THE BLACK HOLE SCIENCE INTERNSHIP PROGRAMME 2025

Rayleigh-Bénard Convection

Project Report

Submitted by:

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Abstract

This report investigates Rayleigh-Bénard (RB) convection. The primary objective of this report is to explore the key physical mechanisms driving this convection, including thermal gradients and multiple forces influencing fluid behaviour, leading to the formation of RB cells. In addition, the report studies how varying experimental parameters such as fluids used, aspect ratio of containers, and temperature differences affect cell formation. Visual observations and theoretical analysis help us better understand the effects of these parameters and the process of convection itself.

Chapter 1: Introduction

1.1 Rayleigh-Bénard Convection

Rayleigh-Bénard Convection is a classical fluid phenomenon that occurs when a temperature gradient is maintained between the bottom and the top layers of the fluid. At low temperatures, the transfer of heat is only carried out by conduction, but as the temperature gradient exceeds a certain value, a density difference between the top and bottom layers of the fluid, leading to organized motion of the particles of the fluid, is formed. Once convection starts, patterns such as hexagonal or roll cells form. This model studies the instability of a fluid upon thermal heat and is used to understand convection as a whole.

Rayleigh–Bénard convection helps model large-scale natural phenomena. In Earth's mantle, it explains how heat from the core causes rock to circulate, driving plate tectonics. In the kitchen, similar convection cells form when heating thin layers of soup or oil in a pan. These visible patterns help visualize thermal instability. In atmospheric convection, sunlight heats the ground, causing warm air to rise and form clouds and weather systems. Despite differences in scale, all these processes are governed by the same buoyancy-driven convection principles studied in Rayleigh–Bénard systems.

1.2 Key Physical Parameters Affecting Rayleigh-Bénard Convection

1.2.1 Viscosity

Viscosity is a fluid's resistance to flow. It describes how much a fluid resists deformation or the movement of its neighbouring particles. For liquids, it corresponds to an informal concept of thickness; for example, syrup has higher viscosity than water. Viscosity mainly depends on temperature. As temperature increases, viscosity decreases. When a liquid's temperature increases, its particles gain energy and move faster. This movement reduces the fluid's viscosity, making it flow more easily. In Rayleigh-Bénard, the lower viscosity from heating makes it easier for convection cells to form. Viscosity also depends on fluid type. Different fluids, such as oil, water, and glycerin have different viscosities. Fluids with long, tangled molecules, like oil, resist flow more due to their higher viscosity. Pressure affects viscosity slightly in liquids but has a significant effect in gases. When we increase the pressure in a liquid, the particles come closer and it increases the viscosity of the fluid. Similarly, if we decrease the pressure, the particles come to their actual viscosity. Adding salt, sugar, or particles to a fluid increases its viscosity.

As mentioned earlier, viscosity is the fluid's internal resistance to flow. Hence, the higher the viscosity, the harder it is for the fluid to flow. Moreover, it suppresses or delays convection. As the viscosity is lower, it is easier for the fluid to flow, so it promotes convection. The Rayleigh number determines whether convection occurs. The higher the viscosity, the lower the Rayleigh number (see section 1.3) and harder it is to reach the critical Rayleigh number (here critical Rayleigh number is the minimum Rayleigh number where thermal convection starts in the fluid layer, i.e, 1708). Similarly lower the viscosity easier it is to trigger convection.

Viscosity affects the cell shape and motion as well. In case of higher viscosity, the fluid forms closer, smoother, and more stable cells. While in lower viscosity, the fluid creates faster, sharper, and more dynamic cells. Highly viscous fluids may suppress convection entirely unless heated sufficiently. The critical temperature difference (ΔT) needed to start convection increases with viscosity. You need to heat more if the viscosity is high.

High viscosity, such as in silicon oil, suppresses convection unless you increase (ΔT) or reduce thickness. Water, being of low viscosity, can start convection more easily at small temperature differences. The critical Rayleigh number is Ra=1708, which is the minimum Rayleigh number at which convection starts in the fluid layer.

1.2.2 Buoyancy

Buoyancy forces arise due to density **(ρ)**variations within a fluid under the influence of gravity and give rise to a wide range of important phenomena in fluid mechanics, Rayleigh-Bénard convection being one. Buoyancy-driven flow is the fundamental principle behind natural convection. The primary driving force for natural convection is the interaction between a fluid’s density difference and the presence of gravity. When a temperature gradient is applied to a fluid (heating from below), the fluid molecules near the hot surface gain energy and move more vigorously, increasing the spaces between them. These spaces increase the volume of fluid near the bottom plate (expansion) and, as a result, decrease the mass per unit volume **(ρ = m/v)**, causing the fluid at the bottom to become less dense. Similarly, the colder fluid on the top has a relatively greater density. Gravity pulls down fluid with greater density **(because of a greater mass per unit volume as compared to the less dense fluid)**. This is because the gravitational force **(F_g)** is directly proportional to mass **($F_g \propto$ m)**. This displaces the less dense fluid, and it moves upwards. As **p = ρgh**, a pressure gradient is formed due to a difference in height. This pressure difference gives rise to an upward force called the buoyant force that drives the fluid.

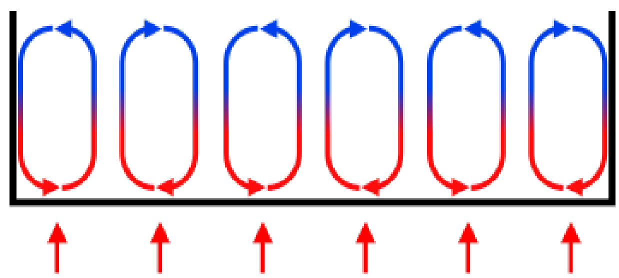


Figure 1.1: *Buoyancy at work when fluid is heated. Warm fluid rises (red), cool fluid sinks (blue), creating circular motion and a repeating pattern across the layer.*

1.2.3 Stability and Instability

Stability and instability are key concepts in understanding Rayleigh-Bénard convection, which is observed in a fluid layer present between two horizontal plates responsible for a temperature gradient. In a stable condition, heat is only being transferred through conduction, creating a thermal equilibrium as a result. In our experiment, initially, the denser fluid is present on the top plate while the lighter and hotter fluid is at the bottom. This system is **stable (Rayleigh number is less than 1708)** because there are not enough buoyant forces to overcome the fluid's viscosity and thermal diffusion. Perturbations or small movements are still taking place but are dampened by viscosity and thermal forces, making the fluid appear at rest as a whole. This is known as a stable state of fluid in which the temperature starts to vary vertically, from bottom to top.

However, as the temperature gradient reaches a specific threshold, the buoyant forces start to increase gradually and work against the stabilizing effects of viscous and thermal forces. When the magnitude of buoyant forces increases to such an extent that it overcomes the resistive forces of the fluid, then the system becomes unstable. When $Ra < 1708$, the system is stable, and conduction takes place. When $Ra > 1708$, the system becomes unstable, and convection begins in the form of organized cellular flow patterns.

