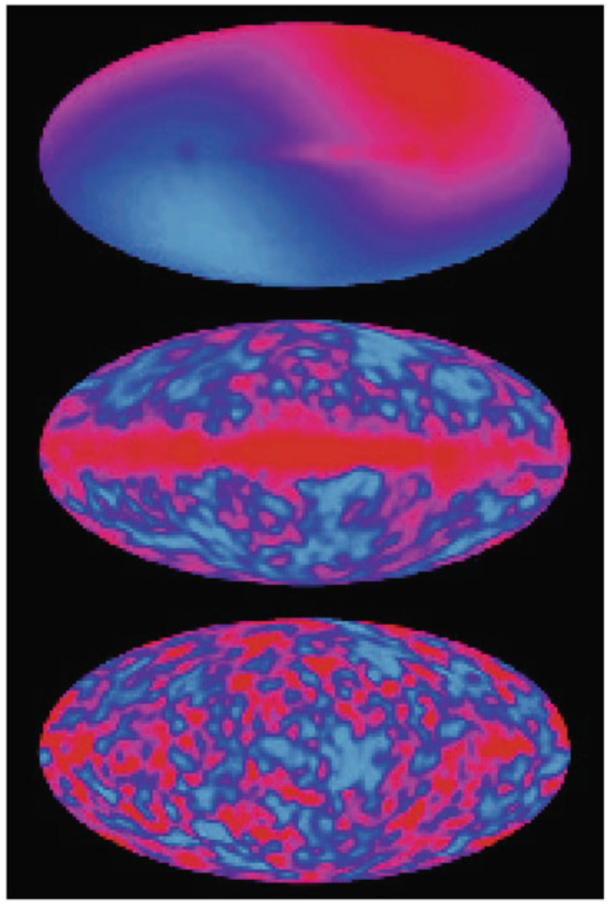

Fig. 15.10 Background radiation measured by COBE. (**a**) One can clearly see the dipole asymmetry (amplitude 10^{-3}), (**b**) with emission from the Galaxy (between 0.3 and 1 cm), (**c**) adjusted; the dark regions are 0.0002 K warmer than the bright ones

At $z = 1\,100$ there were no stars or galaxies; when the universe was 500 million years old it already consisted of 27% dark and normal matter and 73% dark energy. Anisotropies in the range of 10^{-6} K were found. These were shaped by the first young galaxies.

The deviations of the CMB in the range of $10^{-3} \ldots 10^{-6}$ K can be expanded into an angular distribution by spherical surface functions:

$$\frac{\Delta T(\theta, \phi)}{T} = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta, \phi) \tag{15.112}$$

The power spectrum can then be given as follows:

$$C_l =< |a_{lm}|^2 > \tag{15.113}$$

Thus, for each l we get the strength of the l-th moment. Now plot l vs. $\Delta T[\mu\mathrm{K}]$: $\rightarrow$ so-called angular power spectra, i.e. the intensity distribution of the CMB in the sky is

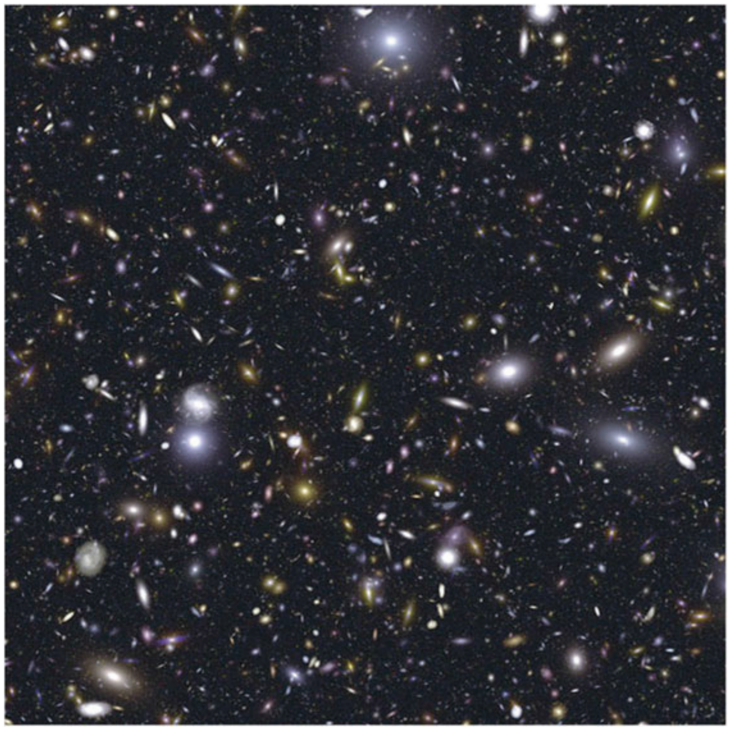

Fig. 15.11 The extreme deep field taken with the Hubble Space Telescope shows galaxies up to a distance of slightly more than 13 billion light-years ($z = 10$). One finds about 5000 galaxies on it (NASA/HST)

expanded into multipoles. At $l \approx 220$ the first acoustic peak occurs, at $l \approx 540$ the second. Due to the Sunyaev-Zel'dovicheffect, CMB photons were scattered by protogalaxies (Compton scattering), and these scattering effects modify the power spectra.