The transition of the system from stable to unstable is dependent on certain factors, including the properties of the fluid as well as the structure of the container used. As the temperature gradient continues to rise between the top and bottom plates, the flow starts to become more and more irregular and may appear deformed. The system could evolve from steady and structured convection into a more complex and turbulent flow behaviour. Understanding this transition of fluid from stable to unstable behaviour is a fundamental part of fluid dynamics and is used in order to understand the relation between thermal heat transfer and flow of fluid in both natural and engineered systems.

1.2.4 Heat Transfer

As mentioned above, the experiment starts when the bottom layer of a fluid is heated. This simple step is crucial for this experiment. The heat is provided through a hot plate, providing thermal energy, which instigates the convection. It is crucial to keep the heat uniform to keep a stable temperature gradient; otherwise, the convection cells will form disorganized cell patterns. The two methods of heat transfer in this experiment are conduction and convection. Conduction is the transfer of heat through direct contact between objects or substances, while convection involves the transfer of heat through the movement of fluids (liquids or gases). Initially, heat transfers from the bottom plate to the adjacent fluid through conduction. As the fluid heats, it rises, carrying heat upwards (convection). At the top, the warm fluid contacts the cooler top plate, transferring heat via conduction. The cooled fluid sinks, completing the cycle.

Convection needs differences of density caused by variations of temperatures, creating circulating currents that form convection cells. Conduction is not plausible for creating these patterns, as it mainly involves the transfer of heat through molecules' direct contact without significant mass movement.

1.2.5 Aspect Ratio

The aspect ratio **(A)** is a fundamental geometric parameter in Rayleigh-Bénard Convection. It is termed as the ratio between the horizontal dimension **(inner-diameter)** of the container and the vertical length of the fluid **(depth of fluid)**.

$$A = \frac{D}{H}$$

Where **D** is the diameter (or width) of the container and **H** is the height (depth) of the fluid.

$\pm\ 0.01\, mm$

Aspect ratio has a fundamental role in determining the behaviour, structure, and the number of convection cells being formed in the fluid. A high aspect ratio indicates that the geometry of the container is wide and shallow, which supports the formation of multiple but small cells in the fluid. In comparison, a small aspect ratio tells us that the used container is narrow and tall, which tends to restrict or minimize the formation of cells in the fluid.

$A < 2$, **then a small number (usually only one) of cells are formed.**

$A = 2 - 10$, **then the cell formation will be average and well-structured.**

$A > 10$, **then cell formation will be ideal, and multiple small cells will be observed.**

The influence of aspect ratio is significant in both experimental and theoretical analyses. It affects the structure of the convection patterns, the interaction with container boundaries, and the stability of the fluid motion. In lower aspect ratios, boundary effects become dominant and can suppress the natural development of convection cells. In higher aspect ratios, such boundary influences are minimized, allowing for more regular and reproducible cell patterns, which are better for observations and analysis.

1.3 Rayleigh Number

When a fluid is heated from below, a small temperature gradient is initially formed. At this stage, convection is absent in the fluid because the buoyant forces generated by the temperature difference are too weak to overcome the stabilizing effects of thermal diffusivity and viscosity. To understand this, we must first grasp the concepts of thermal conductivity and thermal diffusivity. Thermal conductivity is a material's property that describes how a substance conducts heat. It governs the process of conduction (visit modes of transfer for conduction). At a small temperature gradient, the density differences in a fluid are minimal, so buoyant forces are negligible. Thus, the fluid transfers heat between its layers through conduction. The rate at which temperature changes spread between the fluid layers is known as thermal diffusivity. Since conduction is a slow process (especially in fluids), the temperature gradient grows over time. Eventually, the gradient becomes large enough to generate significant density differences, producing buoyant forces strong enough to overcome damping effects produced by thermal diffusivity and fluid viscosity. At this point, the system becomes unstable and convection begins. The Rayleigh Number is a dimensionless number that compares two competing effects in a fluid: thermal buoyancy forces and damping forces (viscous forces, etc.). It is the product of the **Grasshof number (Gr)** and the **Prandtl number (Pr)**, which explain the relation between the buoyant and damping forces.

$$\mathrm{Ra} = \frac{g\alpha d^3 \Delta T}{\eta D_t}$$

$$\mathrm{Ra} = \mathrm{Gr} \,.\, \mathrm{Pr}$$

1.3.1 Thermal Diffusivity

Thermal Diffusivity tells us about how the heat spreads through a fluid. Thermal diffusivity (D_t) determines how rapidly temperature changes propagate through a material.

$$D_t = \frac{k}{\rho C_p}$$

k: thermal conductivity in units of $\frac{W}{mK}$

ρ: density in units of $\frac{kg}{m^3}$

c_p: specific heat capacity at constant pressure = $\frac{J}{kg \cdot k}$

After simplification of units, we find that units D_t are $\frac{m^2}{s}$, i.e., So,

$$D_t = \frac{[distance]^2}{time}$$

$$\text{or } time = \frac{d^2}{D_t}$$

This is known as the characteristic time for heat diffusion **(t_{th})**. It tells us how long it will take for heat to spread over a distance **(d)**.

Now, we take a look at how much time it will take for a molecule of the fluid to rise when heated through the plate and a cold molecule of fluid to fall across a certain distance (d) due to buoyancy. This time can be represented by a secondary characteristic time scale for the motion of the fluid molecules due to buoyancy. We can call this t_m, time of motion or the convective time scale.

Thus, two possible outcomes arise:

1. $t_m < t_{th}$
   In this case, heat diffuses faster than the motion of molecules of fluid due to buoyancy, so only conduction takes place.

2. $t < t_m$
   In this case, buoyancy acting on the fluid makes the fluid move faster than heat can diffuse, so convection occurs.

Let ρ_o = reference (mean) density of the fluid before heating or cooling

Then $\rho = \rho_o\left(1 - \alpha\left(T - T_o\right)\right)$

$\Delta\rho = -\ \alpha\rho_o \Delta T$

Buoyant force per unit volume = $g\alpha\rho_o \Delta T$

After analyzing its units, we get:

Buoyant force per unit volume = $\frac{(mass)}{(length)^2 \times (time)^2}$

Now for viscous forces per unit volume:

$\left[\frac{\eta}{d}\right] = viscous\ stress$

$\eta\ = \frac{F}{A}$

After analyzing the units, we get:

$\eta\ = \frac{kg}{m \cdot s}$

$\left[\frac{\eta}{d}\right] = \frac{kg}{m^2 \cdot s} = \frac{mass}{(length)^2\ (time)}$

And

$\tau_m = \frac{n/d}{buoyant\ force}$

For convection to occur (see section 1.4.1):

$t_m < t_{th}$

$const > \frac{\tau_m}{\tau_{th}}$

$\frac{d^2}{D_t} \times \frac{g\alpha d\Delta T}{\eta} > const$

Ra = $\frac{g\alpha d^3 \Delta T}{\eta D_t} > const$

For a stable system, the proven value of this constant is 1708.

1.3.2 Grashof Number

The Grashof number represents the balance between buoyancy and viscous forces in a fluid layer. In the context of Rayleigh-Bénard convection, it helps determine whether the heated fluid will remain stable or begin to convect. A high Grashof number indicates dominant buoyancy forces, making convection likely if thermal diffusivity is also low. If the Grashof number's magnitude is low, then it indicates that the frictional or damping forces will prevail and the fluid will stay at rest.

$$Gr = \frac{g\beta d^3 \Delta T}{v^2}$$

β: thermal expansion coefficient
d: characteristic length
v: kinematic viscosity
g: gravitational acceleration
ΔT: temperature difference

1.3.3 Prandtl Number

The Prandtl number defines the relative rates of momentum and heat diffusion in a fluid. In our silicone oil-based system, a high Prandtl number implies that momentum spreads more quickly than heat, leading to slowly evolving and well-defined convection patterns during Rayleigh-Bénard convection. But if there's a low Prandtl number, that indicates that heat is diffusing faster than the momentum or the motion of the fluid, leading to no formation of convection cells.

$$Pr = \frac{C_p \mu}{k}$$

C_p: Specific heat capacity
μ: dynamic viscosity
k: thermal conductivity

1.4 Navier-Stokes Equation

<u>BACKGROUND</u>

What is a fluid?

Out of the three states of matter (solid, liquid, and gas), gases and liquids are classified as fluids.

Behavior of Fluids:

Under certain conditions, liquids and gases behave the same and thus, can be interchanged. These conditions are as follows:

- Negligible Compressibility Effects → very little expansion or contraction upon application of pressure due to which density can be applied *constant.*

- No Free Surface → fluid surface is not exposed to air (eg. Water surface exposed to air).

Under these, we can treat liquids and gases the same.

EXAMPLES:

1. A human-powered submarine was tested in a wind turbine.
2. Jet engine exhaust tested in a water tunnel.

EXCEPTIONS:

- Liquids with free surface effects → Waves generated by a boat on a lake surface.
- Gases with high-speed flow → Rockets (significant compressibility effects).

Application Of Normal Stress On A Fluid At Rest.

STRESS:

"Stress is defined as force acting per unit area on a surface."

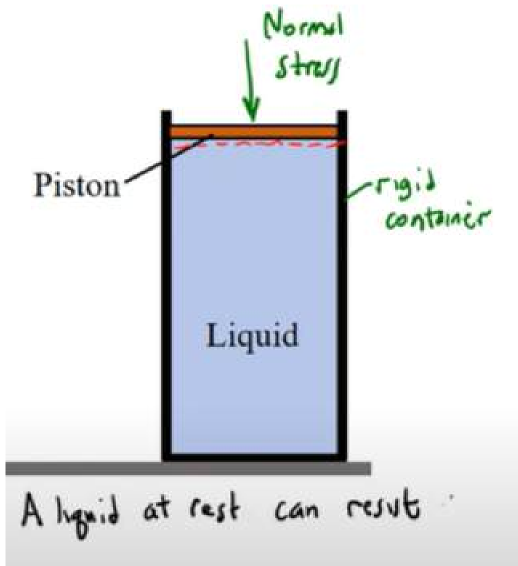
Formula:

S = F/A

Normal Stress:

A normal stress is a stress that is applied perpendicular to the surface of the fluid/object.

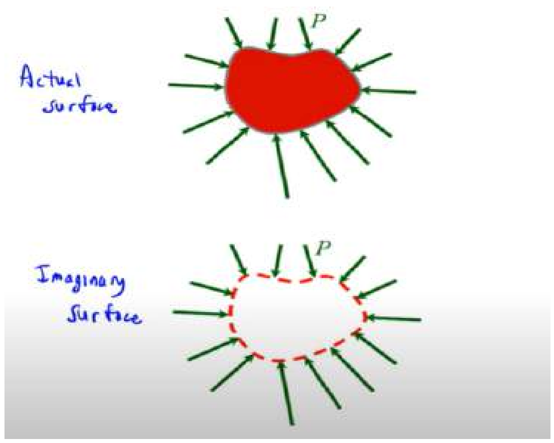
Consider a liquid in a rigid container with normal stress applied to it as shown.



The liquid at rest compresses a little on the application of normal stress but resists it. Liquids are approximately incompressible.

NOTE:

Pressure is an example of normal stress. In the case of fluid at rest, pressure is the only normal stress acting on it. Pressure always acts inward and is normal to the surface

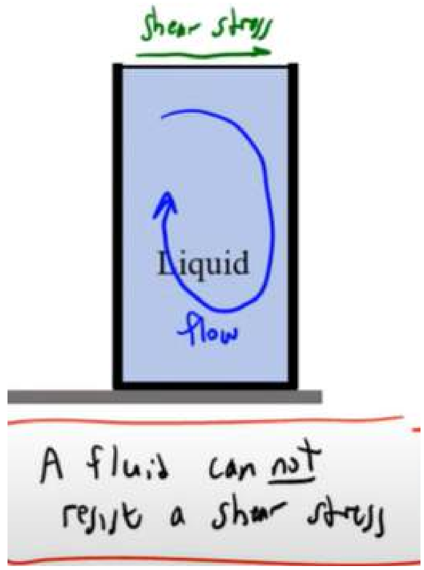


Shear Stress Applied TO A Fluid At Rest.

Shear Stress:

"Shear stress is a stress applied tangentially to the surface."

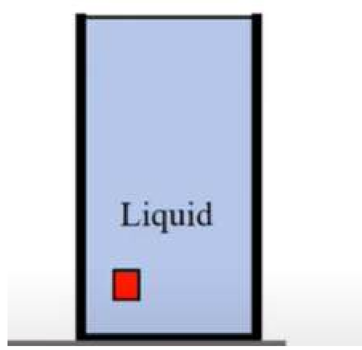
On applying sheer stress to a fluid, the container it is enclosed in doesn't deform. The fluid moves continuously in circular patterns. This means that a sheer stress cannot be resisted by a fluid at rest. It deforms continuously.



Thus, we can define fluid as:

"A substance that deforms continuously under shear stress."

Consider the smallest element of a fluid as shown in red.



- If the element is at rest, we can conclude that there is no sheer stress acting on it, as liquids can't resist sheer stress.
- But the liquid can resist normal stress. Therefore, we can say that normal stress is acting.
- The weight of the fluid element will also act.