Note: Angular distances in the CMB of 1 degree correspond to the size of the horizon at the time of decoupling. Particles farther than 1 degree away in the CMB could not interact, anisotropies on these scales indicate perturbations that existed before the inflationary Phase (next chapter).

Another property of the CMB is polarization. So the background radiation must have been scattered by electrons. Now it was just mentioned that the CMB comes from the recombination era, so there were no more free electrons, so where did the scattering come from? This comes from the reionization era of the universe. The neutral cosmos was re-ionized by the first hot stars. So we get two important pieces of information from the background radiation:

- Temperature fluctuation → Anisotropies
- Polarization.

15.5.3 Acoustic Oscillations

Let us consider an inhomogeneous mass distribution . This leads to an inhomogeneity of the space → can be represented by Fourier expansion as a superposition of many periodic functions. Each periodic potential can be assigned a very specific oscillation frequency. At the time of recombination these oscillations cease. A temperature distribution is generated which depends on the state at the time of decoupling.

First Peak in the Power Spectrum First Frequency reaches its maximum. It is the angle which describes the acoustic horizon. This is the distance that the acoustic wave can just traverse until the moment of decoupling.

The *second peak* is an extremum of the wave with the double frequency and so on. From this one can determine the baryon part of the total matter in the universe. Baryons shift the equilibrium position of the oscillations by their mass.

From the *third peak* follows the radiation driving. If the radiation density is known, then follows the the fraction of dark matter.

15.5.4 Formation of Particles

In Table 15.3 the rest masses of some important particles are given.

Very often energies are given with the help of temperature. If one calculates with

$$E \approx kT$$

and the energy unit 1 eV = 1.60×10^{-19} J from E, then

$$\frac{1\,\mathrm{eV}}{k} = \frac{1.60 \times 10^{-19}\mathrm{J}}{1.38 \times 10^{-23}\,\mathrm{J/K}} \tag{15.114}$$

and

$$1\,\mathrm{eV} = 1.1605 \times 10^{4}\,k\,\mathrm{K} \tag{15.115}$$

Table 15.3 Rest masses of some important particles

Particle	Designation	Mass in MeV/c^2
Proton	m_p	938.3
Neutron	m_n	939.6
Electron	m_e	0.511
Muon	m_μ	140

Calculate which energy to a temperature of 10^{12} K corresponds to! Solution: from $E = kT$ follows $E = 1.38 \times 10^{-23} \times 10^{12} = \ldots \mathrm{J} = \ldots \mathrm{eV}$
Solution: 100 MeV.

Let us consider the universe at $T = 10^{12}$ K which corresponds to 100 MeV. From Table 15.3 it follows that all the baryons we know today were already there. The WIMPS,[9] which possibly represent a part of the dark matter, have rest-masses > 100 GeV. Except for the WIMPs, all Particle are in equilibrium. There are the following reactions:

$$\begin{aligned}
&\text{Compton} - \text{Scattering} \\
&e^{\pm} + \gamma \leftrightarrow \quad e^{\pm} + \gamma \\
&\text{Pair generation, Annihilation} \\
&e^{+} + e^{-} \leftrightarrow \quad \gamma + \gamma \\
&\text{Neutrino} - \text{Antineutrino} - \text{Scattering} \\
&\nu + \bar{\nu} \leftrightarrow \quad e^{+} + e^{-} \\
&\text{Neutrino} - \text{Electron} - \text{Scattering} \\
&\nu + e^{\pm} \leftrightarrow \quad \nu + e^{\pm}
\end{aligned}$$

The mass of the neutrinos is less than 1 eV. These reaction rates are influenced by the number density $n \approx a^{-3} \approx t^{-3/2}$ and the cross section $\sigma \approx a^{-2}$. At early times the particles were in equilibrium, the reaction rates were greater than the expansion rates.

For $T < 10^{10}$ K the neutrinos are no longer in equilibrium with the other particles; a decoupling, or freezing out of the neutrinos occurs, the universe is 1 s old.

Once the temperature has fallen below 5×10^{9} K ($kT \approx 500$ keV), electron-positron pairs can no longer be created efficiently.

15.5.5 Quarks and Quark-Gluon Plasma

In the Standard Model of Particle physics Quarks and Leptons are the fundamental building blocks of matter. Quarks are point-like fermions with spin 1/2 and third-numbered elementary charge of +2/3 or $-1/3$. Furthermore, they have the property of Color charge. Quarks exist in six, *flavors,* and their masses and charges are given in Table 15.4.

Protons and neutrons are not elementary particles, but are made up of *Quarks*. All particles that occur in nature are without *color*: therefore in combinations *bgr* (blue-green-red) or their antiparticles.

[9] Weakly interacting massive particles.

Table 15.4 Properties of quarks

Name	Mass	Charge	Antiparticle	Charge
u (up)	5 MeV	+2/3	$\bar{u}$	+2/3
d (down)	10 MeV	−1/3	$\bar{d}$	−1/3
s (strange)	200 MeV	−1/3	$\bar{s}$	+1/3
c (charmed)	1.5 GeV	+2/3	$\bar{c}$	−2/3
b (bottom)	4.7 GeV	−1/3	$\bar{b}$	+1/3
t (top)	180 GeV	+2/3	$\bar{t}$	−2/3

In addition to these six quarks there are the antiquarks. Hadrons are divided into baryons (consisting of three quarks) and mesons (consisting of two quarks with color and anticolor). Examples:

- Protons: uud,
- neutrons: udd.
- π −Meson: $u\bar{d}$, $\bar{d}$ means antidown quark.