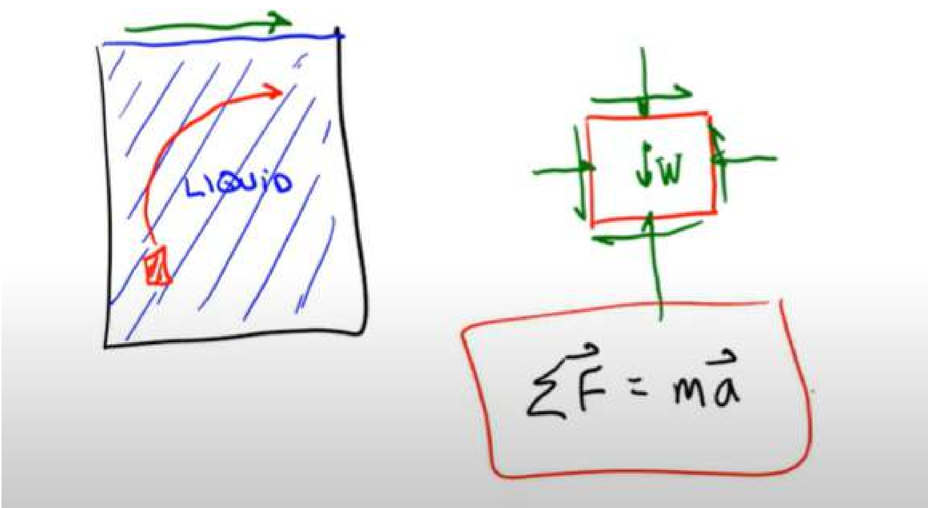
We know fluid is at rest, so, net force acting should be zero.

$$\Sigma F = 0$$

Liquid with applied shear stress.

On applying shear stress, the fluid particles move. This means that they possess some acceleration. So, our equation becomes

$$\Sigma F = ma$$



FORCES ACTING ON A FLUID IN MOTION

When in motion, the following forces act on a fluid.

Fg ⟶ due to the fluid's weight.

Fp ⟶ due to pressure gradient.

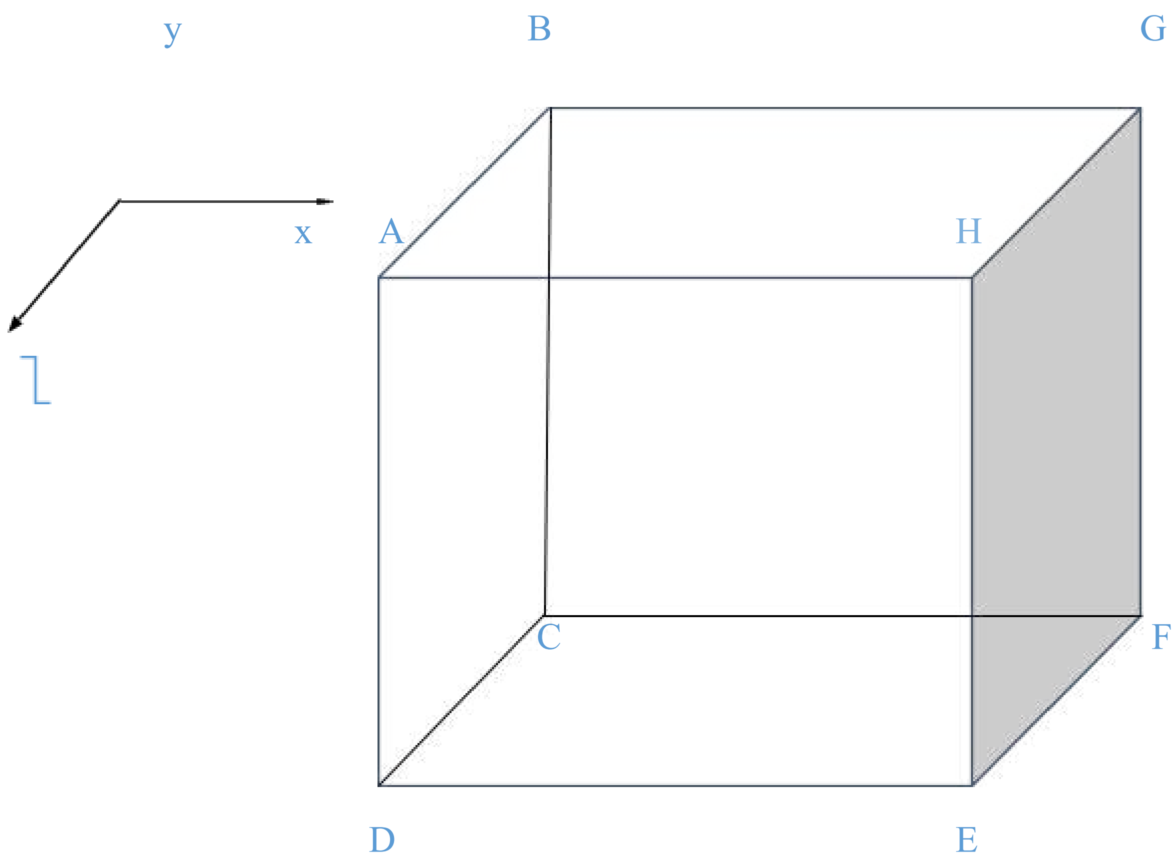
Fv ⟶ due to the viscosity of the fluid.

Ft ⟶ due to turbulence.

Fs ⟶ due to the cohesive property of fluid.

Fc ⟶ compressible force due to the elastic property of fluid (ability to change its volume under pressure).

NEWTON'S SECOND LAW



The figure above is a small mass of a fluid element.

According to the second law, if we consider a force F_x acting on a fluid in the x-direction, it will produce an acceleration in the x-direction (a_x).

As ΣF_x is net fore, we will add all the forces in F_x so,

$$\mathbf{\Sigma F_x = M a_x}$$

$$\mathbf{M a_x = (F g_x) + (F p_x) + (F v_x) + (F t_x) + (F c_x) + (F s_x)}$$

Similarly, we can get equations for the y and z directions.

NEGLIGIBLE FORCES:

1. Forces due to surface tension (cohesive forces) are not significant.
2. Forces due to compressibility are also not significant.

Thus, we are left with

$$Ma_x = (Ft + Fv + Fg + Fp)_x$$

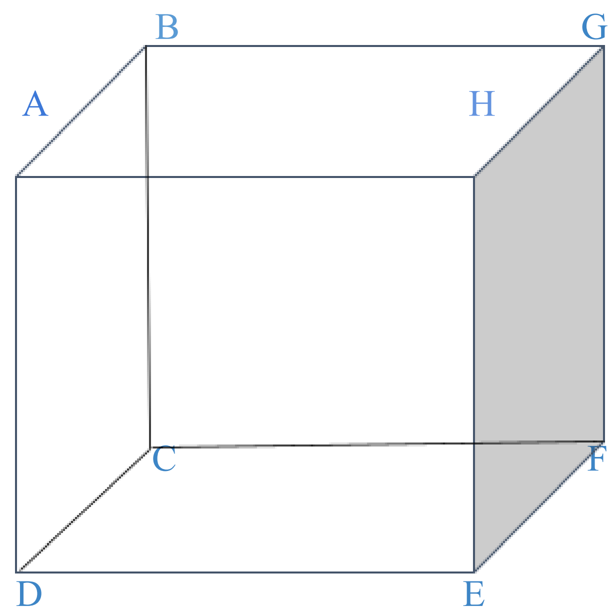
This is the Reynolds equation of motion used in the analysis of turbulent flow.