Other particles consisting of more than three quarks have since been found such as the tetra-quark or the penta-quark.

The interaction between quarks takes place through exchange particles , the so-called gluons.

Under normal conditions quarks are *confined (confinement,* chiral symmetry broken), only above 150 MeV quarks are quasi free (*deconfinement,* chiral symmetry restored). Then a quark-gluon plasma is formed. This plays a role in the formation of the universe, in quark stars, in the interior of neutron stars and in Magnetars.

In the early universe, the phase transition to quark-Gluon plasma occurred at 170 MeV or $T = 10^{12}$ K. Free quarks were found in the laboratory with the RHIC (Relativistic Heavy Ion Collider, Brookhaven National Laboratory, Long Island, USA) in March 2003 experiments with gold-deuteron collisions were performed, the small deuteron (atomic nucleus of deuterium) shoots through the much larger gold atoms and quarks are torn out.

15.5.6 Particle Generation

Consider an elementary particle of m . At some point in the evolution of the universe, there was a time when valid:

$$kT \approx mc^2 \tag{15.116}$$

Prior to this time, a collision between two photons resulted in the creation of a particle-antiparticle pair. Once T below mc^2/k sank, particle pairs could no longer form, and particles and antiparticles annihilated each other, provided the density and thus the collision frequency were large enough. One can also subdivide the elementary particles into:

1. Leptons: Neutrinos, electrons, muons, tauons and their antiparticles.
2. Hadrons:
   (a) Baryons: Protons, neutrons, hyperons (unstable).
   (b) Mesons.

The critical temperature for Hadrons is 10^{12} K. Before this time, therefore, one speaks of the *Hadron era* of the universe. One has the following sections ($t\ldots$ Age of the universe):

1. $T \approx 10^{12}$ K, $t \approx 10^{-4}$ s: Pair annihilation of muons; muon neutrinos and their antiparticles decouple.
2. $T < 10^{11}$ K, $t > 0{,}01\,s$: The mass difference between neutrons and protons (1.3 MeV $\approx T = 1{,}5\times10^{10}$ K) begins to produce more protons and fewer neutrons. The following processes are important in this process:

$$n + \nu_e \leftrightarrow p + e^- \qquad n + e^+ \leftrightarrow p + \bar{\nu}_e \tag{15.117}$$

$$n \leftrightarrow p + e^- + \nu_e \tag{15.118}$$

   The number ratio $n : p$ is determined by the temperature:

$$N_n/N_p = \exp(-1.5 \times 10^{10}/T) \tag{15.119}$$

3. At a temperature of $T \approx 10^{10}$ K when the uniserse had an age of$t \approx 1\,s$ the electron neutrinos and the antineutrinos decouple.
4. $T \approx 5 \times 10^9$ K, $t \approx 4\,s$: the electrons and positrons annihilate each other; neutrino cooling fixes the $n : p$ ratio. One then has only the β decay:

$$n \rightarrow p + e^- + \bar{\nu}_e \tag{15.120}$$

5. $T \approx 10^9$ K; $t \approx 10^2$ s: nuclear synthesis begins, and ^{4}He, deuterium (^{2}H) and ^{3}He,^{7} Li are produced. For example:

$$p + n \leftrightarrow {}^2\mathrm{H} + \gamma \tag{15.121}$$

$${}^2\mathrm{H} + {}^2\mathrm{H} \leftrightarrow {}^3\mathrm{He} + n \tag{15.122}$$

$${}^3\mathrm{He} + n \leftrightarrow {}^3\mathrm{H} + p \tag{15.123}$$

$${}^3\mathrm{H} + {}^2\mathrm{H} \leftrightarrow {}^4\mathrm{He} + n \tag{15.124}$$

As the universe expands, cooling occurs. Heavy particles were created at higher temperatures, light particles at lower temperatures. A temperature of 10^{12} K corresponds to 100 MeV. All baryons known to us were already present.

15.6 Symmetry Breaking in the Early Universe

We first consider the four fundamental forces that describe the interactions between particles today. It turns out that the weakest of these forces, gravity, determines the large-scale structure of the universe, i.e. the shape of galaxies, clusters of galaxies, stars, planets, and so on. Attempts are being made to attribute these forces to one force in what are called GUTs, grand unified theories. String theories and quantum loop gravity extend the standard model of particle physics and can also explain gravity.

15.6.1 The Four Forces of Nature

In the nineteenth century, three basic forces of nature were known: gravitation, electricity, and magnetism. *Maxwell* was able to show that electricity and magnetism can be unified to electromagnetism. Atomic physics and quantum physics led to the discovery of further basic forces (Table 15.5).

The *strong force* (strong interaction) is stronger than gravity by a factor 10^{40}, but acts only at very short distances holding atomic nuclei together. The *electromagnetic force* (EM) is long-range, but can be attractive (between different charges) and repulsive (between charges of the same sign), and only for charged particles. Therefore, the electric forces in the universe neutralize each other.

The *weak interaction* is responsible for radioactive decay and the neutrino interaction. As an example, the decay of a free neutron in Fig. 15.12 is shown in a Feynman diagram (Fig. 15.12). In these diagrams the x-axis is the time.

Table 15.5 The four fundamental forces; K means coupling strength, R the range

Force	K	R [m]	Transducer	Acts on
Strong	1	10^{-15}	Gluons	Quarks
EM	1/137	∞	Photons	All charged Particles
Weak	1×10^{-5}	10^{-18}	W^+, W^-, Z^0	Quarks & Leptons
Gravity	10^{-40}	∞	Gravitons	Particles with mass

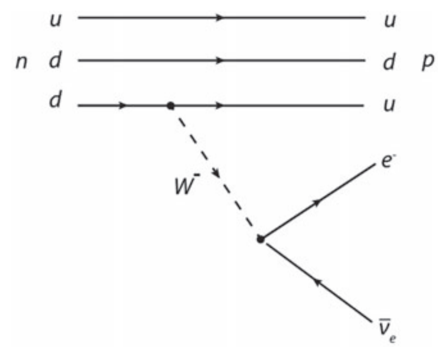

Fig. 15.12 For example, the electroweak interaction explains the decay of a free neutron (consisting of udd quarks) to a proton (consisting of uud quarks)

The *Gravity* is by far the weakest interaction, and yet it dominates the structure of the universe. There are two reasons for this:

- The range is unlimited,
- Between two masses in the universe there is only attraction and no repulsion (but cf. electromagnetic interaction).

It's assumed, all four basic forces result of one single force by so called spontaneous symmetry breaking.

Electromagnetic, weak interaction and strong interaction are described in GUTs *(grand unified theories).* There is experimental evidence for this assumption of unification of the four forces of nature: About 40 years ago it was discovered that the electromagnetic force combines with the weak interaction to form the electroweak force. In the so-called *Standard Model* the interaction can be calculated. In 1979 *Glashow, Salam* and *Weinberg* received Nobel Prize for their theory of the electromagnetic and the weak interaction, 1984 discovered *Rubbia* and *Simon* the From W and Z particles predicted by this model.

Particles can be divided into:

- Bosons: integer spin. $s = 0, 1, 2, \ldots$
- Fermions: half-integer spin; $s = 1/2, 3/2, \ldots$

Fermions bosons are subject to the Pauli Principle The bosons, on the other hand, do not and can act as transmitters of forces.

→ In physics, forces are transmitted by so-called exchange particles. Analogy: How can two ice skaters repel each other? By throwing snowballs at each other; they are attracted by each throwing the snowballs in the opposite direction. In this case the snowballs act as interacting particles.

The photons are the transmitters of the electromagnetic force. If one has thus an electron bound to a proton, photons are constantly exchanged here to transmit the electromagnet attraction. In the weak interaction, these are the W^+-,W^--, Z^0-particles, each of which has 100 times the mass of the proton. In the case of the strong force, the transducers are the Gluons.

The leptons are fermions subject to the electroweak interaction as well as gravity, but not the strong force. These particles include electrons and neutrinos with their corresponding antiparticles.

15.6.2 The Early Universe

So far we have studied the evolution of the early universe from a point in time of 10^{-4} s after the big bang, when its temperature was 10^{12} K. The physics of these processes can be understood and verified today in particle accelerators. For even earlier phases, however, higher energies are required and we must rely on theoretical considerations..

At an age of $t \approx 10^{-6}$ s, at which the temperature was $T \approx 10^{13}\,K$, the quarks annihilate with their antiparticles, you have a soup of quarks and leptons. The electromagnetic force and the weak interaction combine to form the electro weak force. So a phase transition occurs. The theory for this was developed by Weinberg and Salam at CERN with the help of accelerator experiments (W and Z particles were detectee).

Up to 10^{-35} s After the Big Bang, the electroweak force was united with the strong interaction—GUTs *(grand unified theories)*. One of the predictions of the GUTs is the decay of the proton within 10^{31} years.

> At the ever-increasing temperatures of the early universe, a unification of the electromagnetic force with the weak force occurred, and when the age of the universe was only 10^{-35} s the electroweak force unified with the strong force (GUT).

In Fig. 15.13 the behavior of the coupling constants of the four fundamental forces as a function of energy is plotted. One can see how the constants approach each other at high energies.

This splitting or phase transition can be thought of as the phase transition of ice to liquid water. Ice is in a higher state of symmetry (crystals) than water because water, being a liquid, can take any form.

15.6.3 Inflationary Universe

In 1981 *A. Guth* has developed the concept of the *inflationary universe*. When the strong interaction separated from the weak and electromagnetic, a phase transition (symmetry breaking) occurred when the universe was about 10^{-35} s old. This filled the universe with energy, the so called vacuum energy (or false vacuum energy). Thus for about 10^{-32} s gravity became repulsive, and this resulted in extreme expansion of the universe (by a factor of 10^{26}). As soon as the phase transition was completed, the normal evolution continued (Fig. 15.14).