If the flow becomes laminar, there is no turbulence. Thus, neglecting turbulence, we get the following equation.

$$(Fv + Fg + Fp)_x = Ma_x$$

This is called the <u>NAVIER-STOKES EQUATION.</u>

DERIVATION OF THE NAVIER-STOKES EQUATION



Let $d_x d_y d_z$ be the element length.

Let ABCD be the x-side

Let EFGH be the x-side

Let ABGH be the y-side

Let CDEF be the y-side

Let BCFG be the z-side

Let ABEH be the z-side

Considering pressure

As we know that pressure **(p)** acts normally.

$p = \frac{F}{A}$

$F = p \times A$

So, the pressure force on face ABCD = $p \times d_y d_z$

Pressure force on face EFGH = $\left(p + \frac{\partial P}{\partial x} \cdot d_x\right) \times d_y d_z$

Where $\frac{\partial P}{\partial x}$ is the change in pressure as we travel across the fluid, and multiplying it by d_x gives us the change in pressure as we travel distance d_x.

Now, considering the net force of P

On ABCD = $p \times d_y d_z$

On EFGH = $\left(p + \frac{\partial P}{\partial x} \times d_x\right) d_y d_z$

As pressure acts normal to the surface, both of these vectors act in opposite directions. So, we consider one as a positive direction & the other as negative (opposite direction vectors).

$$\Sigma F_p = p \cdot d_y d_z - \left(p + \frac{\partial P}{\partial x} \times d_x\right) d_y d_z$$

$$= -\frac{\partial P}{\partial x} \times d_y d_z d_x \rightarrow 1$$

Gravity force due to the mass of the fluid (in x direction):

$$\rho = \frac{m}{v}$$

$$m = \rho \times v$$

To find the force due to mass, we need to find mass.

We use the formula of density fluids, because in fluids, mass is usually dealt with per unit volume, which is:

$$\frac{m}{v} = \rho$$

So we get mass by $\rho \times$ v

In the fluid particle drawn,

$$m_x = \rho \times \left(d_y \times d_z \times d_x\right) \rightarrow 2$$

Multiplying all lengths will give us volume

As F =mg,

$$F = g \times \rho \times \left(d_y \times d_z \times d_x\right)$$

Now, consider the sheer force that is generated due to viscosity. Let this force be τ_x (in x direction).

AS PER THE SECOND LAW

As per Newton's 2nd Law of motion, all forces acting on the fluid.

$$x\left(F_g + F_p + F_v\right) = max$$

Now plugging in the forces,

$$g\left(\rho d_x d_y d_z\right) + \left(-\frac{\partial P}{\partial x} \times d_x d_y d_z\right) - \tau_x = \rho d_x d_y d_z \times \frac{d_u}{d_t} \rightarrow 3$$

Shear Stress Due To Viscosity

Shear stress **(** τ_x **)** due to viscosity is ***proportional to the rate of change of velocity in the direction normal to the surface***. This is not an external force applied from the outside. Instead, it is an internal shear stress that arises due to the fluid's behaviour.

How Is This Shear Stress Generated?

This internal shear stress develops in response to a ***velocity gradient*** within the fluid. Unlike applied external shear stresses that cause a fluid to flow (as mentioned above), this shear stress is a result of motion, not a cause.

In fluid flow scenarios such as pipe flow or convection-driven flow like Rayleigh-Benard convection, a velocity gradient is present. This means that different layers of fluid are moving at different speeds.

- In pipe flow, the fluid moves slowest near the boundaries (due to the no-slip condition and adhesion with the pipe wall) and fastest near the centre, creating a velocity gradient throughout the fluid layers.
- In Rayleigh-Bénard convection, the fluid motion is circular, so the velocity gradient is more complex and multidirectional.

Role Of Viscosity

Viscosity is a fluid's internal resistance to motion. As a velocity gradient is generated, viscosity acts to smooth out this difference by:

- Slowing down the fast-moving layers.
- Speeding up the slow-moving layers.

This action creates an internal shear stress that tries to resist the motion and reduce the gradient.

In The Context Of Rayleigh-Benard Convection

In Rayleigh-Benard convection:

- Before the Rayleigh number exceeds 1708, no convection occurs; only conduction occurs where the fluid remains static and heat is diffused.

- Once the Rayleigh number > 1708, convection begins. Fluid starts moving in organized patterns, and velocity gradients appear.
- These gradients cause the fluid to develop internal shear stresses due to viscosity.

Thus, the Navier-Stokes equation becomes relevant only once convection begins, because only then does the fluid start flowing, and velocity gradients are created.

We take τ_x negative because it acts in the opposite direction to the motion in the d_x direction, as it is due to viscosity. (Viscosity resists motion).

By definition, shear stress = $\mu \times \frac{\partial u}{\partial n}$

Where $\frac{d_x}{d_n}$ is the rate of change of velocity normal to the direction.

Let us consider 2 faces, ABCD and EFGH

As shear stress = Resistance force due to viscosity/ area

Resistance force = shear force × area

Shear stress on face ABCD = $\mu \times \frac{\partial u}{\partial x} \times (d_y d_z) \rightarrow 4$

Shear stress on face EFGH = $\mu \times (d_y d_z) \times \frac{\partial(u + \frac{\partial u}{\partial x} \times d_x)}{\partial x}$

Where $\frac{\partial u}{\partial x} \times d_x$ is the change in velocity as the fluid moves through a distance d_x

$$= \mu \times (d_y d_z) \times \left[\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2} \times d_x\right] \rightarrow 5$$

Net shear force from equations 4 and 5:

$$\tau_x = \mu \times \frac{\vartheta u}{\vartheta x} \times (d_y d_z) - \left[\mu \times (d_y d_z) \times \left[\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2} \times d_x\right]\right]$$

$$\tau_x = -\mu \cdot \frac{\partial^2 u}{\partial x^2} \cdot d_y d_z d_x \rightarrow 6$$

Similarly, for face BCFG and ADEG (z-direction):

$$\tau_x = -\ \mu \cdot \frac{\partial^2 u}{\partial z^2} \cdot d_y d_z d_x \rightarrow 7$$

And for face ABGH and CDEF (y-direction)

$$\tau_x = -\ \mu \cdot \frac{\partial^2 u}{\partial y^2} \cdot d_y d_z d_x \rightarrow 8$$

Total shear force on all six faces of the element in the x-direction (equations 6, 7, 8):

Taking $\left(-\ \mu \cdot d_y d_z d_x\right)$ **common**

$$\tau_x = \left(-\ \mu \cdot d_y d_z d_x\right)\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] \rightarrow 9$$