So the energy density of the vacuum transformed into matter and radiation. The theory of the inflationary universe is needed to explain peculiarities of the universe:

- Horizon problem: Consider the universe in the microwave range. Two opposite points in the sky can never have been physically related, but inflationary expansion can explain this.
- Flatness problem: The density parameter Ω_0, which describes the ratio of actual matter density to critical matter density (the density required to maintain a closed universe), is

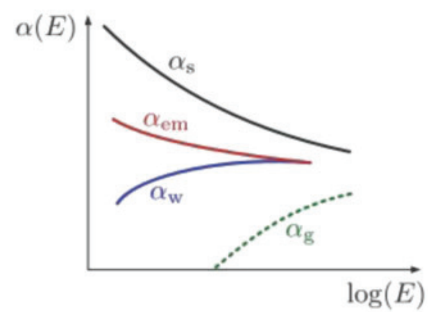

Fig. 15.13 At higher energies, the coupling constants of the four forces (s strong, w weak, em electromagnetic and g gravitational) approach each other

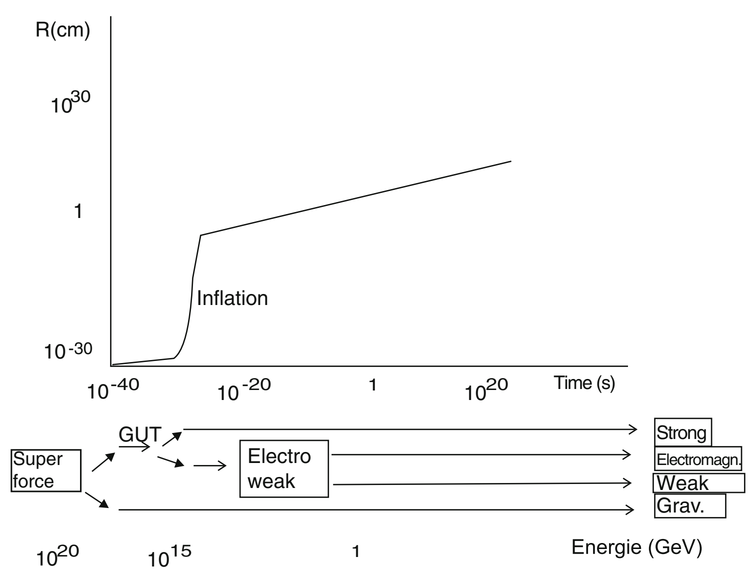

Fig. 15.14 Evolution of the universe

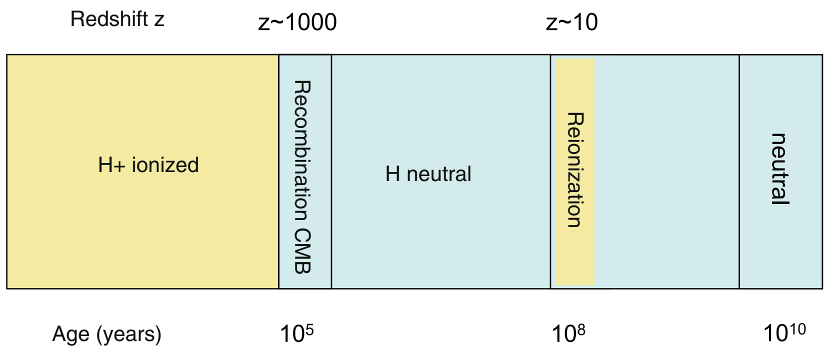

Fig. 15.15 Evolution of the universe: At $z = 1000$ the universe becomes transparent, from this time comes the cosmic background radiation (CMB). Then the universe was neutral (mainly neutral H). Due to massive, luminous stars and quasars, a new ionization at $z = 10$ took place when the age was about 10^8 years

between 0.03 and 2. But already 1 s after the big bang was $1 - \Omega_0$ was only 10^{-15}, so the universe was extremely flat.[10]
- Rapid expansion caused magnetic monopoles that may have formed earlier to become very widely dispersed and therefore undetectable today.
- Inflation created small density fluctuations, which were then the condensation nuclei for later matter structures (galaxies, galaxy clusters, ...).

In Fig. 15.15 the thermal evolution of the universe is sketched.

Even earlier than 10^{-43} s (Planck time) one needs a quantum theory of gravity. Here there are many speculations like *Superstring theories* of *Green* and *Schwartz*.

The inflationary phase occurred due to the phase transition that led to the splitting of the strong force from the electroweak force; the universe expanded by a factor of 10^{26} from the size of a proton to the size of 10 cm.

15.6.4 String Theory

We have already discussed the problem of unification of the four fundamental forces. The so-called standard model is unsatisfactory on this point because gravity cannot be incorporated. In string theory one assumes that particles consist of strings, which do not

[10] For a flat universe $\Omega_0 = 1$.

Fig. 15.16 Interaction between strings; this does not happen at a well-defined point

Fig. 15.17 Interaction in the Standard Model; this happens at a point and does not make mathematical sense for gravitons

show any further structure, but can appear in different states of vibration. Particles are thus vibrations of a string, and one finds:

- Particles that form matter: Electrons, muons, neutrinos and elementary quarks,
- transmitters of interactions: Photons, W and Z bosons, gluons,
- Graviton: this particle, which is supposed to transmit gravitation, is also postulated, and thus gravitation would be integrated by string theory (Fig. 15.16).

The four-dimensional space-time structure of the universe should naturally follow from the string theories. Relativistic quantum theory describes the properties of elementary particles in the presence of very weak gravity, and particle theory works only in the absence of gravity. General relativity describes the orbits of planets, big bang and black holes, and gravitational lensing. However, here one mostly calculates with a classical universe, i.e. quantum effects are not considered. Therefore, string theory would fill this gap.

Gravitons are the transmitters of gravity. Suppose a graviton is placed in a quantum field, the particle interactions happen at a point in space-time with zero distance between the interacting particles (Fig. 15.17). This does not make sense. In string theory, strings collide at a small finite distance, and the mathematical solutions appear to make sense.

In string theory, elementary particles are described as excitation modes of elementary strings. The strings are free in space-time. The tension of the strings is described by:

$$\frac{1}{2\pi a'} \tag{15.125}$$

whereby a' is equal to the square of the string length scale. Strings are of size in the range of the Planck length (10^{-35} m). There are two kinds of strings: open and closed strings. In order to integrate also the fermions into the theory, one needs the so-called *Supersymmetry:* For every boson (which are the particles that transmit forces) there exists a corresponding fermion (which are the particles that make up matter). And for every fermion there should be a supersymmetric boson. These heavy supersymmetric particles should be found by particle accelerators of the coming generation.

String theories can thus be formulated in different ways:

- open or closed strings,
- boson strings only: Boson strings,
- for bosons and fermions $\rightarrow$ Super symmetry, super strings.

For a string theory of bosons one needs 26 space-time dimensions, for super strings only ten. For the former, there is a particle with an imaginary mass, the tachyon. The reduction of, say, ten space-time dimensions to our known four is called *compactification.* M-theory *(mother of all theories)* unifies the many different string theories.

According to string theories, there must be more than the four space-time dimensions we know.

The idea of more than four dimensions is not new. *Klein* and *Kaluza* around 1921 were the first to advocate the idea that our universe could have more than three spatial dimensions. But why do we observe only the known three? Here is an analogy: If we look at a garden hose from a distance, it looks like a one-dimensional object. Only up close do we realize that it is a three-dimensional object. It could also be the case below the current measurement limits of 10^{-16} cm there are further dimensions. According to the theory of Kaluza-Klein, gravity and Maxwell's equations can be described together in a 5-dimensional universe. In addition to the three space dimensions we know, there should be one more. The problem was, however, that this theory could not be unified with quantum mechanics at all, and so it was abandoned.

String theory now says that specific particle properties such as mass or electric charge are determined by the size, number, shape of the holes, conditioned by the higher dimensions.

In *Topology* one examines the properties of geometric space, which do not change under:

- Space dilation,
- twist,
- bending.

Example

A doughnut and a sphere are topologically distinct, there is no way to create a doughnut from a sphere by twisting or bending. A doughnut and a teacup look different. However, these two bodies are topologically the same.

In general relativity, space-time is assumed to be constantly changing its size and shape (expansion of the universe). However, the topology of the universe remains the same.

Let us consider quantum mechanical processes on the smallest size scales: At smallest scales there are fluctuations, and therefore one must assume such fluctuations in the space-time structure as well. These fluctuations average out when we look at the universe on larger scales. Mathematically, this can be represented by shrinking to a singular point followed by expansion in an orthogonal direction. The singularity cannot be described by general relativity.

The boundary conditions of strings can be different: Closed strings have periodic boundary conditions, open strings have two types of boundary conditions: (a) Neumann boundary conditions, the endpoint is free to move but has no momentum, (b) Dirichlet boundary conditions, where the endpoint is fixed on a manifold, this is called a Dp-brane (p indicates the number of spatial dimensions of this manifold). In Fig. 15.18 one has open strings with one or both endpoints fixed on a D2-Brane.

Let us examine the interaction of D-branes with gravity. In Fig. 15.19 one has a graviton, represented by a closed string, interacting with a D2-brane. The string thereby becomes an open string, and at the time of the interaction the endpoints are on the D-brane.

A good application to test this concept is found in Hawking black hole radiation. If one assumes open strings, then radiation follows in the form of closed strings. The *Beckenstein-*

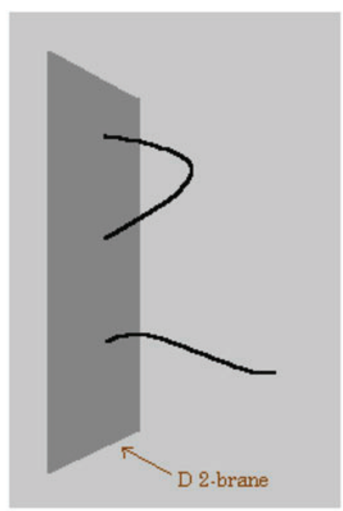

Fig. 15.18 D2 brane of an open string

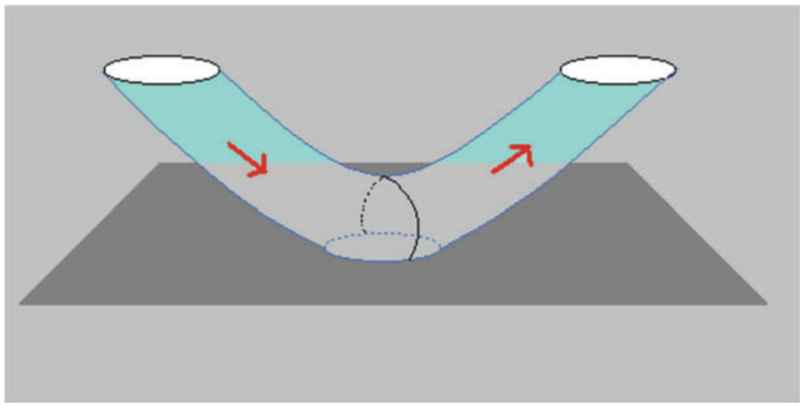

Fig. 15.19 D-brane interaction with a graviton. Note the transition to an open string at the time of the interaction

Hawking entropy formula states that the entropy S of a black hole can be expressed by:

$$S = A/4, \tag{15.126}$$

where A is the event horizon. Once an object has penetrated the event horizon, it can never escape from it. The above formula can be explained using string theoryren.