Putting equation 9 in equation 3:

$$g\left(\rho d_x d_y d_z\right) + \left(-\frac{\partial P}{\partial x} \times d_x d_y d_z\right) - \left(-\ \mu \cdot d_y d_z d_x\right)\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] = \rho d_x d_y d_z \cdot \frac{d_u}{d_t}$$

$$g\left(\rho d_x d_y d_z\right) - \left(\frac{\partial P}{\partial x} \times d_x d_y d_z\right) + \left(\mu \cdot d_y d_z d_x\right)\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] = \rho d_x d_y d_z \cdot \frac{d_u}{d_t}$$

Dividing both sides by $d_x d_y d_z$

$$\rho g_x - \frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \rho a_x$$

$$\rho g_x - \frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)$$

This equation represents the x-component of the Navier-Stokes equation. Similarly, we could derive the y and z components in the same manner as well:

$$\rho g_y - \frac{\partial P}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) = \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right)$$

$$\rho g_z - \frac{\partial P}{zx} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) = \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right)$$

1.5 Convection cells

The fluid does not behave randomly during the experiment; rather, it organizes into unique patterns called convection cells. The shape and structure of convection cells varies with the geometry of the container and the temperature gradient across the container. Convection cells are the circular patterns of moving fluid that result from a temperature difference. In convection currents, the warm, less dense fluid rises and the cool, dense fluid sinks, causing circulation and the resulting circular motion. We can see convection cells in a wide range of systems, from the atmosphere and mantle of the Earth to more practical examples, such as boiling water. The convection cell motion can be observed in forms such as rolls and hexagons, depending on the fluid properties and heating conditions.

1.5.1 Convection Rolls

Convection rolls are structured circulation patterns created when convection starts in a fluid that is heated from below and cooled from above. Within a convection roll, warm fluid rises in the center and spreads out horizontally at the top. This warm fluid then cools and sinks at the edges of the container, creating cylindrical and continuous patterns of circulation. Thus, the convection rolls can optimally transport heat through the fluid with consistent structure and stability. The ideal container geometry for reproducible convection rolls is a shallow, wide, rectangular container. The large aspect ratio and linear boundaries enhance the likelihood of shade convection rolls forming that are consistently parallel and uniform in behavior.

1.5.2 Convection Hexagons

When convection starts in a fluid heated from below and cooled from above, and in a region close to the onset of instability, convection hexagons can form as cellular flow patterns. In this flow pattern, the warm fluid rises at the center of each hexagonal cell, leaves the cell at the top, spreads out, cools, and sinks at the edges. Hexagons are observed when the heating is strong enough to destabilize the fluid, but the flow is not yet turbulent. Hexagonal convection displays well in circular or square containers

1.5.3 Turbulent Flow

Any disordered and complex form of motion observed in Rayleigh-Bénard Convection is known as turbulent flow. It usually occurs when the temperature gradient between the bottom and top surfaces exceeds a certain value. This provides the experimental system with more thermal energy than required to form simple and regular convection cells such as rolls and hexagons (Check 1.5.1 and 1.5.2). These patterns are replaced gradually by the highly irregular and constantly changing motion of the fluid.

Furthermore, the turbulent flow is directed by certain factors, with the Rayleigh number being the most crucial one. When the Rayleigh number becomes extremely large in magnitude, typically above 10^6 or 10^7, depending on the fluid properties and the system geometry, the smooth and simple motion of the fluid transforms into a turbulent flow.

In this state, the fluid motion becomes unpredictable, fast, and unsteady. The organized structures no longer persist over time. Instead, the fluid moves in many directions at once, often changing speed and direction rapidly. Unlike the earlier convection cells, which were stable and regularly repeated across the fluid surface, turbulent flow has no clear symmetry or repetition.

Once this state is achieved, the motion of the fluid transitions into an unpredictable, fast, and unsteady flow. The organized structures disappear, and instead, the fluid starts moving in many directions at once with a constantly changing speed. Unlike the convection cells, which were stable and repeated across the surface of the fluid, this flow has no clear structure, symmetry or repetition.

This transition to turbulence is also influenced by:

- A large temperature gradient between the bottom and top surfaces
- A greater depth of the fluid layer
- A low viscosity of the fluid, which allows easier movement

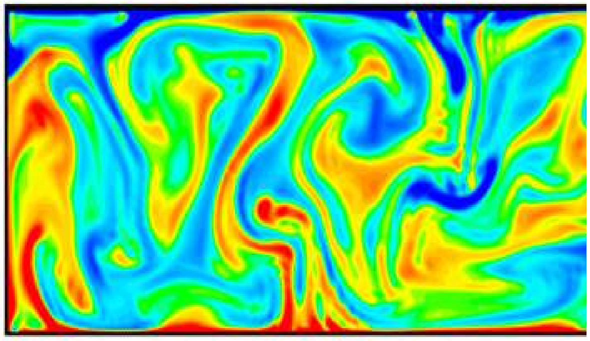


Figure 1.2*: Visual representation of turbulent flow in a Rayleigh–Bénard convection system. The image illustrates the chaotic, disordered motion of fluid occurring at high Rayleigh numbers, where structured convection cells break down into irregular, unpredictable patterns*

Chapter 2: Experimental Setup

2.1 Fluid

Rayleigh-Bénard Convection is greatly affected by the choice of fluid used. Fluid properties such as thermal expansion, thermal conductivity, and viscosity play a key role in convection. Low viscosity, high thermal expansion, and thermal stability tend to produce more stable and clearer convection cells. Three common options that we used were water, cooking oil, and silicone oil. Each has its unique physical properties, which influence the Rayleigh number, the key criterion for the initiation of convection, as well as practical considerations like clarity and stability under heat.

2.1.1 Silicon Oil

Silicone oil is one of the most effective substances for Rayleigh–Bénard convection. Silicone oil has a wide range of viscosities (viscosity can be chosen based on what is needed for an experiment), low thermal conductivity, and very low volatility (thermally stable). Its thermal stability makes it ideal for creating clear convection cells that last long enough to observe. Silicone oil is transparent, contributing to better observation of cell formation, and it supports a much higher

Rayleigh number, so it is easier to induce convection with smaller (but still significant) temperature differences.
In the table below, there are values for quantities of silicon oil.

Property	Symbol	Typical range	Unit
Thermal expansion coefficient	β	(1.0 - 1.2) * 10^-3	1/K
Kinematic viscosity	ν	(1*10^-6) - (1*10^-4)	m^2/s
Dynamic viscosity	μ	(1*10^-3) - (1*10^-1)	Pa.s
Thermal conductivity	k	0.13 - 0.16	W/m.K
Thermal diffusivity	α	(5*10^-8) - (1.5*10^-7)	m^2/s
Grashof number	Gr	10^3 - 10^7 (depends on ΔT and depth)	Dimensionless
Prandtl number	Pr	50 - 10,000 (varies with viscosity)	Dimensionless
Rayleigh number	Ra	10^5 - 10^9 (must exceed 1708 for onset)	Dimensionless

Table 2.1: *Typical thermal and physical properties of the fluid used in Rayleigh–Bénard convection experiments, including viscosity, thermal expansion, conductivity, and key dimensionless numbers.*

2.1.2 Cooking Oil

Cooking oil possesses moderate viscosity and low thermal conductivity, and is suitable for creating convection cells, but has impurities, is not transparent (may affect observations), can break down or smoke when heated (especially over longer periods), and is therefore not always reliable for good reproducible measurements. In the table below, there are values for quantities of cooking oil.