15.6.5 Quantum Foam

The two major theories of 20th century physics are general relativity and quantum theory. If one applies these two theories to scales of Planck length, i.e., about 10^{-35} m, then bubbles of space-time are continuously formed, which decay again. Due to the uncertainty principle, the uncertainty of location (Δx) and momentum (Δp) of a particle:

$$\Delta x \Delta p \geq \hbar/2 \tag{15.127}$$

The momentum can therefore fluctuate by $\Delta p < \frac{\hbar}{2\Delta x}$ without being able to measure it. The uncertainty of the energy is:

$$mc^2 = \Delta E = \Delta pc = \frac{\hbar c}{2\Delta x} \tag{15.128}$$

A black hole is determined by the Schwarzschild radius: $r = \frac{2Gm}{c^2}$.

Objects whose extent is equal to the Planck length would have to possess a mass of 10^{-8} kg , because:

- $m < 10^{-8}$ kg: spatial uncertainty $\rightarrow$ greater expansion.
- $m > 10^{-8}$ kg: Schwarzschild radius > Planck length.

15.6.6 Quantum Vacuum

As already shown, the vacuum is not empty, but filled with virtual particles. According to the Heisenberg uncertainty principle, a particle of energy ΔE a short time

$$\Delta t < \hbar/\Delta E \tag{15.129}$$

exist. Thus, virtual particle/antiparticle pairs are created. If the time is very short, high energy or large masses are created because $m_0 = E/c^2$. Large masses in turn bend space-time, and bubbles are created in space-time. The quantum foam is chaotic in structure.

The quantum vacuum represents a state of lowest energy $\rightarrow$ However, the energy is never exactly zero. Into the above formula enters:

- Planck length: 10^{-35} m,

$$l_P = \sqrt{\frac{\hbar G}{c^3}} \tag{15.130}$$

- Planck time: 10^{-43} s.

$$t_P = \sqrt{\frac{\hbar G}{c^5}} \tag{15.131}$$

Time needed for light to travel one Planck length.

The location and time of a particle can never be determined more precisely than these two quantities.

$\rightarrow$ Space and time are not continuous, but are a composite of quanta of space-time .

15.6.7 Loop Gravity, Quantum Loop Gravity

We have used string theory as a way of to describe gravity with the other forces of nature. The singularity of the big bang could be avoided. Any object smaller than the Planck scale would immediately collapse into a black hole due to the uncertainty principle, as it has

Fig. 15.20 The MAGIC telescope at the Observatorio Roque de los Muchchos in La Palma

extremely high energy or mass. In loop gravity, one assumes (as in string theory) that there cannot be arbitrarily small structures. Space itself obeys quantum mechanics, it can be described by quantum states. Such a quantum state is described by a network of nodes connected by lines. The distances between nodes correspond to the Planck length. A cubic centimeter would thus contain 10^{99} nodes. The universe contains about 10^{85} cm^3. Thus, one could obtain much more information with a perfect microscope that could zoom into a cubic centimeter than with a perfect telescope that resolves the universe to cm-length!

Where is this mesh embedded? The mesh itself defines the space. There is nothing between the links and nodes. Analogy: consider a sand dune. It is made up of many grains of sand. Between the sand grains there is nothing. The elementary particles are then net nodes. The movement of the particles is a displacement of the nodes in the net.

Some effects can be predicted with loop gravity: long wavelength gravitational waves, Hawking radiation, black hole entropy, positive cosmological constant. Other effects would be a wavelength dependence of the speed of light. This would be especially apparent when the wavelength becomes comparable to the distances between nodes, i.e. the Planck length. If detectable at all, this could only be detected for highest energy cosmic rays, here the difference would amount to 10^{-9}. This would therefore mean differences in transit time at different wavelengths, e.g. during a gamma-ray burst. The blazar Markarjan 501, is 500 million light-years away. This object was studied with the MAGIC telescope (diameter about 15 m, Fig. 15.20)[11] and showed transit time differences.

[11] MAGIC stands for Major Atmospheric Gamma Imaging Cerenkov Telescopes.

15.6.8 The First Stars

We have seen that the universe consists of three components:

1. Baryonic matter: visible, observable; about 4%.
2. Dark matter: not observable, gravitational effect only! About 23%.
3. Dark energy: explains accelerated expansion of universe.

On large scales, the universe exhibits a filamentary structure. This cannot be explained by baryonic matter alone. Baryonic matter is subject to thermal pressure, i.e. gas pressure $\rightarrow$ No clumping of matter. Dark matter is subject only to gravity and not to pressure.

So dark matter explains the filamentary distribution of baryonic matter. Giant clouds formed at the network nodes and collapsed. Population III stars formed: they contained only hydrogen and helium, which affects opacity. This is why they could be more massive than the Stars.