Property	Symbol	Typical range	Unit
Thermal expansion coefficient	β	(6.5 - 9.0) * 10^-4	1/K
Kinematic viscosity	ν	(3.0 - 6.0) * 10^-5	m ^2/s
Dynamic viscosity	μ	(0.03 - 0.07)	Pa.s
Thermal conductivity		0.15 - 0.20	W/m.K

Property	Symbol	Typical range	Unit
Thermal diffusivity	α	(6.0 - 9.0) *10^-8	m^2/s
Grashof number	Gr	10^3 - 10^7 (depends on ΔT and depth)	Dimensionless
Prandtl number	Pr=ν/α	400 - 1000 +(very high)	Dimensionless
Rayleigh number	Ra=Gr×Pr	10^6 - 10^9 (must exceed 1708 for onset)	Dimensionless

Typical thermophysical characteristics of cooking oil used in convection experiments, including moderate viscosity, limited transparency, and dimensionless numbers influencing flow behavior.

2.1.3 Water

Water has many advantages due to its wide availability and clarity. Water is a substance with relatively high thermal conductivity and low viscosity, but it can only develop convection cells up to a point. Because of the relatively high conductivity and low viscosity, heat can diffuse too quickly, and thus disallow a more distinct cell formation. Additionally, water readily evaporates, develops vapor phases, and can boil at smaller temperature differences, which can make it more troublesome to maintain a consistent surface temperature, and thus affect accurate measurement and observation of convection cells.

In the table below, there are values for quantities of water.

Property	**Symbol**	**Typical Range**	**Unit**
Thermal expansion coefficient	β	$(2.0 – 3.0) \times 10^{-4}$	1/K
Kinematic viscosity	ν	$(0.89 – 1.0) \times 10^{-6}$	m^2/s
Dynamic viscosity	μ	$(0.89 – 1.0) \times 10^{-3}$	Pa.s
Thermal conductivity	k	0.58 – 0.62	W/m.K
Thermal diffusivity	α	$(1.4 – 1.5) \times 10^{-7}$	m^2/s
Grashof number	Gr	$10^3 – 10^6$ *(depends on ΔT & depth)*	Dimensionless
Prandtl number	Pr=ν/α	6 – 8	Dimensionless
Rayleigh number	Ra=Gr×Pr	$10^4 – 10^7$ *(must exceed 1708)*	Dimensionless

Table 2.3: *Key thermal and physical parameters of water in Rayleigh–Bénard convection, emphasizing high conductivity, low viscosity, and challenges in maintaining distinct convection patterns.*

2.2 Mica Powder

Mica powder is a fine, shiny powder made of mica, a natural mineral capable of splitting into thin, mirrored layers. Mica is chemically stable, heat resistant, and lightweight, so it is often used in a wide range of industry and scientific applications. Mica powder is composed of tiny but flat particles that can reflect light, providing shiny or pearlescent qualities. Mica powder also does not dissolve in water and does not dissolve in oils, and will stay suspended in a water solution for long periods, which makes it easier to visualize fluid motion in experiments.

In Rayleigh–Bénard convection experiments, mica powder can help in visually seeing the flow of convection cells. When heating a fluid from below and cooling a fluid from the top, mica powder can follow the movements of the fluid. The mica will follow the path of the warm rising fluid and the cool sinking fluid as it moves vertically throughout a convection roll or hexagonal cell. The movement will visually represent the convection roll or hexagonal cells of the fluid in motion. The reflective quality of the mica allows and provides visibility of these flow structures if lit properly, so an observer can view convection behavior in real time without altering the fluid motion dynamics.

2.3 Variation of Aspect Ratio

In our experiment, we varied the aspect ratio by changing the container diameter while keeping the fluid volume relatively constant. This allowed us to observe how different aspect ratios influence the number, size, and structure of the convection cells. The table below summarizes the range of aspect ratios we tested and the corresponding convection patterns that were observed.

Serial No.	**Container Type**	**Container Thickness (mm)** $\pm\ 0.01mm$	**Fluid Volume (ml)** $\pm\ 0.01mm$	**Fluid Depth (mm)** $\pm\ 0.01mm$	**Aspect Ratio** $\pm\ 0.01mm$
1	Small Beaker (100 ml)	4.26	50	25.35	1.81

2	Big Beaker (250 ml)	3.62	50	14.14	4.59
3	Petri Dish (Small)	1.95	50	8.95	9.46
4	Petri Dish	5.10	50	7.21	12.77

*Images of Beakers

A ***B*** ***C*** ***D***

Figure 2.1: *These figures display the actual containers 100 mL beaker, a 250 mL beaker, and Petri dishes, which were used to vary the aspect ratio in Rayleigh-Bénard convection trials.*

2.4 Heat Source and Temperature Measurement

Thermal heat is a basic requirement for Rayleigh benard convection so we need a heat source which can be set to a constant temperature and for its effective measurement we need a thermometer which is precise and accurate. So, we used the following heat source and thermometer:

2.4.1 Hot Plate

A hot plate is a flat heated surface used to heat substances like liquids, solids or containers, often used in labs, kitchens, and experiments. A lab hot plate is used to heat beakers, flasks or samples. It controls a maintained temperature. It often comes with a magnetic stirrer to stir while heating.

A hot plate works based on the principle of heating through conduction. When the sample is placed on a hot plate, the heat is transferred from the plate to the sample through direct contact. Hot plates are typically operated using a variable temperature control, which allows the user to set and adjust the required temperature of the sample. The temperature control is connected to a heating element that is embedded in the plate. So, when the user sets the desired temperature, the heating element is activated, and the plate begins to heat up. The sample is then placed on the hot plate, and the heat is transferred from the plate to the sample through conduction.
In this experiment, we used an electric hot plate, which is a hot plate powered by electricity.

2.4.2 Infrared Thermometer

During the experiment, it is necessary to constantly measure the temperature. An infrared thermometer measures temperature by detecting the infrared radiation emitted by an object. This type of radiation, part of the electromagnetic spectrum, is collected through a lens and directed onto a detector, usually a thermopile. The thermopile transforms the radiation into an electrical signal, which is then processed and converted into a temperature reading displayed on the device. Infrared thermometers are ideal for Rayleigh–Bénard convection because they are non-invasive, fast, and accurate for surface temperatures, which are the most critical measurements in driving and analyzing convection patterns.