Population III stars: some 100 solar masses
Diameter $\approx 10\,R_\odot$
Surface temperatures: some 100,000 K
"Dark" stars could have been formed during the collapse of dark matter. However, they shone due to annihilation of dark matter particles and antiparticles. Mass determined according to

$$E = mc^2 \tag{15.132}$$

is converted into energy. Dark stars were therefore extremely bright. Lifetimes were only a few million years. Now there are two scenarios:

- After this Dark Matter dispersion due to anihilation, the star shines due to normal nuclear fusion. However, because of the large mass, this means also a short lifetime, and the star explodes into a supernova. A black hole remains, and the interstellar matter is enriched with heavier elements.
- Massive dark stars do not reach normal stellar stage (i.e. thermonuclear fusion) and become heavy black holes of several thousand or more solar masses. This may have given rise to the Supermassive Black Holes (SMBH) in the nuclei of galaxies.

The successor to the Hubble telescope, the James Webb Space Telescope, will be used to search for such supernovae. Since they originate from the early phase of the universe, they show an extreme red shift and should therefore be observable in the IR.

15.6.9 Parallel Universes

There are two ways to think of the creation of parallel universes:

The universe originated from quantum foam; if our universe originated from it, then any number of universes with possibly quite different laws of nature may have originated.

Many-worlds interpretation of quantum mechanics: During observations, the world splits into multiple worlds. The totality of all parallel worlds is also called multiverse. The difference to above is, that here the same laws of nature must be valid in all universes.

15.7 Time Scale

We scale the evolutionary history of the universe from the origin to the present to one year. Then we find the data given in Table 15.6 given data.

15.8 Further Literature

We give a small selection of recommended further reading.

Bergström, L., Goobar, A., Cosmology and Particle Astrophysics (Springer Praxis Books) (Paperback) Springer, 2006

Table 15.6 History of the evolution of the universe from the origin to the present day scaled to one year

Time	Event
January 1 0^h0^m	Big bang, creation of H, He
January 1 0^h14^m	Decoupling of radiation and matter
5 January	First stars and black holes, C, N, O, ...
16 January	oldest known galaxy, quasar
9 September	Formation of solar system and earth
28 September	First life on Earth (cyanobacteria)
December 16–19	Vertebrate fossils, plants
20–24 December	Forest, fish, reptiles
December 25	Mammals
28 December	Extinction of the dinosaurs
December 31, 20^h00^m	First humans
December 31 23^h55^m	Neanderthals
December 31, $23^h55^m56^s$	Year 0
January 12	Earth becomes too hot
April 7	Sun becomes a red giant
April 16	Collision Milky Way with Andromeda Galaxy

Sexl, R.U., Urbantke, H.K.: Gravitation and Cosmology, Spektrum Akademischer Verlag, Heidelberg, 5th edition 200
Introduction to General Relativity and Cosmology, C.G. Böhmer, World Scientific, 2016.
Gravitation and Cosmology, S. Weinberg, WSE, 2008
Modern Cosmology, A. Liddle, Wiley, 2015
Extragalactic Astronomy and Cosmology, P. Schneider, Springer, 2014
Foundations of Modern Cosmology, J.F. Hawley, K.A.Holcomb, Oxford Univ. Press, 2005
Astrophysics and the Evolution of the Universe, L.S. Kisslinger, World Scientific, 2016

Tasks

15.1 Under which simplification does $\tau = 1/H$ gives the actual age of the world?

Solution
Uniform expansion.

15.2 At what distance is there a galaxy from us whose red shift is $z = 0.1$ is?

Solution
If we calculate in the units for the speed of light [km/s] or for the Hubble constant [$\mathrm{km\,s^{-1}\,Mpc^{-1}}$]: $d = cz/H_0 = 3 \times 10^5 \times 0.1/(50 \times 1.1) \rightarrow d = 600\,\mathrm{Mpc}$.

15.3 When cosmic ray particles collide, muons are produced in the Earth's atmosphere. These travel at 99.4% of the speed of light. Their half-life is, for particles at rest 1.5×10^{-6} s. By what amount does its half-life increase?

Solution
One first calculates $\gamma = 1/\sqrt{1 - 0.994^2} \rightarrow \Delta t' = \gamma \Delta t = 13.7 \times 10^{-6}$ s. Fast-moving muons decay more slowly, travel preserves young.

15.4 Determine the components of the metric tensor in the case of Minkowski space!

Solution
From $ds^2 = dx^0 dx^0 g_{00} + dx^0 dx^1 g_{01} + dx^0 dx^2 g_{02} + dx^0 dx^3 g_{03}$
$+dx^1 dx^0 g_{10} + dx^1 dx^1 g_{11} + dx^1 dx^2 g_{12} + dx^1 dx^3 g_{13}$
$+dx^2 dx^0 g_{20} + dx^2 dx^1 g_{21} + dx^2 dx^2 g_{22} + dx^2 dx^3 g_{23}$
$+dx^3 dx^0 g_{30} + dx^3 dx^1 g_{31} + dx^3 dx^2 g_{32} + dx^3 dx^3 g_{33}$ it follows that $g_{00} = -1$, $g_{11} = 1$, $g_{22} = 1$, $g_{33} = 1$ and all other components are zero.

15.5 Could a dark matter planet entering our solar system be observed?

Solution
No, only by its gravitational effects; you couldn't even observe a transit!

15.6 Show how to calculate from the formula 15.55 find the classical Doppler effect!

Solution
Set $c \to \infty$.

15.7 Show that, unlike the classical Doppler effect, there is a red shift in the relativistic one even when the source is moving transversely!

Solution
Set $\cos \Theta = 0$. A consequence of time dilation.