2.5 Experimental Setup

To observe Rayleigh-Bénard convection and vary different parameters, we used the following apparatus.

- Tripod
- Camera

Chapter 3: Observations and Experiment

3.1 Silicone oil

The Petri dish used in the first experiment had an aspect ratio of approximately 12.77. The initial temperature of the fluid was 28°C, while the hot plate was maintained at 55°C. The fluid remained motionless, gradually heating from the bottom. Convection began once the fluid reached a temperature of 55°C. At this point, the warmer, less dense fluid at the bottom rose, while the cooler, denser fluid sank to occupy the space left behind. This characteristic circulation pattern is observed in all experiments involving convection. As the temperature continued to rise, coherent flow structures became visible. These structures were made observable due to the presence of mica powder in the fluid. The convection cells that formed were organized and well-defined.

Aspect Ratio: 12.77

Initial Temp of Silicone Oil: 28°C
Initial Temp of Hot Plate: 55°C
Convection starts: 55°C

During the second experiment, we gently heated silicone oil mixed with mica powder on a hot plate. The container used was a 100 ml beaker with an aspect ratio of approximately 1.81. Initially, the oil temperature was at 29°C, and the hot plate was at 55°C. As the temperature of the oil increased, it stayed calm until around 54°C, when we began to see the first signs of convection, subtle movements in the fluid. A few degrees later, at about 57°C, distinct convection cells started to appear, forming visible, organized patterns across the surface. As heating continued, these patterns gradually lost their shape. By the time the temperature reached 82°C, the cells had become distorted and irregular.

Aspect Ratio: 1.81

Initial Temp of Silicone Oil: 29°C
Initial Temp of Hot Plate: 55°C

Convection starts: 54°C
Cells start forming: 57°C
Deformed shape: 82°C

In the third trial, we used a larger beaker of 250 ml with an aspect ratio of approximately 4.59 but kept the same silicone oil and mica powder mixture. The oil started at 29°C, while the hot plate was once again set to 55°C. At first, the oil appeared motionless, just like before. However, this time, convection began much earlier (around 42.5°C) as gentle movements started to appear in the fluid. It wasn’t until the temperature reached 60°C that well-defined convection cells became visible, forming organized patterns across the surface. This setup showed how the shape and size of the container can affect when and how convection begins, even when the materials and heating conditions are the same.

Aspect Ratio: 4.59

Initial Temp of Silicone Oil: 29°C
Initial Temp of Hot Plate: 55°C

Convection starts: 42.5°C
Cells start forming: 60°C

In the final setup, we used a petri dish with a smaller diameter and an aspect ratio of approximately 9.46 with the same mixture of silicone oil and mica powder. The oil started at 28°C, while the hot plate remained at 55°C. At first, the fluid stayed completely still. As it warmed up, convection began at around 45°C, marked by the slow rise of warmer fluid and the sinking of cooler fluid. A few minutes later, when the temperature reached 55°C, convection cells started to take shape, forming neat, visible patterns on the surface. This experiment highlighted how a wider and shallower container influences the timing and appearance of convection, even though all other conditions stayed the same.

Aspect Ratio: 9.46

Initial Temp of Silicone Oil: 28°C
Initial Temp of Hot Plate: 55°C

Convection starts: 45°C
Cells start forming: 55°C

3.2 Cooking Oil

The Petri dish used for the cooking oil experiment had a diameter of 92.04 mm and an aspect ratio of about 12.7. The vegetable oil started at 28°C, while the hot plate was maintained at 61.9°C. Initially, the fluid remained still, gradually heating from the bottom. Convection was initiated once the temperature reached 31°C. The warmer, less dense fluid at the bottom rose, while the cooler, denser fluid sank to take its place, thereby establishing convective circulation. The mica powder used in the experiment enabled visualization of the flow structures. At first, the convection cells appeared disordered with poorly defined boundaries, most likely due to the high aspect ratio and the physical properties of the oil. After reducing the aspect ratio, the cells began to define themselves more clearly and became increasingly organized. It was also observed that the formation of convection cells began at approximately 33°C. This indicates that container geometry is a key factor in controlling the stability and visibility of convection.

3.3 Water

The Petri dish used during the water experiment has a diameter of 92.04 mm and an aspect ratio of approximately 12.7. Initially, the water was at a temperature of 26°C while the hot plate was set to 31°C. The water began at rest and started to heat only from the bottom. Mica powder was involved in the fluid to allow the visibility of convection cells. However, water's high surface tension prevented the mica powder from spreading evenly across the surface, causing it to clump rather than float or dissolve. To reduce this surface tension, a small amount of soap was introduced. While this successfully lowered the surface tension, it caused the mica powder to sink to the bottom, due to its relatively high density and lack of solubility in water.

Theoretically, convection was taking place in the water but no convection cells were observed. Mica powder, which was effective in every other experiment either gathered from the bottom or the surface of the water. This is why no convection or cells were observed. In conclusion, the experiment indicated that an effective solvent was needed to observe convection cells.

Chapter 4: Results and Discussion

4.1 Final Observations

From these experiments, we made several important deductions about the factors that influence convection. One of the key insights was the role of the container's aspect ratio, which significantly affected when convection began and how the convection cells developed. Wider and shallower containers, which have higher aspect ratios, triggered convection at lower temperatures compared to taller containers. This behaviour is closely linked to the Rayleigh number, a dimensionless value that predicts the onset of convection. Since the Rayleigh number depends on factors such as temperature difference, depth, thermal expansion, and fluid viscosity, our observations showed that even with the same heating conditions and fluid type, variations in geometry can shift the system above or below the critical Rayleigh number(≈1708). Additionally, the high viscosity of silicone oil helped slow down the fluid motion, allowing us to clearly observe the gradual development and deformation of convection cells. These deductions are essential for understanding how geometry, temperature, and fluid properties work together to control convection behaviour

4.2 Summary

In this experiment, we successfully observed Rayleigh-Bénard convection, where a fluid layer heated from below and cooled from above developed visible convection patterns due to temperature-induced density differences. As the temperature gradient increased, regular convection cells (often hexagonal and roll-like) were formed, indicating the onset of buoyancy-driven flow.

The results confirm the theoretical prediction that when the Rayleigh number exceeds the critical value of 1708, convection begins. The emergence of organised patterns such as rolls or hexagonal cells highlights the role of buoyancy-driven instability in the fluid system.

This behaviour is important for understanding various natural and industrial processes, including convection mechanisms such as atmospheric circulation, ocean currents Overall, the experiment helped deepen the concept of thermal instability, fluid dynamics, and pattern formation in heated fluid layers. While offering practical insight into the onset and structure of thermal convection.

While the setup was effective, minor limitations such as uneven heating and boundary disturbances may have influenced pattern clarity. Future improvements could involve using transparent enclosures and high viscosity fluids for better visualization. Overall, the experiment was a valuable hands-on demonstration of buoyancy-driven flow, with direct relevance to both natural phenomenon and industrial applications.